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CAMAS

COMPUTER AIDED MIGRATION OF APPLICATIONS SYSTEM

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F2SAD - prediction capabilities

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Chapter 1
Introduction

This report describes an evaluation of the F2SAD tool with several well known basic algorithms. In this report we consider some sorting algorithms. With this report we respond to the specific request of the review commission to make a more detailed validition of the prediction capabilities of the UvA workbench tools.

The intention of this document is to provide confidence in the ability of the tools (that implement the models developed in SAD and PARASOL) to estimate the execution time of some well known algorithms. The algorithms described here have a performance behaviour that is common knowledge. Despite this fact, we are still able to come up with some points that are interesting and not plain textbook knowledge.

In this report, various figures depicting measured and predicted execution times of Fortran programs will pass. The main part of the annotated algorithms has been included in appendix A.

At the time of the CAMAS review 5 we will present additional results on numerical relaxation algorithms.

Chapter 2
Sorting algorithms

Sorting algorithms are symbolic algorithms. Rather than performing heavy computations they compare and manipulate (swap or reorder) data. Sorting algorithms are quite well understood in their complexity behaviour. Despite this fact, few textbooks do actually compare the execution time of the algorithms. Two algorithms can be of the same order of complexity and still differ in their performance because a different number of instructions is executed.

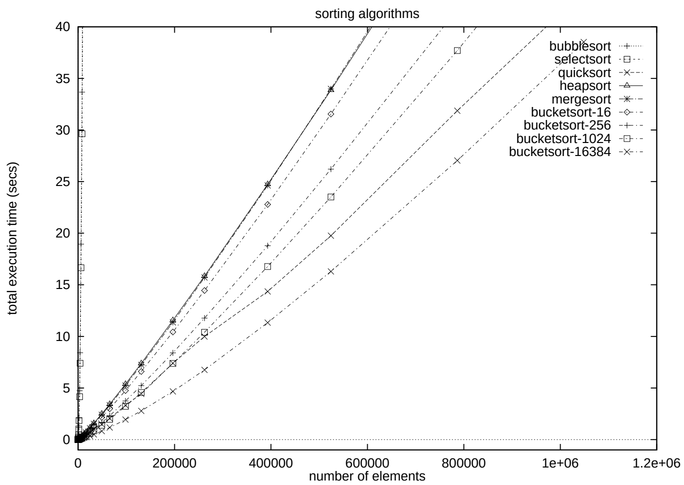

Figure 2.1: This figure shows pure times of the sorting algorithms. The algorithms are applied to an array of uniform random numbers. It has been included here only to give an indication of the real execution time, since it is clearly not very informative when comparing the algorithms. All other figures therefore include either a logarithmic vertical axis and/or the execution time divided by the number of elements. This last type of figure works well to view the scalability of an algorithm and the crossover points for selecting between algorithms, but the overhead introduced by any algorithm is rather obscured. The logarithmic vertical axis plots still show the overhead, but the crossover points and behaviour is less visible. When viewing the other figures one has to keep in mind this figure, which tells you that the execution time of the sorting algorithms of the same complexity are actually very similar.

If we classify the sorting algorithms by their average-case time complexity, we can distinguish three classes. The exponential order $O(n^2)$, the logarithmic based sorting algorithms $n\log(n)$ and the linear time sorting algorithms $O(n)$. The exponential order algorithms are sometimes still (unjustifiably) used if a programmer is lazy and works on small arrays or under the disguise of a parallel computer. Shell-sort is such a variant used on parallel computers because of its parallel nature.

