The Logic of Negation in Boethius

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Introduction

Boethius' *de Hypotheticis Syllogismis* is by far the most extensive account of the conditional and its logic to have survived from antiquity. A rather obscure and tedious work, it has puzzled commentators from Peter Abaelard to Jonathan Barnes. Most of the difficulties that they have had in extracting the principles of Boethian logic seem to me to follow from the assumption that what he offers is an account of the application of propositional operators to propositional contents. Though generally not made explicit by modern historians, the concepts of propositional content and propositional operation are nevertheless presupposed by the symbolic apparatus which they typically use to represent the claims of ancient and mediaeval logics. I will try to show that an examination of Boethius' theory of language forces us to give up the assumption that his logic is propositional and that when we do so his remarks on compound propositions turn out to be rather less mysterious than they have seemed.*

Let me begin by saying a little about Boethius, his importance for the later development of logic, and the treatment of his work by modern commentators.

* Abbreviations:

DHS   Anicii Manlii Severini Boetii, *De Hypotheticis Syllogismis*, Luca Obertello (ed.), Brescia, 1969.
2IDI   Boethius' greater commentary on *de Interpretatione*, vol. 2 of Anicii Manlii Severini Boetii, *Commentarii in Librum* Περὶ Ἑρμηνείας, Carl Meiser (ed.), Leipzig, 1880.
Earlier editions of both of these are printed in volume 64 of J.-P. Migne's, *Patrologia Latina*, Paris, 1860, as well as:
ITC   *In Topica Ciceronis Commentaria.*
TD   *De Topicis Differentiis*
For a recent and very full bibliography of works by and on Boethius see Henry Chadwick, *Boethius: The Consolations of Music, Logic, Theology and Philosophy*, Oxford, 1981.

Boethius was active at the very end of classical antiquity. Born in 480, he was executed on a charge of treason in 524. At his death he had barely begun his extraordinary project of instructing the Latin world in the wisdom of the Greeks. To this end he proposed to translate and comment upon all of the works of Aristotle and Plato. It is not clear quite how far he got but his translations of Porphyry's *Isagoge*, the *Categories* and *de Interpretatione* along with commentaries on them and two brief paraphrases of part of the *Prior Analytics* were never lost. His translations of the remaining works of the *Organon*[1] seem to have been recovered in the middle of the twelfth century.

Prior[2] showed that Boethius, apparently for the first time, presents a development of the theory of the categorical syllogism to take account of negative terms. This is really only the most trivial extension, however, and almost all of Aristotle's work is ignored. Boethius' brief summaries are entirely elementary and cover only the treatment of the assertoric syllogism in the first seven chapters of Book 1 of the *Prior Analytics*[3]. With the rediscovery of the original his paraphrases became redundant. The same cannot be said of his commentaries on the *Isagoge*, *Categories* and *de Interpretatione*. The latter in particular, much more than Aristotle's own work, shaped mediaeval thinking about philosophical semantics. Boethius also provided the Middle Ages with something which they could not recover from Greek sources, an extensive account of the logic of the conditional and a treatment of the discovery and defence of non-syllogistic arguments. One of the great achievements of early twelfth-century logic is Abaelard's combination of these two into a unified theory of inference.

Modern work on Boethius began with the publication in 1951 of Karl Dürr's monograph *The Propositional Logic of Boethius*[4]. Although it has been the

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[1] Apart from *An. Post.* of which only fragments seem to have survived into the twelfth century. See Sten Ebbesen, 'Manlius Boethius on Aristotle's Analytica Posteriora', *Cahiers de l' Institut du Moyen Âge Grec et Latin*, Copenhagen, 8 (1973), pp. 3-32; Margaret Gibson, 'Latin Commentaries on Logic Before 1200', *Bulletin de Philosophie Médiévale*, 24 (1982), pp. 54-64.

[2] Arthur Prior, 'The Logic of Negative Terms in Boethius', *Franciscan Studies*, 13 (1953) pp. 1-6.

[3] For a summary of the extent of Greek commentaries on *An. Pr.* see N. Rescher, *Al Farabi's Short Commentary on Aristotle's Prior Analytics*, University of Pittsburgh Press, 1963, pp. 23-27. Note especially the quotation on p. 18 from al-Farabi on the history of logic where he refers to an earlier period in which *An. Pr.* beyond I.7 was called 'the part which is not read'.

