UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Advanced Topics in Set Theory

2003/2004; 1st Semester
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Homework Set # 8. Deadline: NONE

This homework set will not count for the final grade.

Exercise 25 (Determinacy in $\mathbf{L}$).

Show that there is a nondetermined projective set in $\mathbf{L}$. Compute its complexity in the projective hierarchy (try to minimize the complexity in the construction).

Exercise 26 (Wellorderable sets of reals under AD).

Show that AD implies that there is no injection from $\aleph_1$ into $\mathbb{N}^{\mathbb{N}}$.

Hint. If $f : \aleph_1 \to \mathbb{N}^{\mathbb{N}}$ is an injection, look at $f[\aleph_1]$. Why does this set violate the fact that all sets have the perfect set property?

Exercise 27 (Determinacy for $\wp(\mathbb{R})$ moves).

Show that $\mathsf{ZF} + \mathsf{AD}_{\wp(\mathbb{R})}$ is inconsistent.

Hint. $\mathsf{AD}_{\wp(\mathbb{R})}$ implies AD. Use Exercise 26 and construct a game in which a winning strategy would give an injection from $\aleph_1$ into the reals.

Exercise 28 (Games without alternating moves and games with rules).

We call a tree on $\mathbb{N}$ a **finitary rule**, and we say that a sequence $s \in \mathbb{N}^{<\mathbb{N}}$ violates the rule T if $s \notin T$.

We can define a game $G(T, A)$ as the game $G(A)$ with the extra condition that the first player to violate the rule T loses.

We call a function $\mu : \mathbb{N}^{<\mathbb{N}} \to \{\mathrm{I}, \mathrm{II}\}$ a **moving function**, and define a game $G(T, \mu, A)$ as the game $G(T, A)$ with the difference that instead of having player I play at sequences of even length and player II play at sequences of odd length, the player to make the next move is determined by μ.

We say that two games G and H are **equivalent** if player I (II) has a winning strategy in G if and only if he has a winning strategy in H.

Show that for each A, T, and μ there is a set $A_{T,\mu}$ such that $G(T, \mu, A)$ and $G(A_{T,\mu})$ are equivalent.

Call a pointclass $\mathbf{\Gamma}$ **game-closed** if for each $A \in \mathbf{\Gamma}$ and each T and μ, the set $A_{T,\mu}$ is in $\mathbf{\Gamma}$. Give conditions for $\mathbf{\Gamma}$ such that you can show that $\mathbf{\Gamma}$ is game-closed.