The most common sorting algorithms are the $O(n\log n)$ order sorting algorithms like quick-, heap- and mergesort. Although all three have the same average-case order of

algorithm	*average*	*worst-case*	*implementation*
bubblesort	$O(n^2)$	$\Theta(n^2)$	array (list also possible)
selectsort	$O(n^2)$	$\Theta(n^2)$	array (list also possible)
quicksort	$O(n \log n)$	$\Theta(n^2)$	array
heapsort	$O(n \log n)$	$\Theta(n \log n)$	array
mergesort	$O(n \log n)$	$\Theta(n \log n)$	list
bucketsort	$O(n)$	$\Theta(n^2)$	list

Table 2.1: Sorting algorithms; their time complexity as average case time complexity $O(\ldots)$ and worst-case time complexity $\Theta(\ldots)$. The implementation can either be in linear array or a linked list form.

performance, they are inherently different. Quicksort has very bad worst-case performance, namely $\Theta(n^2)$. It is almost impossible to implement mergesort on arrays and the natural way to implement heapsort is on arrays. It is controversial whether quicksort is the fastest (on average), mergesort or heapsort. Comparing these algorithms is an interesting test case for our tool, F2SAD.

The theoretical complexity limit to sorting is $O(n \log n)$, but also in this case the practical knowledge of contraints to the input of the sorting algorithms can be exploited. To this albeit linear order sorting algorithms have been developed. These include radix, counting and bucketsort, in which the latter is sometimes used as a classifier or find-algorithm. These sortings algorithms do not base their algorithm on comparisons, like the sorting algorithms earlier mentioned, but rather on classification or decision trees.

The predicted times in these sections were produced using F2SAD which will now also incorporate the Parasol II tool in the same program. The tool produces a time-complexity formula in which the machine constants and the algorithmic parameters are still abstract. When not specified, we have used a Sun Classic LX model as test machine.

```
Memory            : 32 MB
Model             : SPARCstation LX
At frequency      : 50 MHz
CPU               : Texas Instruments TMS390S10 (MicroSparc)
Data cache        : 2 Kb blocksize=16 1-way associative
Intruction cache : 4 Kb blocksize=32 1-way associative
```

2.1 Randomly distributed input data

Figure 2.2 shows the measured execution timings of the exponential and $O(n \log n)$ performance as well as the predicted execution timing. The input to the sorting algorithms is uniformly randomly distributed. Also the parameters in the time complexity formula have been set in such a way that they reflect this condition.

The reason for taking random input data is obviously that sorting algorithms will than expose an almost average-case behaviour. Some algorithms have a worst-case behaviour which is of a different complexity order or, less dramatic, have different minor terms in the complexity formula.

How the parameters are set we will come to later, but first we have a look at the measured and estimated execution time and compare them. Figure 2.3 shows the expected and estimated execution time for the bucketsort algorithm, compared to a mergesort implementation, while figure 2.2 gives the same data for the other sorting algorithms.

Figure 2.4 shows the error of the predicted versus the expected value.