[4] Karl Dürr, *The Propositional Logic of Boethius*, North Holland, Amsterdam, 1951.

standard authority for even the most curious and critical of recent historians, Dürr's work turns out to be quite useless as a guide to Boethian logic. While he might be said to offer a logic for compound propositions Boethius in no way provides a propositional logic of the sort for which Lukasiewicz proposed to write the history[5]. Dürr's method is quite extraordinarily crude. He simply symbolizes Boethius' schemata for hypothetical syllogisms with the conditional construed first as material and then as strict implication. The test of Boethius' worth as a logician is whether or not the result of the translation is a theorem of *Principia Mathematica* or of Lewis' S5.

It seems from Dürr's presentation that Boethius held a very striking thesis about conditionals. Translated into the language of a propositional calculus he is forced to claim, for example, both that from 'if P, then (if Q, then R)', and '(if Q, then not R)', there follows 'not P' and that from 'if (if P, then not Q), then R' and 'not R' there follows 'if P, then Q'. He is thus committed to the contradictory opposition of 'if P, then Q' and 'if P, then not Q'. Robert Stalnaker[6] has recently advocated the principle of Conditional Excluded Middle for the logic of conditionals but Boethius would, I think, be unique among ancient and mediaeval philosophers if he supposed it to be true of a genuinely conditional connection[7]. I will try to show that in fact he not only does not but could not embrace C.E.M. The problem is not, as Jonathan Barnes suggests[8], that Boethius' logic is not *classical* propositional calculus but rather that it is not propositional at all.

In the early 1960s Storrs McCall[9] appealed to Boethius' account of the hypothetical syllogism to motivate the development of a non-classical prop-

[5] Jan Lukasiewicz, 'Zur Geschichte der Aussagenlogik', *Erkenntnis*, 5 (1933), pp. 111-131.

[6] See for example the introduction and the various essays appearing in W. L. Harper, R. Stalnaker & G. Pearce (eds.), *Ifs*, Reidel, Dordrecht, 1981.

[7] '(P→Q) v (P→-Q)' holds if the arrow is the material conditional and the disjunction truth-functional and inclusive. Later mediaeval logic has material disjunction and material implication in the so called *ut nunc* consequence but I do not know if this thesis is anywhere stated or discussed. Mediaeval logic only very infrequently distinguishes between indicative and counterfactual conditionals.

[8] Jonathan Barnes, 'Boethius and the Study of Logic' in Margaret Gibson (ed.), *Boethius: His Life, Thought and Influence*, Basil Blackwell, Oxford, 1981, pp. 73-89.

[9] Storrs McCall, 'Connexive Implication', *The Journal of Symbolic Logic*, 31 (1966), pp. 415-433. For a critical exploration of connexive logic see Richard Routley and Hugh Montgomery, 'On Systems Containing Aristotle's Thesis', *Journal of Symbolic Logic*, 33 (1968), pp. 82-96.

ositional logic. McCall calls his formal system a *connexive logic* after what he takes to be the Stoic Chrysippus' requirement for the truth of a conditional that there be a *connexion* between the antecedent and the consequent. Such a connexion exists, McCall suggests, if and only if the antecedent is incompatible with the contradictory of the consequent but compatible with the consequent itself. The characteristic feature of connexive logics is their inclusion of propositional theses which McCall thinks can be found in Aristotle and Boethius. I will argue, however, that what he calls Boethius' Thesis: 'if (if P, then Q), then not (if P, then not Q)', is again not something which Boethius would, or could, accept. Elsewhere I have shown that the connexive principles were regarded as beyond doubt by Peter Abaelard and that a crisis in the history of logic followed upon the proof by Alberic of Paris that they are incompatible with simplification[10].

More recently Eleonore Stump[11] has attempted to show that Boethius should be treated as a plausible and historically accurate commentator on Stoic logic. This is certainly a bold suggestion since he himself claims to have known of no Stoic work on compound propositions, there are almost no traces of any such knowledge in his works, and his only remark on the Stoic theory of negation seems to entirely misrepresent it. Stump has nevertheless appealed to Boethius for an account of the Stoic Third Indemonstrable. Cicero gives the Indemonstrables in his *Topics* in a passage which Boethius discusses at great length in his commentary on the work. If Boethius' formulation of the argument schema is again symbolized propositionally, he appears to insist that 'not (P and not Q)' is equivalent to 'not (if P then not Q)' and this to 'if P, then Q'. C.E.M. once more. I will try to show that it is inappropriate to construe his remarks in this way and, I am afraid, that we cannot employ him as a guide to Stoic logic.

