Core Logic, ILLC / Universiteit van Amsterdam

Temporal Logic and the Logic of Agency

8 December 2004

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Outline

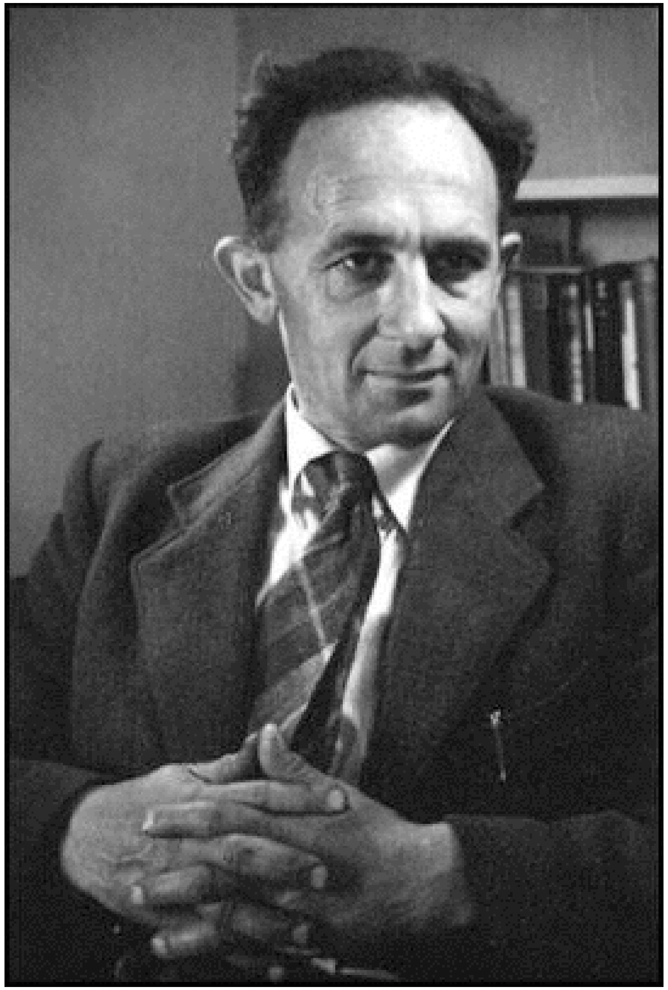
Temporal Logic

* Arthur Prior and the development of (tense) logic after 1950
* Tensed *vs* tenseless talk
* Hybrid logic
* Semantics for the future tense

Logic of Agency

* Review of branching time
* Agents and choices
* “Seeing to it that”
* Some further developments

Arthur Prior



Arthur Prior

1914 born in Masterton, New Zealand
1946 Lecturer, Canterbury University College, NZ
1956 John Locke Lectures, Oxford; initiated British Logic Colloquium
1958 Professor in Manchester
1960 Editor, *The Journal of Symbolic Logic*
1966 Fellow and Tutor, Balliol College, Oxford
1969 died in Trondheim, Norway

Main works:

1957 *Time and Modality*
1967 *Past, Present and Future*
1968 *Papers on Time and Tense* (new ed., 2003)
1971 *Objects of Thought* (ed. P.T. Geach and A.J.P. Kenny)
1977 *Worlds, Times and Selves* (ed. K. Fine)

Arthur Prior and the development of (tense) logic

Technical developments in logic:

* among the first explicitly semantic approaches to modal logic

* among the earliest expressiveness results (Hans Kamp)

* earliest developments towards “hybrid logic”

Other fields:

* Philosophy of language: phenomenology of “essential indexicality”

* Metaphysics: logical analysis of the problem of *futura contingentia*

Prior on logic and natural language

* Foundational problem: How do we know what the logical connectives mean?

* Prior's argument (The runabout inference-ticket): Giving introduction- and elimination-rules alone cannot give the meaning of a connective

* Logic as a certain (formal) way of studying natural language / the world:

* Logic is about the real world;
* No fixed boundary between logic and other sciences

Time and tense in natural language

(1) Socrates is sitting.

* English (and other Indo-European languages): *tensed* language

* natural language sentences are complete without dates

* ancient and medieval discussion: propositions are complete without dates

* 20th century (Frege, Russell): explicit dates needed, or token-reflexive analysis:

(2) Socrates is* sitting at t. (“is*” a tenseless copula)

(3) Socrates is* sitting while this sentence is uttered.