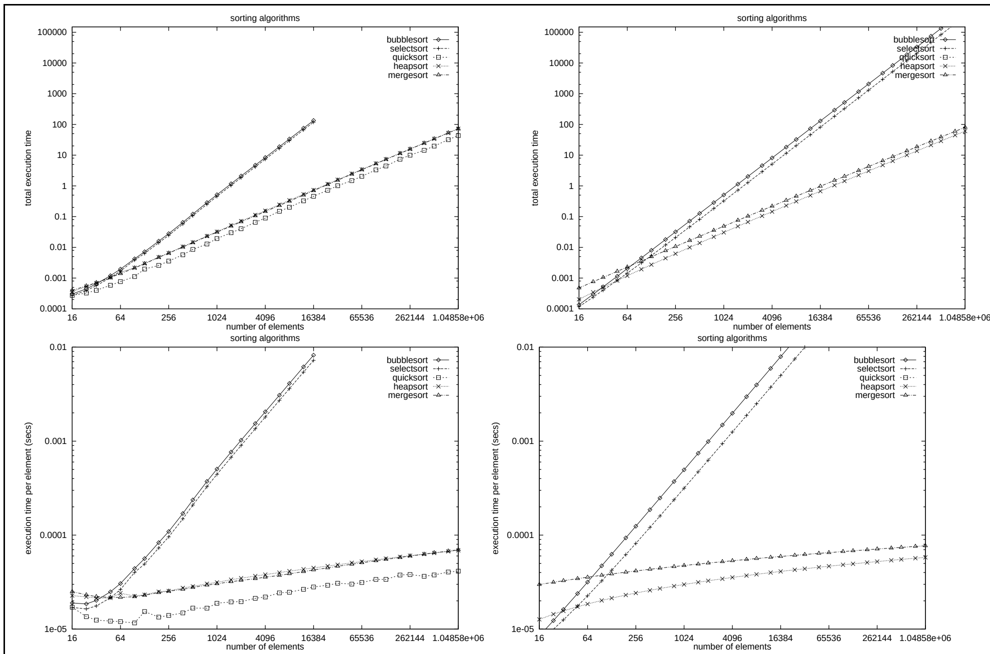

Figure 2.2: Measured and predicted execution times for the select-, bubble-, quick-, heap- and mergesort under the constraint that the input data to the sorting program is randomly distributed. The actual numbers are in given in table **??**.
The figures on the left show the measured execution time, on the right is the predicted execution time.

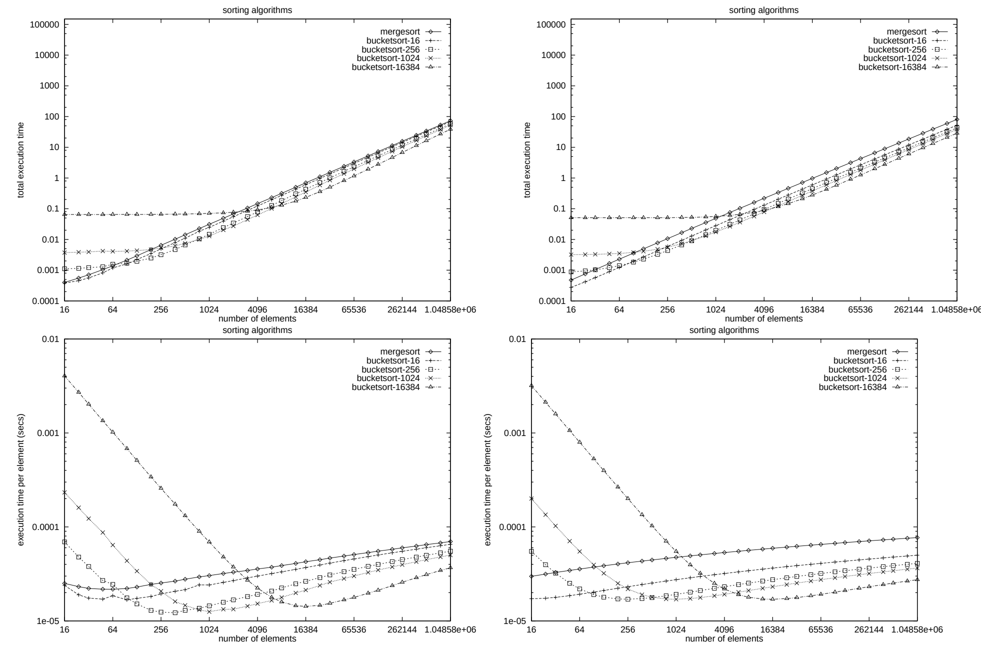

Figure 2.3: Measured and predicted execution times for the bucketsort algorithm with different numbers of buckets (16, 256, 1024 and 16384) compared against the underlying mergesort algorithm. The input data to the sorting programs is assumed to be uniform randomly distributed. The actual numbers are in table **??**. The figures on the left show the measured execution time, on the right is the predicted execution time. The upper two graphs plot the time against a logarithmic axis, the lower graphs show the execution time spend (on average) per element in the array.