Even if we cannot rely on Boethius for information about the Stoics, however, he certainly does deserve to be studied for what he can tell us

[10] The theses characterising conditional simplification are: $\vdash (P \& Q) \rightarrow P$ and $\vdash (P \& Q) \rightarrow Q$, the rules of inference for simplification are: $\vdash P \& Q/ \vdash P$, $\vdash P \& Q/ \vdash Q$. Following Everett Nelson, McCall's connexive logic includes the latter but not the former and has no deduction theorem. See Everett J. Nelson, 'Intensional Relations', *Mind*, 39 (1930), pp. 440-453. The twelfth century version of connexivism rejected both the theses and the rules of inference. See Christopher J. Martin, 'Embarrassing Arguments and Surprising Conclusions in the Development of Theories of the Conditional in the Twelfth Century' in J. Jolivet & A. De Libera (eds.), *Gilbert de Poitiers et ses Contemporains, 1985*, Bibliopolis, Naples, 1987, pp. 377-401.

[11] Eleonore Stump, 'Boethius's In Ciceronis Topica and Stoic Logic' in John F. Wippel (ed.), *Studies in Medieval Philosophy*, Washington, D.C., 1987, pp. 1-22.

280

about late Peripatetic and Neo-Platonic logic. Much more importantly, I think, an understanding of his writings is essential if we are not to misrepresent great achievements of twelfth century logic as no more than the anticipation of a few of the theorems of PM or S5.

Boethius on the Conditional

A theory of sentential meaning has two basic tasks to perform. It must connect together the various uses to which sentences can be put and it must derive the meanings of sentences from the meanings of their components. In modern times this insight is due to Frege as is the distinction between the propositional content of a sentence and its force. Employing Frege's distinction we can account both for the differences between speech acts in which the same propositional content is presented and for the role of the sentential connectives in the construction of compound propositional contents from simpler propositional contents. Negation, for example, is a function mapping any given propositional content to another which is true if the content negated is false and false if it is true. Peter Abaelard knew this operation as 'destructive', or 'extinctive negation'.

Peter Geach has suggested that insistence on the distinction between force and content be called the Frege Point[12]. That is a good name for it but we should note that it was already known to the Latin logicians of the twelfth century and no doubt also to the Stoics[13]. By a propositional logic I will mean any account of compound propositions and the arguments based upon them which, cognisant in effect of the Frege point, treats at least some sentential connectives as propositional content forming operations on propositional contents. The operations need not be defined truth-functionally in the manner of Frege and Tarski but they must take propositional contents as arguments and yield propositional contents as values. Propositional logics are characterised by a principle of substitution: If 'p' and 'F(p)'

[12] Peter Geach, 'Assertion', Philosophical Review, 74 (1965). The distinction is developed in detail by Michael Dummett in Frege: Philosophy of Language, 2nd edn. Cambridge, Mass., 1981, Chapter 10, 'Assertion'.

[13] For the Stoics see G. Nuchelmanns, Theories of the Proposition, North Holland, 1973, chs. 4-5. To the contrary see W. Kneale and M. Kneale, The Development of Logic, Oxford, 1962, ch. III.4. The Arabs were perhaps aware of the Frege point. Their work on logic has barely been studied but it is clear that they had a sophisticated account of the hypothetical syllogism. See Nabil Shehaby, The Propositional Logic of Avicenna: A translation from al-Shifta: al-Qiyas, Dordrecht, Holland, 1975. After this paper was written there appeared M. Maroth, Ibn Sina und die Peripatetische 'Aussagenlogik' Leiden, 1989.

are propositional contents then so is 'F(q)' where 'q' is any propositional content[14]. Boethius, I claim, does not have a propositional logic and consequently no such principle of substitution. He does, however, have what we might call a *logic for compound propositions*. He provides us with rules for the manipulation of a relatively small number of varieties of compound propositions without an appeal to propositional substitution. The concepts of propositional form, propositional substitution, and propositional connective are inseparable. Boethius has none of them.

Boethius has little to say about the relations between different kinds of speech acts[15] but thanks to Aristotle's work in the theory of knowledge he has an easy time in relating the meaning of at least one sort of simple sentence to the meanings of the words that it contains.