Essential indexicality

* Many uses of indexicals like “I”, “now”, and (maybe) “here” cannot be eliminated

* Famous example (John Perry, 1979): The sugar trail in the supermarket

* Indexicals are vital for explaining actions and emotions

* Names can be mis-applied, “I” cannot

* Prior (1959): Tense is essentially indexical

Prior’s “thank goodness” argument: The essential indexicality of tense

> [. . .] half the time I personally have forgotten what the date *is*, and have to look it up or ask somebody when I need it for writing cheques, etc.; yet even in this perpetual dateless haze one somehow communicates, one makes oneself understood, and with time-references too. One says, e.g. ‘Thank goodness that’s over!’, and not only is this, when said, quite clear without any date appended, but it says something which it is impossible that any use of a tenseless copula with a date should convey. It certainly doesn’t mean the same as, e.g. ‘Thank goodness the date of the conclusion of that thing is Friday, June 15, 1954’, even if it be said then. (Nor, for that matter, does it mean ‘Thank goodness the conclusion of that thing is contemporaneous with this utterance’. Why should anyone thank goodness for that?)

Formalising the tenses

* Tense is essential $\Rightarrow$ take atomic sentences to be tensed

* Introduce (modal) operators F (future) and P (past)

* Iterability argument for use of operators

* P and F are weak operators;
* duals G (always **g**oing to be) and H (**h**as always been)

* Prior considers propositional *and* quantified languages

* Problems of contingently existing individuals; modal system Q

Prior’s syntax: Polish notation

corresponding to *CUbanCUcanIbc*, is expressed by *CPnNpNPnp*. Neither of these is derivable from the other, but we may concentrate on the one in *F*. Among the theses which become derivable when we add *CFnNpNFnp* to the minimal metric system are

T1. *CFnpNFnNp*
T2. *CPnFnpp*
T3. *CPnFSnmpFmp*
T4. *CFnpCFnqFnKpq*
T5. *CPFpAApPpFp*
T6. *CKFpFqAAFKpqFKpFqFKqFp*.

The intuitive connexion between each of these theses and non-branching in the future is straightforward; we can see how each of them could have exceptions if there were no such thing as ‘the’ future but only a number of alternative possible futures. As a specimen, we prove T5, as follows:

(1)	*CImnCPmFnpPnFnp*	
(2)	*CImnCPmFnpp*	(1, T2)
(3)	*CImnCPmFnpAApPpFp*	(2, p.c.)
(4)	*CImSlnCPmFnpPSlnFnp*	
(5)	*CImSlnCPmFnpPlPnFnp*	(4, A5.2)
(6)	*CImSlnCPmFnpPlp*	(5, T2)
(7)	*CΣlImSlnCPmFnpΣlPlp*	(6, quantification theory)
(8)	*CΣlImSlnCPmFnpPp*	(7, Df.*P*)
(9)	*CΣlImSlnCPmFnpAApPpFp*	(8, p.c.)
(10)	*CInSmlCPmFnpPmFSmlp*	
(11)	*CInSmlCPmFnpPmFmFlp*	(10, A5.1)
(12)	*CInSmlCPmFnpFlp*	(11, T2)
(13)	*CΣlInSmlCPmFnpΣlFlp*	(12)
(14)	*CΣlInSmICPmFnpFp*	(13, Df.*F*).
(15)	*CΣlInSmlCPmFnpAApPpFp*	(14, p.c.)
(16)	*AAImnΣlImSlnΣlInSml*	(number-theory)
(17)	*CPmFnpAApPpFp*	(16, 3, 9, 15)
(18)	*CΣmΣnPmFnpAApPpFp*	(17, *Σ*1*n*, *Σ*1*m*)
(19)	*CΣmPmΣnFnpAApPpFp*	(18, A4.2)
(20)	*CPFpAApPpFp*	(19, Df.*P*, Df.*F*).