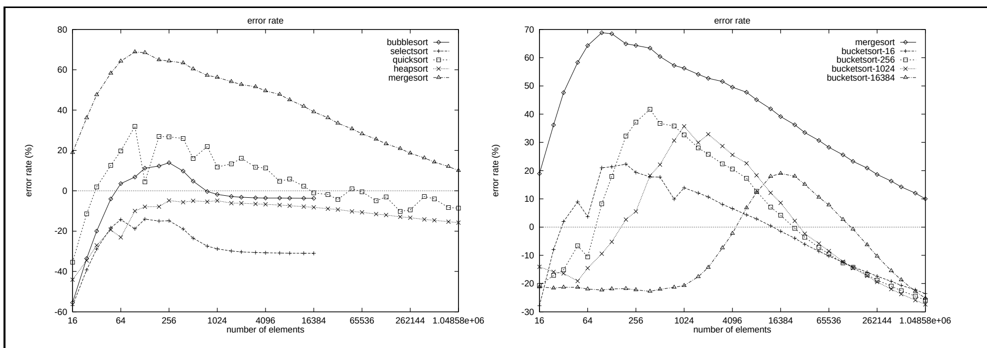

Figure 2.4: The error as a percentage of the expected value for the data shown in figures 2.2 and 2.3.

2.2 Setting the parameters

For the algorithmic parameters —the number of times loops and conditions are taken— the F2SAD/ Parasol II toolset has basically two ways actualizing. One is by using a profile files to determine the parameters and the other is by defining them by hand. The programs which have been analyzed in this report have been deliberately analyzed by hand. Below we give an example of theoretical complexity parameters for the selection sorting algorithm.

2.2.1 Selection Sort

In appendix A, the algorithm considered here, can be found. The number of times the outer most loop at line 9 is executed is clearly $n-1$, the size of the array to be sorted. The inner loop at line 12 has different properties. In the first iteration of the outermost loop it is executed $n-2$ times, the second time $n-3$ times continuing until it is executed only once. This leads to the summation:

$$\sum_{i=1}^{n-2} i = \frac{1}{2}(n-2)\left((n-2)+1\right) = \frac{1}{2}(n-1)(n-2)$$

Since the outer loop iterates $n-1$ times, the inner loop will iterate $\frac{1}{2}(n-2)$ times on average.
The conditional on line 13 is true whenever an element a_i in the list $a_0, a_1 \ldots a_n$ is smaller than all its predecessors $(a_{i-1}, a_{i-2} \ldots a_0)$ For $i = 0$ the conditional is always true, for $i = 1$ with a uniform random list this will be $\frac{1}{2}$, for $i = 2$ it will be $\frac{1}{4}$. We will not go into any detail, but the the idea between this logic is that each predecessor has a probability of $\frac{1}{2}$ to be smaller and in this way each predecessor will half the chance that the element a_i is smaller than all its predecessors.
Each element a_i will be subject to the conditional i times, which leads to the following formula for the chance that the condition evaluates to true:

$$\frac{\frac{1}{i} + \frac{1}{i-1} + \ldots frac11}{n-i}$$

The nominator approaches 2 thus we get:

$$\sum_{i=0}^{n-1} \frac{2}{n-i} = 2\ln(n) + C$$

In which we can ignore C, and since there are n elements we have to divide this by n

To recapulate we are left with the following parameters:

```
N.1  n-1          (the outer loop)
N.2 0.5*(n-2)     (the inner loop)
P.1 2*ln(n)/n     (the constant)
```

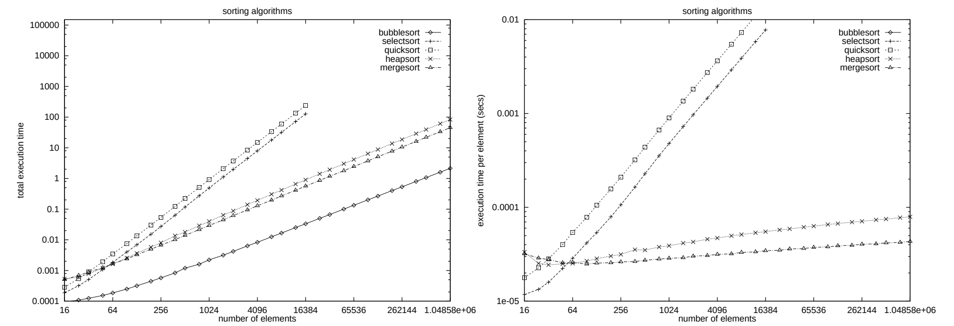

Figure 2.5: For already sorted input data.

