Anuj Dev, Ben J. A. Kröse, Frans C. A. Groen

Novel Functions (SNN) Laboratory
Faculty of Mathematics and Computer Science
University Of Amsterdam
Kruislaan 403, NL-1098 SJ Amsterdam
Real World Computing Partnership
email: anuj@fwi.uva.nl

'90?

1. Type of Presentation

- The type of presentation: Poster presentation
- The number of booth units: Zero

2. Introduction

The estimation of image motion $\boldsymbol{v}(\boldsymbol{x}, t)$ from a sequence of images $I(\boldsymbol{x}, t)$ is a well addressed problem in computer vision. The assumption of conservation of image intensity results in the well known optic flow constraint [1]:

$$\nabla I(\boldsymbol{x}, t) \cdot \boldsymbol{v}(\boldsymbol{x}, t) + I_t(\boldsymbol{x}, t) = 0 \tag{1}$$

Since the optic flow constraint (1) gives only one equation to the image motion vector at every point, an additional equation is needed to estimate the two components of the image motion vector. The method reported in [3] uses a parametric model on the optic flow constraint (1) to produce the image motion from the first order space time image derivatives. The underlying assumption of the model is that the image motion is constant in a very small region $\mathcal{S}$. Every optic flow constraint in region $\mathcal{S}$ eliminates one free parameter of the image motion vector. Since the vector has 2 free parameters, theoretically 2 such constraints are enough if the spatial derivatives are non coplanar. However, since the image (time) derivatives may be noisy, a Least Mean Square approach is more robust to noise. This can formally be translated in to minimizing the functional $E_{\mathcal{S}}$ over the M image locations in $\mathcal{S}$:

$$E_{\mathcal{S}} = \sum_{\boldsymbol{x} \in \mathcal{S}} \left(\nabla I(\boldsymbol{x}) \cdot \boldsymbol{v}_{\mathcal{S}} + I_t(\boldsymbol{x})\right)^2. \tag{2}$$

Since this functional is linear in its parameter the image motion in this region $\boldsymbol{v}_{\mathcal{S}}$ follows from the normal equations:

$$\boldsymbol{v}_{\mathcal{S}} = \left(\boldsymbol{A}^T \boldsymbol{A}\right)^{-1} \boldsymbol{A}^T \boldsymbol{b} \tag{3}$$

where $\boldsymbol{A}$ is the $M \times 2$ array of spatial image derivatives and $\boldsymbol{b}$ is the $M \times 1$ vector of image time derivatives:

$$\boldsymbol{A} = \begin{pmatrix} I_x(\boldsymbol{x}_1) & \dots & I_x(\boldsymbol{x}_M) \\ I_y(\boldsymbol{x}_1) & \dots & I_y(\boldsymbol{x}_M) \end{pmatrix}^T$$

$$\boldsymbol{b} = -\left(I_t(\boldsymbol{x}_1) \dots I_t(\boldsymbol{x}_M) \right)^T$$

Although theoretically sound, the estimation of the image motion is known to be difficult due to:

1. *The derivative property*: the estimate of the image motion is noisy. The image motion is computed from the derivatives of the image luminance. Since the derivative operator amplifies noise, the noise in the luminance will be amplified. Related to this is:
2. *The aperture problem*: due to the limited observability of the image motion from the luminance, the image motion can only be estimated accurately in a small part of the image.
3. *The assumption of intensity conservation*: The optic flow constraint is only valid for Lambertian reflectors and smooth surfaces [2]. If there is specular reflection and or depth discontinuities in the viewed surface, the assumption of conservation of intensity is not valid.

In this paper we will detect the possible error sources as described above. If we are able to derive a measure of the error in the estimated image motion vector from the measurements alone, we can discard erroneous image motion vectors from estimated image motion. If these measures are reliable this is a trade-off: higher accuracy with low density versus lower accuracy and higher density. A reliable confidence measure will optimize this trade-off.

We address the problem of finding confidence measures by an error analysis of the image motion model (3) and show that this results in two measures that attach a relative confidence to the estimated image motion. The first is the sensitivity of the image motion model to noise in the measurements of the image derivatives. The second is computed from the residual error E_S of the model (2) and validates the optic flow constraint. We show how to combine these two measures into a estimate of the image motion variance hence into *one* absolute confidence measure.