4. *Comparisons with earlier systems*. Given the additions for infinity, we may abridge the A1’s to *CFnCpqCFnpFnq* and its image, and

The intuitive connexion between each of these theses and non-branching in the future is straightforward; we can see how each of them could have exceptions if there were no such thing as ‘the’ future but only a number of alternative possible futures. As a specimen, we prove T5, as follows:

1.	$m=n \supset (P(m)F(n)p \supset P(n)F(n)p)$	
2.	$m=n \supset (P(m)F(n)p \supset p)$	[1, T2]
3.	$m=n \supset (P(m)F(n)p \supset (p \vee Pp \vee Fp))$	[2, p.c.]
4.	$m=(l+n) \supset (P(m)F(n)p \supset P(l+n)F(n)p)$	
5.	$m=(l+n) \supset (P(m)F(n)p \supset P(l)P(n)F(n)p)$	[4, A5.2]
6.	$m=(l+n) \supset (P(m)F(n)p \supset P(l)p)$	[5, T2]
7.	$\exists l\, m=(l+n) \supset (P(m)F(n)p \supset \exists lP(l)p)$	[6, quantification theory]
8.	$\exists l\, m=(l+n) \supset (P(m)F(n)p \supset Pp)$	[7, Df.*P*]
9.	$\exists l\, m=(l+n) \supset (P(m)F(n)p \supset (p \vee Pp \vee Fp))$	[8, p.c.]
10.	$n=(m+l) \supset (P(m)F(n)p \supset P(m)F(m+l)p)$	
11.	$n=(m+l) \supset (P(m)F(n)p \supset P(m)F(m)F(l)p)$	[10, A5.1]
12.	$n=(m+l) \supset (P(m)F(n)p \supset F(l)p)$	[11, T2]
13.	$\exists l\, n=(m+l) \supset (P(m)F(n)p \supset \exists lF(l)p)$	[12]
14.	$\exists l\, n=SmI \supset (P(m)F(n)p \supset Fp)$	[13, Df.*F*]
15.	$\exists l\, n=(m+l) \supset (P(m)F(n)p \supset (p \vee Pp \vee Fp))$	[14, p.c.]
16.	$m=n \vee \exists lm=(l+n) \vee \exists ln=(m+l)$	[number-theory]
17.	$P(m)F(n)p \supset (p \vee Pp \vee Fp)$	[16, 3, 9, 15]
18.	$\exists m \exists nP(m)F(n)p \supset (p \vee Pp \vee Fp)$	[17, $\exists 1n$, $\exists 1m$]
19.	$\exists mP(m) \exists nF(n)p \supset (p \vee Pp \vee Fp)$	[18, A4.2]
20.	$PFp \supset (p \vee Pp \vee Fp)$	[19, Df.*P*, Df.*F*].

4. *Comparisons with earlier systems*. Given the additions for infinity, we may abridge the A1s to

A1.1. $F(n)(p \supset q) \supset (F(n)p \supset F(n)q)$

and its image, and derive the rules to infer $\vdash F(n)\alpha$ and $\vdash P(n)\alpha$ from $\vdash \alpha$

Semantics for modal logics

* use a modal object language, what about the semantics?

* models: time-flow as a binary relation (earlier than/later than)

* language of the earlier-later-relation: “U-calculus” ($m < m'$ etc.)

* tension: if the tenses *are* basic, the formalism should reflect this
* the models cannot be more fundamental than the tense operators

* Prior on the status of models: “handy diagrams”
* no metalanguage

* aim: interpreting the U-calculus within tense logic

* expressiveness: irreflexivity (easy in U-calculus, no tense-logical analogue)

Hybrid logic I: Standard translation

* Modal logic as a fragment of first order logic: mimic the semantic clauses

$$
\begin{array}{lll}
\mathfrak{M}, w \models p & \text{iff} & w \in P \\
\mathfrak{M}, w \models \neg\phi & \text{iff} & \mathfrak{M}, w \not\models \phi \\
\mathfrak{M}, w \models \phi \wedge \psi & \text{iff} & \mathfrak{M}, w \models \phi \text{ and } \mathfrak{M}, w \models \psi \\
\mathfrak{M}, w \models \langle R \rangle \phi & \text{iff} & \text{there is } w' \text{ s.t. } R(w, w') \text{ and } \mathfrak{M}, w' \models \phi
\end{array}
$$

via standard translation:

$$
\begin{array}{lll}
ST_x(p) & = & P(x) \\
ST_x(\neg\phi) & = & \neg ST_x(\phi) \\
ST_x(\phi \wedge \psi) & = & ST_x(\phi) \wedge ST_x(\psi) \\
ST_x(\langle R \rangle \phi) & = & \exists y \, (R(x, y) \wedge ST_y(\phi))
\end{array}
$$

* Other direction?

