UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2004/2005; 1st Semester
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Homework Set # 10 *Deadline:* November 24th, 2004

Exercise 30 (3 points total).

Compute the following binary numbers (¼ point each):

(1) $\texttt{1011010} + \texttt{1101} = ?$.
(2) $\texttt{1101101} \times \texttt{10} = ?$.
(3) $\texttt{1001101} \times \texttt{1000} = ?$.
(4) $\texttt{1100110} \times \texttt{1101} = ?$.

Describe in words an algorithm for the multiplication of two binary numbers (2 points).

Exercise 31 (9 points total).

Let $2^{\mathbb{N}}$ be the set of all infinite 0-1 sequences. For $x \in 2^{\mathbb{N}}$, we define $\widehat{x}(n) := 1 - x(n)$. We call $\mathcal{C} \subseteq 2^{\mathbb{N}}$ a **symmetric class** if for every $x \in \mathcal{C}$, we also have $\widehat{x} \in \mathcal{C}$. A function $F : \mathbb{N} \to \mathcal{C}$ is called a $\mathcal{C}$-good parametrization if the sequence $\langle F(n)(n) \,;\, n \in \mathbb{N}\rangle$ is an element of $\mathcal{C}$ and F is a surjection.

(1) Show that no symmetric class $\mathcal{C}$ can have a $\mathcal{C}$-good parametrization (4 points).
(2) Derive Cantor's Theorem ("there is no bijection between $\mathbb{N}$ and $2^{\mathbb{N}}$") as a corollary (2 points).
(3) Give an example of a (non-symmetric) class $\mathcal{C} \subseteq 2^{\mathbb{N}}$ that has a $\mathcal{C}$-good parametrization (3 points).

Exercise 32 (9 points total).

Find three academic (PhD) students of **Alonzo Church** (1903-1995) and give:

- the student's name (½ point each),
- the student's year of birth and death (if applicable; ½ point for each student),
- the institution from which the student received his or her PhD (½ point each),
- one theorem that the student proved (just the statement, no proof; 1 point each).

Name an ILLC staff member who is an academic grandchild of Church (*i.e.*, the PhD student of a PhD student of Church's; 1½ point).

Exercise 33 (4 points total).

Find two respectable authors who believe that the Church-Turing thesis is false and give a brief description (one to three sentences) of their position with a reference to a published paper in which they hold this view (2 points each).

(**Note.** By "published", we mean "published in a scholarly journal or monograph", not "stated on some webpage". What "respectable" means is of course open for some interpretation; in general, research logicians and computer scientists with a university affiliation can be assumed to be "respectable". The more "respectable" the authors are that you find, the better.)

`http://staff.science.uva.nl/~bloewe/2004-I-CL.html`