We have used here the notation `N.`x and `P.`x, for the control flow parameters, which is also used by F2SAD. The numbers after the `N.` and `P.` have no real meaning, but are distributed according to the flow of the program. They are the same each time the program is run through F2SAD.

For sorted data the conditional `P.2` will be nearly 0 (actually it will be $\frac{n}{\frac{1}{2}(n-1)(n-2)}$, all other parameters remain the same.

Figure 2.5 gives results in the hypothetical case that all input data is sorted. In that case obviously for example the bubblesort algorithm displays a very friendly execution time behaviour.

2.3 A note on linear sorting

As was mentioned above the linear order sorting algorithms use some other sorting algorithm to sort the classes they have build. As we have seen, the overhead and the usage of an other algorithm do not make it attractive for sorting purposes, since it is only very slightly better than the underlying sort. But, the linear order sorting algorithms have also a very different purpose. If it is necessary to classify the input in ranges, resulting in a list of only roughly sorted lists, there is no need for the underlying comparison sort mechanism. And therefor these algorithms have their separate usefulness, especially in parallel computers in which data has be redistributed. The bucket method can be used to classify the data, and to distribute each class to a processor.

Appendix A
Source code

This appendix includes all the source code of the algorithms studied in this report. The main program is not included since it is generated in order to provice multiple input data sets.

A.1 Bubblesort

```
      SUBROUTINE bubblesort(asize, a)
      IMPLICIT NONE
      INTEGER asize
      DOUBLE PRECISION a
      DIMENSION a(*)
      DOUBLE PRECISION swap
      INTEGER i, size
      LOGICAL flag

      size = asize
 10   flag = .FALSE.
      DO 20, i=1, size-1
         IF(a(i) .GT. a(i+1)) THEN
            PRINT *, i, a(i), a(i+1)
            swap   = a(i)
            a(i)   = a(i+1)
            a(i+1) = swap
            flag   = .TRUE.
         END IF
 20   CONTINUE
      size = size - 1
      IF(flag) GOTO 10
      END
```

A.2 Selectsort

```
      SUBROUTINE selectsort(size, a)
      IMPLICIT NONE
      INTEGER size
      DOUBLE PRECISION a
      DIMENSION a(*)
      DOUBLE PRECISION smallest
      INTEGER i, j, index

      DO 20, i=1, size-1
         index    = i
         smallest = a(index)
         DO 10, j=i+1, size
            if(smallest .GT. a(j)) THEN
               index    = j
               smallest = a(index)
            END IF
 10      CONTINUE
         a(index) = a(i)
         a(i)     = smallest
 20   CONTINUE
      END
```

A.3 Heapsort

The heapify routine is the key to the heapsort algorithm. The parameters to the heapify routine are an array A and an index i into that array. The precondition for the heapify routine is that the left binary subtree and the right binary subtree are both heaps. $A(i)$ howevery may be larger than the elements in both subtrees, thus violating the heap property. The heapify routine will "sift down" this element A(i) and by this way both subtrees and $A(i)$ will become one larger heap.