According to Aristotle, a proposition like any other significant expression, is a sign for a mental item called in Latin an *intellectus*, an understanding. Nouns and verbs are causally but conventionally associated with the production by the faculty of intelligence of understandings of the forms in virtue of which things have the features which these words are imposed to name. Propositions differ from other sorts of sentences in being true or false, where truth and falsity arise with the composition and separation of understandings. Boethius maintains that the very same sort of composition takes place in the mind as outside it. On hearing, for example, 'a man is running' we construct in our minds a composite understanding of a substance modified by an accident. Thus:

> When I simply understand man, the substance itself, I hold in cognition nothing true or false. And if I consider running in a cogitation of my mind, that very cogitation because it involves the consideration of a simple thing is separated from truth and falsity. But when I join running and man and from them I make something with my intelligence (and that if I should pronounce it with my voice would be something of this kind: 'a man is running'), then from the composition and conjunction of this substance and accident there comes about the sort of understanding in which there is truth and falsity.
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> <*2IDI*, 1, 42>

Boethius thus supposes that the understanding signified by a simple affir-

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[14] This is not the principle of uniform substitutivity. It captures a feature of well-formedness while uniform substitutivity is, perhaps, a feature of theoremhood. See Krister Segerberg, *Classical Propositional Operators*, Oxford, 1982; section 1.5 on substitution and section 2.2 on substitutivity.

[15] On the very limited work on speech acts done by the Peripatetics and its relation to the theories of the Stoics see: D. M. Schenkeveld, 'Stoic and Peripatetic Kinds of Speech Act and the Distinction of Grammatical Moods', *Mnemosyne*, XXXVII (1984), pp. 291-353.

mation is that of a thing disposed in a certain way and not as we might say an understanding *that* things are such and so. In his commentary on *Metaphysics*, *E*, Christopher Kirwan[16] notices that Aristotle's words suggest just this picture of propositional meaning but he dismisses it as a confusion of concept and belief. To make the distinction that Kirwan wants, however, beliefs and judgements must be construed as propositional attitudes. To do this propositions must be regarded as the expressions of propositional contents and devices must be available for indicating these contents. Just the same machinery is needed if facts and states-of-affairs are to be contrasted with things. The combination in Aristotle's philosophical semantics of an almost exclusive concentration on one-place predicates with the claim that truth and falsity arise with composition and division guaranteed that this way of thinking was slow to develop.

That is not to say, incidentally, that anything in Aristotle's account of meaning is opposed in principle to the notion of facts and it has certainly often been glossed in terms of them. Kenneth Olson in his recent *Essay On Facts*[17] oddly supposes that facts can have no place in Aristotle's world since they cannot be located in one of the ten categories. If they are to be catalogued anywhere, however, it is certainly not there! The *Categories* is concerned only with things said without combination. Facts and states-of-affairs will appear with, or shortly after, the appearance of propositional contents in order to explain their truth-values and logical relations. They play no role in Boethius' account of meaning but along with a propositional logic they are found, I think for the first time in the Middle Ages, in the writings of Peter Abaelard.

Boethius has little to say about negation but what he does say makes it perfectly clear that he does not think of it as a content forming operation on propositional contents. *'Negatio'* is either the name of the negative particle or of one of the two species of simple, or predicative, assertions. In the latter case we might just as well translate the word as 'denial'. A simple affirmation is an assertion of something of something <*alicuius de aliquo*>, a simple denial an assertion of something (as apart) from something <*alicuius ab aliquo*>. Compound assertions are not of anything of anything in this sense and so they are neither affirmations nor negations. We will see latter, however, that Boethius is prepared to assign them a quality.

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[16] Christopher Kirwan, *Aristotle's Metaphysics*, Books Gamma, Delta, Epsilon, Oxford, 1971, pp. 198-200.

[17] Kenneth Olson, *An Essay On Facts*, Stanford, California, 1987.

Following Aristotle in *de Int.* 1, Boethius maintains that a denial signifies the mental act of dividing the understanding signified by the predicate term from that signified by the subject term. Abaelard calls this operation *separative* negation. Boethius interprets Aristotle's description of the verb as 'always a note of something said of something' <*de Int.*, 3, 16b7> as meaning that it is the verb which marks assertion. To alter the kind of assertion made it is thus the verb which must be acted upon and so it is the verb to which the negative particle must be applied.

The association of negation with the verb in the 'basic combination' of subject and predicate is a familiar point which Geach and Strawson have emphasized in their discussions of categorical propositions. Strawson, for example, proceeds from negation as a 'natural' operation on propositions to the assertion of the complementary predicate of the same subject[18]. From this association of negation with the predicate he argues that the mark of propositionality must lie with the verb. Boethius' explanation exactly reverses the order. In a simple assertion the work is done by the verb and so to indicate a different kind of assertion it is the verb which one must act upon[19].