Hybrid logic II: Prior on "world-states"

* Wp ("p is the world state"): $Wp \to p$ and $(Wp \wedge q) \to \Box(p \to q)$
* ("The world is everything that is the case", Wittgenstein, *TLP* 1)

* sorted language: ordinary propositional variables ($p, q, r, \ldots$) and world-variables ($a, b, c, \ldots$); for world-variables, have $\Diamond a$ and $\Box(a \to p) \vee \Box(a \to \neg p)$

* "p holds at a" as $\Box(a \to p)$, "a is earlier than b" as $\Box(b \to Pa)$

* need for a modality $\Diamond$ ("somewhere in the model") and $\Box$ ("everywhere")
* linear models: $p \vee Pp \vee Fp$; branching time: $p \vee Pp \vee Fp \vee PFp$
* generally, not definable (generated submodels!)

Hybrid logic III: Modern hybrid logic

* sorted language: propositional variables $p, q, r, \ldots$; nominals $i, j, k, \ldots$

* semantics for nominals: true at exactly one node

* introduce various binders for nominals ($\downarrow, \exists, \Sigma, \Downarrow$) and logical modalities ($\Diamond$)

* hierarchy of languages w.r.t. expressive power: $\downarrow \leq \exists \leq \Downarrow$; $\Diamond \leq \Sigma \leq \Downarrow$

* strongest hybrid language recaptures first-order expressivity:

$$
\begin{array}{lcl}
HT(v_1 = v_2) & = & \Downarrow x.(v_1 \wedge v_2) \\
HT(P(v_1)) & = & \Downarrow x.(v_1 \wedge p) \\
HT(R(v_1, v_2)) & = & \Downarrow x.(v_1 \wedge \langle R \rangle v_2) \\
HT(\exists y\, \phi) & = & \exists y.\, HT(\phi)
\end{array}
$$

Semantics for the future tense I: Paying one's gambling debts

* We assert future-tensed statements in the face of indeterminism

* Betting as a prime example: "The coin will show heads"

* If the sentence was true (or false) at the time of utterance, then the world must be deterministic, contrary to assumption ($\Rightarrow$ "logical determinism")

* If the sentence was neither true nor false, then why should I pay my gambling debts? After all, neither I nor my opponent said something true.

Semantics for the future tense II: Branching time

* Metaphysical question about the nature of time

* *Descriptive metaphysics* (Strawson): Focus on *actual* conceptual scheme;
* use natural language and the way we act as guidelines

$\Rightarrow$ Overwhelming support for clear distinction between open future/fixed past

* (*Revisionary metaphysics* might urge to revise our attitude (Spinoza, Russell))

* Formally, tree-like structure of time:

* no backward branching: $\forall x, y, z\,((x \leq z \wedge y \leq z) \rightarrow (x \leq y \vee y < x))$

* historical connection: $\forall x, y\, \exists z\,(z \leq x \wedge z \leq y)$

Semantics for the future tense III: Occam *vs* Peirce

* $F\phi$ for "it will be the case that ϕ", evaluate at moment m

* semantics for F analogous to alethic modal logic: basic tense logic K_t
* $m \models F\phi$ iff there is $m' > m$ s.t. $m' \models \phi$

* this semantic definition does not reflect our use of "it will be that"

* histories: maximal chains; $H_{(m)}$ the set of histories through m
* moving along histories backward and forward is unproblematic (linear order)

* Peircean: $m \models F\phi$ iff in every $h \in H_{(m)}$ there is $m' > m$ s.t. $m' \models \phi$

* Occamist: relative to h: $m, h \models F\phi$ iff there is $m' > m$, $m' \in h$, s.t. $m', h \models \phi$

* *Prior-Thomason semantics* for F: Occamist

Semantics for the future tense IV: Stand-alone sentences

* Take Occamist approach seriously. Assertion problem:

* Context of utterance supplies moment of evaluation m
* Context of utterance does not supply history of evaluation h

$\Rightarrow$ Sentence $F\phi$ cannot be evaluated in given context, no truth value

* Solution: Later moment m' singles out set of histories through m;
* at m' the previous assertion will then be vindicated (or not)

* Assertions about the future share the pragmatics of betting

B R E A K

visit the branching space-times conference website at

http://confer.uj.edu.pl/branching

Recap: branching time

* Tree-like partial ordering of moments $\langle T, < \rangle$

* no backward branching: $\forall x, y, z \, ((x \leq z \land y \leq z) \rightarrow (x \leq y \lor y < x))$

* historical connection: $\forall x, y \, \exists z \, (z \leq x \land z \leq y)$