3. Conclusion

We analyzed two confidence measures to select valid image motion. These are the sensitivity and the residual error of the image motion model. Experiments show that these are excellent criteria for selection of valid image motion. The experiments show a dramatic improvement, a $4 - 30$ times smaller mean square error (MSE) in the estimated image motion against 50% density. Furthermore experiments show that the confidence measure based on the residual error is much better at discarding erroneous image motion vectors. The confidence measure which combines the previously described measures decreases the MSE even further by a factor 3 and results in a total decrease of a 90 times smaller MSE at 50% density.

References

[1] Horn, B.K.P. and Schunck, B.G.: "Determining optical flow", Artificial Intelligence, 17, 1981.

[2] Verri, A. and Poggio, T.: "Motion Field and Optical Flow: Qualitative Properties", IEEE transactions on Pattern Analysis and Machine Intelligence, 11, 1989.

[3] Lucas, B. and Kanade, T.: "An iterative image restoration technique with an application to stereo vision", Proceedings of the DARPA Image Understanding workshop, 1981.

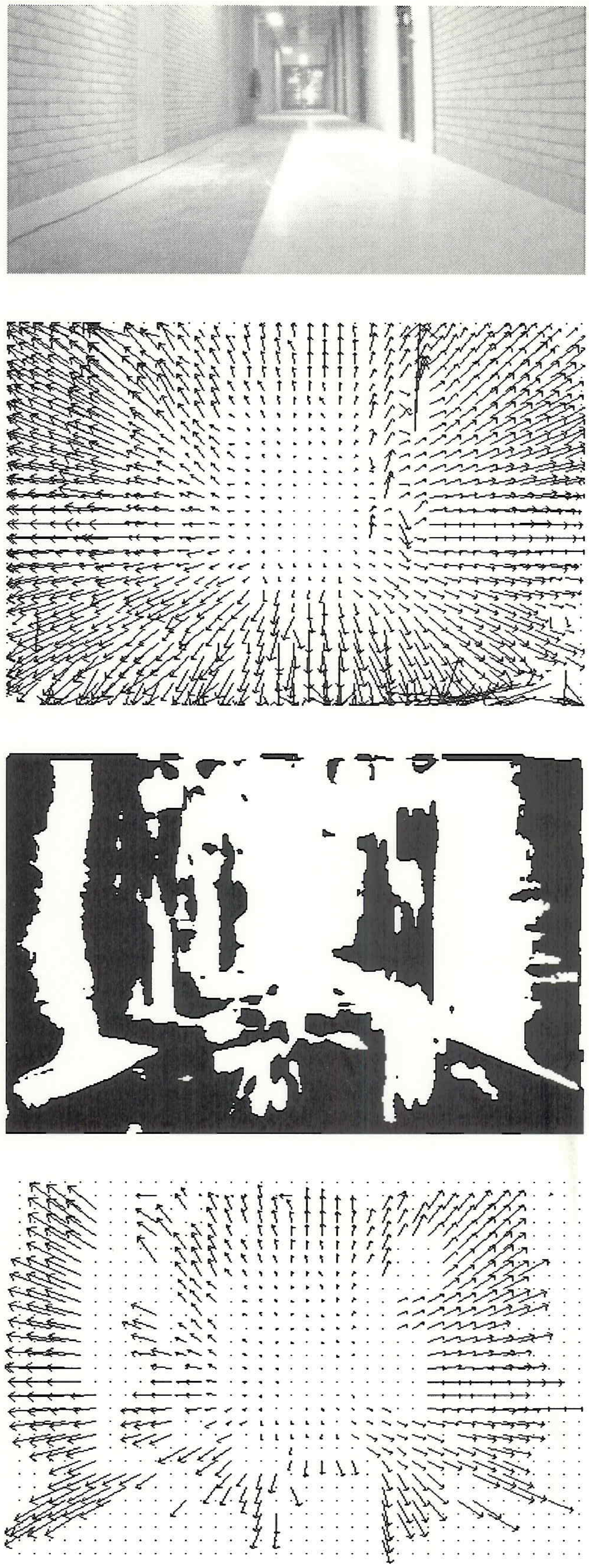

Figure 1: Top to bottom: A typical frame from the mobile robot platform. Estimated image motion. Selected parts (at a density of 50%) of the image (white) where the variance of the image motion is below a predefined threshold. Estimated image motion at selected locations.