Aristotle introduces negative terms as 'indefinite' names and verbs predicable of everything, whether existent or not, of which the corresponding positive terms are not predicated. With such terms available but parentheses yet to be invented Boethius can mobilise a very simple argument against the Stoics. If we follow them, he says, and prepose negation to get, say, '*non homo ambulat*' we will not be able to tell whether the proposition is an affirmation with an indefinite subject or a negation with a definite subject[20]. Granted that the indication of scope can be difficult, he thus misrepresents the Stoics by implying at least that they agreed with the Peripatetics on the semantics of negation but disagreed on a point of syntax. He is not alone, however, for exactly the same argument is found in Ammonius[21].

Nothing said so far determines the logical connection between affirmations and denials. They are associated with quite distinct mental operations and an argument is needed to show how the truth-values of the signified understandings are related. Chapter 6 of *de Interpretatione* thus becomes the crux

[18] See for example Peter Strawson, *Subject and Predicate in Logic and Grammar*, London, 1974.
[19] E.g. *2IDI*, 5, p. 105.
[20] *2IDI*, 10, pp. 261-2.
[21] Ammonius, 'In Aristotelis De Interpretatione Commentarium', *Commentaria in Aristotelem Graeca*, Vol. IV, Pars V, Berlin 1897, p. 87 ad *de Int.* 7, 17a38-b12.

of the work for Boethius since he construes it as containing a proof that a singular affirmation and the corresponding negation divide truth and falsity between themselves. The affirmation is true if and only if the mental composition is true and that is true if and only if the combination of substance and accident occurs in the world. The negation is true just in case the mental separation is true and that is so if and only if the subject and predicate things are actually separated. Not surprisingly, Boethius is rather vague on this last point. What is important, however, is that the proof of chapter 6 applies only to simple assertions and there is no suggestion that it might be extended to compounds.

Boethius' account of affirmation and negation is exactly the kind of theory which Frege criticized so unmercifully in his essay *Negation*[22]. Without a distinction corresponding to that between force and content Boethius cannot construct the meaning of compound propositions from the meanings of simple propositions. In a simple proposition an unqualified verb signifies combination, negated it signifies separation. In a conditional it cannot do either without turning one compound assertion into two simple assertions. Lacking propositional operations and substitution, Boethius must account for all of the various kinds of compound proposition in turn and he must do so for all combinations of quality among their components[23]. Though he is certainly not clearly aware of these constraints, his treatment of compound propositions does to some extent conform to them. Hence the 'striking and tedious feature' of the exposition noted by the Kneales. In giving his account of compound propositions Boethius rings all possible changes on their components.

My account of Boethius' theory of affirmation and negation is confirmed by his remarks on the copulative connective 'and'. It does not produce one proposition from two, he claims, but rather is simply expletive. As usual Ammonius provides a pleasant analogy: the copulative conjunction yields only an accidental, or syntactic, unity 'in the manner of a bundle which is

[22] Translated by P. T. Geach and R. H. Stoohoff, in Gottlob Frege, *Logical Investigations*, Oxford, 1977.
[23] Exactly the same problem arises, for example, for the account of affirmation and negation offered by Russell in 'On Propositions: What they are and how they mean'. By taking negation to be part of the force rather than the content of a proposition he precludes the truth-functional account of propositional connectives which he typically appeals to elsewhere. Russell's paper is reprinted in Bertrand Russell, *Logic and Knowledge*, R.C. Marsh (ed.), London, 1956, pp. 285-320.

said to be one in virtue of the wrapping containing many plants'[24]. This observation ought to be embarrassing to historians of logic since the categorical syllogism is sometimes presented by Aristotle, and always by Boethius, in the form of a conditional with a copulative antecedent. From what I have said it is clear that it is inappropriate to translate it uncritically into the symbols of a propositional calculus. Boethius says nothing about conditionals with copulative antecedents and so we must suppose that he did not regard them as logically significant compound propositions.

In presenting the theory of the hypothetical syllogism Boethius introduces four kinds of simple conditional, thirty two kinds of compound conditional and twenty four kinds of what he calls *mediate conditionals*. He refers to 'it's X' and 'it's not X' as simple propositions. 'If A, then B' is a simple conditional if both 'A' and 'B' are simple propositions in this sense and a compound conditional if one or both are simple conditionals. Mediate conditionals consist of pairs of simple conditionals and their theory is that of the transitivity of the simple conditional connection between predicates[25].