* histories h: maximal linear subsets of T

* historical modalities quantifying over h: $Poss$ (possible) and $Sett$ (settled)

* undividedness at m (for $h, h' \in H_{(m)}$, m not maximal):

$$h \equiv_m h' \text{ iff there is } m' \in h \cap h' \text{ s.t. } m < m'$$

* $\equiv_m$ an equivalence relation; partition Π_m: *elementary possibilities* at m

Agents and choices in a branching framework

* partition Π_m describes nature's indeterminism

* m is an *indeterministic point* iff Π_m has more than one element

* descriptive metaphysics: sometimes we are in control of nature's indeterminism

* formally: $Agents$ a set of labels for agents

* for $\alpha \in Agents$; partition $Choice_m^\alpha$ describes α's choices at m

* $Choice_m^\alpha(h) \subseteq H_{(m)}$; "$=$" means that α has no choice

* no choice between undivided histories:

$$(h' \in Choice_m^\alpha(h) \wedge h'' \equiv_m h') \rightarrow h'' \in Choice_m^\alpha(h)$$

Multiple agents and independent choices

* agents' choices at m are simultaneous, so should be independent

$\Rightarrow$ for any function f_m that maps $Agents$ to elements of $Choice_m^\alpha$,

$$\bigcap_{\alpha \in Agent} f_m(\alpha) \neq \emptyset$$

* strong constraint on $Choice_m$

* implausible if, e.g., two agents can manipulate the same object

* spatial separation as a precondition for independence

* branching time not a theory of space

$\Rightarrow$ need to use *branching space-times* as a formal basis for agency

Seeing to it that I: Stit normal form

* many natural language expressions are *agentive* for some α; contrast

(4) Ishmael sailed over the seven seas (agentive)

(5) Ishmael sailed over the side of the *Pequod* (not agentive)

* some operators need agentive complements, e.g., imperatives, deontic notions

* normal form for agentives: α sees to it that ϕ ([$\alpha stit$: ϕ])

* thesis: ϕ is agentive for α iff it can be paraphrased as "α sees to it that ϕ"

* $stit$ as a family of agent-indexed modal operators; allow nesting

Seeing to it that II: Semantics

* various stit operators in the literature

* consider $dstit$, the “deliberative stit”: current choice secures outcome

* two conditions: (i) positive: secure outcome, (ii) negative: non-trivial

$m, h \models \alpha\, dstit : \phi$ iff

(i) for all $h' \in Choice^{\alpha}_{m}(h)$, we have $m, h' \models \phi$
(ii) there is $h'' \in H_{(m)}$ for which $m, h'' \not\models \phi$

* nobody sees to it that $2 + 2 = 4$

* usually, ϕ will be of the form $F\psi$ for contingent ψ

Seeing to it that III: Refraining

* refraining both an action (refrainings are attributed to agents; one can be praised or blamed for refrainings) and a non-action (after all, refraining means *not acting*)

* negated $stit$ is inappropriate

* von Wright: refraining = ability plus negation of action

$$[\alpha\, ref : \phi] \text{ as } \neg[\alpha\, stit : \phi] \wedge Poss : [\alpha\, stit : \phi]$$

* for $dstit$, equivalent to nested $stit$:

$$[\alpha\, ref : \phi] \text{ as } [\alpha\, stit : \neg[\alpha\, stit : \phi]]$$

* refraining from refraining equivalent to acting

Further developments I: Doxastic logic within the logic of agency (Wansing 2001)

* Doxastic interpretation: $\Box_\alpha \phi$ as "α believes that ϕ" (KD45)

* $\Box_\alpha$ a normal modal operator: belief closed under logical consequence

* but belief is *not* closed under logical consequence!

* view belief acquisition as an action

* "α believes that ϕ": at some past moment, α acquired the belief that ϕ, and she hasn't given up that belief since

Further developments II:
Rich deontic logic (Brown 2004)

* Deontic interpretation: $\Box\phi$ as “it is obligatory that ϕ”

* various problems (“paradoxes of standard deontic logic”), e.g.

* Ross’s paradox: If $\vdash A \rightarrow B$, then $\vdash \Box A \rightarrow \Box B$,
* so if I ought to mail your letter, I ought to mail it or burn it

* problems of conflicting and contrary-to-duty obligations;
* problems of dynamical nature of obligations

* employ $stit$ models, distinguish logical from causal consequences of actions

* personalize obligations

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