$l \leftarrow Left(i)$
$r \leftarrow Right(i)$
if $l \leq HeapSize[A]$ *and* $A[l] > A[i]$
    then $largest \leftarrow l$
    else $largest \leftarrow i$
if $r \leq HeapSize[A]$ *and* $A[r] > A[largest]$
    then $largest \leftarrow t$
if $largest \neq i$
    then *exchange* $A[i] \leftrightarrow A[largest]$
        Heapify(A*,*$largest$*)*

```

      SUBROUTINE heapify(size, a, parent)
      IMPLICIT NONE
c     left(index)  = index*2
c     right(index) = index*2 + 1
      DOUBLE PRECISION a
      INTEGER size, parent
      DIMENSION a(*)
      INTEGER i, l, r, largest
      DOUBLE PRECISION swap

      i = parent
 10   l = i*2                                                      left(i)
      r = i*2+1                                                    right(i)
      IF ((l .LE. size) .AND. (a(l) .GT. a(i))) THEN
         largest = l
      ELSE
         largest = i
      END IF
      IF ((r .LE. size) .AND. (a(r) .GT. a(largest))) THEN
         largest = r
      END IF
      IF (largest .NE. i) THEN
```

Most paramters of the heapsort are

```
         a(i)       = a(largest)
         a(largest) = swap
         i = largest
         GOTO 10
      END IF

      END

```

for $i \leftarrow \frac{Size[a]}{2}$ ***downto*** *1*
do *Heapify(A,i)*

```
      SUBROUTINE buildheap(size, a)
      IMPLICIT NONE
      DOUBLE PRECISION a
      INTEGER size
      DIMENSION a(*)
      INTEGER i

      DO 10, i=size/2, 1, -1
         CALL heapify(size, a, i)                                   SX/2
 10   CONTINUE

      END

```

BuildHeap(A)
for $i \leftarrow length[A]$ ***downto*** *2*
do *exchange* $A[1] \leftrightarrow A[i]$
decrease $HeapSize$ *by 1*
Heapify(A,1)

```
      SUBROUTINE heapsort(asize, a)
      IMPLICIT NONE
      INTEGER asize
      DOUBLE PRECISION a
      DIMENSION a(*)
      DOUBLE PRECISION swap
      INTEGER i, size

      size = asize
      CALL buildheap(size, a)
      DO 10, i=size, 2, -1                                          SX-1
         swap = a(1)
         a(1) = a(i)
         a(i) = swap
         size = size - 1
         CALL heapify(size, a, 1)
 10   CONTINUE

      END

```

A.4 Mergesort

```
     SUBROUTINE mergelsort(a, lstptr, head, tail)
     IMPLICIT NONE
     INTEGER stacksize
     PARAMETER (stacksize = 256)
     INTEGER lstptr, head(*), tail(*)
     DOUBLE PRECISION a(*)
     INTEGER stackindex, stack(stacksize)
     INTEGER size, list1, list2, run, hsize, x, y, z

     x = 0
     y = 0
     z = 0

     size = 0
     run  = lstptr
10   IF(run .GT. 0) THEN
        size = size + 1
        run  = tail(run)
        GOTO 10
     END IF
     sv = size

     stack(1)   = 0
     stackindex = 2

1    IF(size .LE. 1) GO TO 4

     list1 = lstptr
     list2 = lstptr
     hsize = size/2
20   IF(hsize .GT. 0) THEN
        hsize = hsize - 1
        list2 = tail(list2)
        GOTO 20
     END IF

     stack(stackindex)   = size
     stack(stackindex+1) = list2
     stack(stackindex+2) = 1
     stackindex          = stackindex + 3
     size                = size/2
     lstptr              = list1
     GO TO 1
2    list1               = lstptr
     stackindex          = stackindex - 3
     size                = stack(stackindex)
     list2               = stack(stackindex+1)

     stack(stackindex)   = size
     stack(stackindex+1) = list1
     stack(stackindex+2) = 2
     stackindex          = stackindex + 3
     size                = size - size/2
     lstptr              = list2
     GO TO 1
3    list2               = lstptr
     stackindex          = stackindex - 3
     size                = stack(stackindex)
     list1               = stack(stackindex+1)