Boethius holds that the affirmation '*homo est*' conjoins man and existence and is true just in case a man is among the things which exist[26]. If we were allowed to read '*si est homo, est animal*' propositionally it would thus connect the existence of a man with the existence of an animal. That this cannot be the intended interpretation is clear from the way in which Boethius speaks of a consequence as holding with respect to an individual, as in 'Tully, if he's a man, is an animal', and from examples of true conditionals such as 'if it's a man, then it's not a horse'. Furthermore, in describing simple conditionals Boethius repeatedly speaks of the *term* B following from the term A or of the being of the term B following from A. He clearly thinks of the consequence as connecting A and B as predicates, or what we would call the properties of being (an) A and being (a) B. The only exception in his presentation of the theory of the conditional seems to be the single example given in *DHS* in which the antecedent and consequent are, perhaps, events: 'if there should be an interposition of the earth, an eclipse of the moon would follow' <*DHS* I, iii, 7>. Boethius also often cites as examples stock Stoic conditionals such as 'if it's day, then it's light'

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[24] Ammonius, *op. cit.* p. 73, *ad de Int.* 5, 17a15-17.
[25] From what I have said about propositionality it follows that the theory of mediate hypotheticals cannot be that of the wholly hypothetical syllogism: $\vdash P{\rightarrow}Q$, $\vdash Q{\rightarrow}R/\vdash P{\rightarrow}R$. This is how Jonathan Barnes construes it in his *Terms and Sentences: Theophrastus on Hypothetical Syllogisms*, British Academy, London, 1984.
[26] E.g. *2IDI*, 3, p. 77.

286

and 'if it's day, then it's not night'. He seems to suppose that such meteor-ological impersonals are to be explicated in exactly the same way as conditionals such as 'if it's a man, then it's an animal'.

Boethius speaks of the truth of conditionals on only a couple of occasions and never of false conditionals. Since they are propositions, however, conditionals must have truth-values. Failure to mention them may simply reflect the habit that ancient logicians have of taking as examples only true propositions and sound arguments. A necessary condition for the truth of the simple conditional 'if it's A, then it's B', or as Boethius says, for the existence of an immutable consequence, is that being B is inseparable from being A:

> Those alone are opposed to hypotheticals which destroy their substance. The substance of hypothetical propositions lies in this: that the necessity of their consequence is strong enough to persist. If someone therefore would properly oppose a conditional, he should bring it about that he destroys the consequence. Just as when we say 'if it's A, then it's B', we will not resist this either by showing A not to be or B not to be but rather <by showing that> if A is posited it does not follow that B is but that A may be even though the term B is not.
> <DHS, I, ix, 5-7>

Boethius seems to have regarded inseparability as both necessary and sufficient for the truth of a conditional. Various scattered remarks that he makes about the antecedent and consequent of consequences 'coming together in a single understanding' suggest, however, that he thought that some sort of conceptual connection is associated with the *consequence of nature* expressed in a true conditional. He distinguishes the accidental inseparability of fire's being hot and the heavens' being spherical from the inseparability found in a consequence of nature such as that connecting being a man with being an animal. The accidental conditional 'if fire is hot, the heavens are spherical' is, as far as I know, the only example that he gives in which antecedent and consequent are complete indicative propositions with genuine, and different, predicates and subjects. It fails to express a consequence of nature since fire's being hot does not explain the spherical-ness of the heavens nor their sphericalness the heat of fire <DHS, I, iii, 6>. It just so happens that the heavens are spherical and fire is hot at all the same times.

The distinction between accidental conditionals and consequences of na-ture of course recalls that between the accidental and the *per se* which plays

such a crucial and varied role in Aristotle's philosophy. The addition to inseparability of the requirement of conceptual connection suggests a distinction made in Porphyry's *Isagoge* between what we might call strict and conceptual inseparability. Inseparable accidents or properties are strictly inseparable from their subjects in that although they cannot be physically removed, their subjects can be conceived without them. Species, genera, differentiae and so definitions are in addition conceptually inseparable in that their subjects cannot even be conceived without them. It is this contrast which will form the basis of Abaelard's distinction between the theory of argument and the theory of the conditional.

It is important to emphasise that the truth-conditions of conditionals must be stated generally. 'If it's an ass, then it's rational' is false and so false with respect to Cicero. If inseparability were construed as a relation between the truth-values of propositions and true conditionals as strict implications then, since it is impossible for Cicero to be an ass, the conditional 'if Cicero is an ass, then he is rational' would be true. For Boethius there is no possibility of vacuous truth and no possibility of Lewis' arguments. Both of them appear for the first time in Latin logic at some time in the twelfth century[27].

So far I have argued that Boethius cannot treat sentential connectives as a propositional content forming operator on propositional contents. I turn now to the various claims that have been made about his so called propositional logic.

The Commentators

To begin with there is one place at which Boethius does seem to employ extinctive negation and where he describes himself as preposing negation to a compound proposition. It is in his commentary on Cicero's *Topics* in a discussion of what he calls the *locus conditionalis*[28].

Boethius' advertisement for the theory of the dialectical topics is irresist-

[27] See Christopher J. Martin, 'William's Machine', *Journal of Philosophy*, vol. 83, 1986, pp. 564-572. Avicenna does discusses the effect of impossible antecedents upon consequence but his account of the conditional will also not permit vacuous truth. See Shehaby, *op. cit.*, pp. 70-72.

[28] *ITC*, II, 1075C-1078B; IV, 1124B - V, 1145B.

ible. It will show us how to easily find justifiable answers to dialectical questions. That is to questions of the form: 'is A a B or not ?'. We need what Cicero calls an *argumentum*: 'a reason yielding conviction with respect to something in doubt'[29]. To find one we must consider certain features of A and B. From the properties of their definitions, integral parts, and the things to which they stand in certain specified relations we will be able to infer probably that A is or is not B[30]. As a last resort we can appeal to something which has no intrinsic connection with A or B but can still give a probable account of their relationship. For example where the question is about the stars we might cite the authority of astronomers.

The sorts of things that we should consider in trying to answer the question constitute the topical differences. They are the *logical* places to which we go in search of *argumenta*. What we find there are general principles called maximal propositions connecting topical differences to the things about which questions are asked. They can thus warrant inferences from premisses to a conclusion. For Aristotle there is no need for such warrants since a dialectical syllogism is a syllogism in the canonical sense of the *Prior Analytics* and dialectical in so far as its premisses are probable. Boethius on the other hand claims that dialectics is concerned with probable *argumenta* and is, and was, most plausibly read as locating probability in the connection between premisses and conclusion.

To answer our dialectical question we might, for example, cite the definitions of A and B and the maximal proposition that things with different definitions are different <*TD* II, 1185C>. Alternatively we might draw attention to the efficient cause of A, notice that it has the property of being B, and appeal to the maximal proposition that whenever an efficient cause is B so is its effect <*TD* II, 1189C>. Although Boethius could not do so, we can represent the use of topical difference and maximal proposition in an

[29] *ITC*, II, 1048A-1054B. Although Boethius also allows conditional dialectical questions of the form 'is it a B if it's an A or not ?' in *TD*, in his discussion of the topics he shows only how they are applied to answer predicative questions.

[30] '*Probabilis*' and '*verisimilis*' are Boethius' translations of 'ἔνδοξος'. A probable, or reputable, *argumentum* is one 'which seems to be so to everyone or to the majority or to the wise, and of these either to all or to the majority or to the best known and leading ones amongst them, or one which seems to be so to a skilled person in his own field, as in medicine to a doctor or in the pilotage of ships to a pilot, or, finally, that which seems to be so to him with whom one is speaking or to him who is judging' <*TD* 1181C-D>.

argument as follows[31]:

> Whenever two things stand in the topical
> relationship T if one of them is related   [Maximal
> in way P to a third thing D, then the other   Proposition]
> is related in way Q to a fourth thing B.

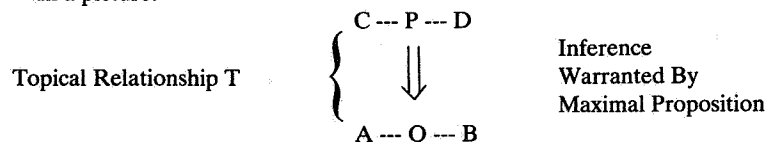> A and C stand in the topical relationship T.   [Major Premiss]

> C is related in way P to D.   [Minor Premiss]

> ∴A is related in way Q to B.

Here 'A' and 'B' may each be either the subject or predicate term of the dialectical question. 'Things' may include collections such as the integral parts of a whole or all the species of a genus. The theory of the topics thus embraces some of the valid forms of argument notoriously ignored by Aristotle. It is limited, however, by the list of topical relations and Boethius argues that this list is complete. There is no logical place, for example, in which we might seek support for a relational argument such as 'C is bigger than B, B is bigger than A; therefore C is bigger than A'. The theory of topics also competes with the theory of the syllogism in the sense that *argumenta* like that *from definition* mentioned above are expressed in arguments which may be construed as categorical syllogisms.