```

```

      IF(a(head(list1)) .LT. a(head(list2))) THEN
         lstptr = list1
      ELSE
         lstptr = list2
      END IF

      run = 0
 30   IF(list1 .GT. 0 .AND. list2 .GT. 0) THEN
         z = z + 1
         IF(a(head(list1)) .LT. a(head(list2))) THEN
            IF(run .GT. 0) THEN
               tail(run) = list1
            ELSE
               lstptr = list1
            END IF
            run   = list1
            list1 = tail(list1)
         ELSE
            IF(run .GT. 0) THEN
               tail(run) = list2
            ELSE
               lstptr = list2
            END IF
            run   = list2
            list2 = tail(list2)
         END IF
         GOTO 30
      END IF
      IF(list1 .GT. 0) THEN
         IF(run .GT. 0) THEN
            tail(run) = list1
         ELSE
            lstptr = list1
         ENDIF
      ELSE IF(list2 .GT. 0) THEN
         IF(run .GT. 0) THEN
            tail(run) = list2
         ELSE
            lstptr = list2
         ENDIF
      END IF


 4    IF(size .EQ. 1) tail(lstptr) = 0

```

F2C isn't able to process vector-if statements, that is why the following IF-statement is commented out and two replacement IF's are dropped in.

```
      y = y + 1
      IF(stack(stackindex-1).EQ.1) GO TO 2
      x = x + 1
      IF(stack(stackindex-1).EQ.2) GO TO 3

      END

```

A.5 Quicksort

```
      SUBROUTINE quicksort(asize, a)
      INTEGER stacksize
      PARAMETER (stacksize = 256)
      INTEGER asize
      DOUBLE PRECISION a
      DIMENSION a(*)
      DOUBLE PRECISION aux
      INTEGER start, size, front, back, stack, idx
      DIMENSION stack(stacksize)

      size  = asize
      idx = 0
      start = 1
 20   IF(size .GT. 1) THEN
         front = start+1
         back  = start+size-1

 30      IF(front .LE. back) THEN
            IF (a(start) .GT. a(front)) THEN
               front = front + 1
            ELSE IF(a(start) .LE. a(back)) THEN
               back  = back  - 1
            ELSE
               aux      = a(front)
               a(front) = a(back)
               a(back)  = aux
            END IF
            GOTO 30
         END IF

         IF(front .NE. start+1) THEN
            aux        = a(start)
            a(start)   = a(front-1)
            a(front-1) = aux
            stack(idx+1) = front - start - 1
            stack(idx+2) = start
            idx          = idx + 2
         END IF
         size  = size - back + start - 1
         start = front
         GOTO 20
      END IF

      IF(idx .GT. 0) THEN
         idx = idx - 2
         size   = stack(idx+1)
         start  = stack(idx+2)
         GOTO 20
      END IF

      END
```

A.6 Bucketsort

```
      SUBROUTINE bucketlsort(a, lstptr, head, tail)
      IMPLICIT NONE
      DOUBLE PRECISION a(*)
      INTEGER lstptr, head(*), tail(*)
      INTEGER numbuckets
      PARAMETER (numbuckets = 16384)
      INTEGER buckets(numbuckets), bucketnum, i, aux

      DO 10, i=1, numbuckets
         buckets(i) = 0
 10   CONTINUE
 20   IF(lstptr .GT. 0) THEN
         bucketnum = INT(a(head(lstptr)) * numbuckets) + 1
         aux               = tail(lstptr)
         tail(lstptr)      = buckets(bucketnum)
         buckets(bucketnum) = lstptr
         lstptr            = aux
         GO TO 20
      END IF

      DO 30, i=1, numbuckets
         CALL mergelsort(a, buckets(i), head, tail)
 30   CONTINUE

      lstptr = 0
      DO 40, i=1, numbuckets
         IF(buckets(i) .GT. 0) THEN
            IF(lstptr .EQ. 0) THEN
               lstptr = buckets(i)
            ELSE
               tail(aux) = buckets(i)
            END IF
            aux = buckets(i)
 50         IF(tail(aux)) THEN
               aux = tail(aux)
               GO TO 50
            END IF
         END IF
 40   CONTINUE

      END
```