Boethius refers to the connection between the premisses of syllogism and its conclusion as he does to that between the antecedent and consequent of a true conditional as a *consequence*. Without argument he sometimes in effect adopts a principle of conditionalization and states only the maximal proposition and the conditional formed from the minor premiss and the conclusion. Maximal propositions thus appear as principles which may be cited in support of conditionals. This at any rate is how Abaelard understands them and since for him true conditionals express 'laws of nature', maximal propositions become the first principles of science. They provide axiom schemata for the theory of natural kinds. Abaelard is forced to deny this elevated status to many of Boethius' maximal propositions, however,

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[31] In a picture:

Topical Relationship T   {   C --- P --- D   Inference
                                ⇓   Warranted By
                          A --- Q --- B   Maximal Proposition

since the inferences that they warrant, though probable, are not necessary. The hard work lies in settling just which maximal propositions do express truths about natures.

The *locus conditionalis* contains three topics said by Cicero to be the peculiar property of dialecticians. Going to it we find that *argumenta* may be drawn from an antecedent, from a consequent, and from a repugnant. According to the theory, a topical argument based on the first of these will look something like this:

The consequent belongs to whatever the antecedent belongs.	[Maximal Proposition]
A is antecedent to B.	[Major premiss asserting topical connection]
A belongs to X.	
∴ B belongs to X.	

In Boethius' gloss on Cicero's example of the locus from an antecedent all these elements do indeed appear. In stating the argument, however, he ignores the maximal proposition and gives the major premiss as a simple conditional[32]. The result is in effect an instance of the Stoic First Indemonstrable justified with an appeal to the maximal proposition! Boethius seems completely unaware of perhaps the most basic claim of Stoic logic. The presentation of *modus ponens* in this way in conjunction with the appeal to the topics in the demonstration of conclusions for which categorical syllogisms are available led in the twelfth century to considerable disagreement over the relative priority of two branches of logic. Boethius' remark that topics are suited to syllogisms <*TD* 1173C> was quite reasonably understood as making the theory of the syllogism depend upon that of the topics.

The argument from repugnants given by Cicero is a version of the Third Indemonstrable which has for its major premiss a proposition of the form 'not both P and not Q'. In his account of the argument Boethius does two things. Firstly, as we would expect, he takes repugnance to be a relationship between properties. Secondly he insists that since we are dealing with the *locus conditionalis* the conditional connective should appear in the major premiss of the argument. He thus replaces Cicero's copulative conjunction with the 'causal' connective 'if' in order, as he says, to show whi· ·nd of proposition is being used. He goes on to remark that this is justified since

[32] Eg. *ITC*, 1076A-C.

"nothing is the equal of the conjunction 'if' for showing connection, though the copulative conjunction 'and' may be used to the same effect" <*ITC*, V, 1140C>. Given his views on the conditional and copulation this remark must be understood simply as an identification of the truth-conditions of the conditional with those of the negated conjunction. There is a consequence just in case the antecedent is inseparable from the consequent.

As far as I know Boethius is the only classical Latin writer who discusses the meaning of a conditional to which a negation has been preposed. The construction is certainly uncommon, it is not mentioned by the grammarians, and we cannot assume that the negative particle is naturally read as a propositional operator with a particular scope[33].

For Boethius two things are repugnant if one of them is a contrary to something which follows from the other. Since being awake and being asleep are contrary and if you are snoring, then you are asleep, being awake and snoring are repugnant to one another. The definition suggests that the Third Indemonstrable provided the inference rule for eliminating a subdisjunction represented as a negated conjunction. The subdisjuncts may be false together but they cannot be true together. Given the truth of the subdisjunction one may validly argue only from the assertion of one disjunct to the denial of the other and so the Third Indemonstrable stands alone where the rest are paired[34].

Boethius introduces a *repugnant proposition* as a conditional proposition in which the antecedent and consequent terms are repugnant to one another. In a true conditional the relationship between antecedent and consequent is that of a consequence of nature. In a false repugnant proposition, and of course all repugnant propositions are necessarily false, the relationship is that of repugnance. It is impossible for the antecedent and the consequent to apply to the same thing at the same time. Thus 'if it's awake, then it's snoring' is a repugnant proposition as is 'if it's a stone, then it's an animal'. A repugnant proposition can always be formed from a consequence by negating its consequent. From the consequence 'if it's day, then it's light,

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[33] In Boethius' translation of Porphyry's *Isagoge*, we find '*neque si animal est, homo est, neque si animal est risibile est*' used to deny that the consequents follow from the antecedents. In his commentary, however, B. does not mention the conditionals. See *In Porphyrium Commentaria*, PL 64, p. 143D sq.

[34] Is Boethius' account of repugnance here taken from the Stoics? For a different theory of the Third Indemonstrable - as a negated material copulation - see Michael Frede, *Die Stoische Logik*, Gottingen, 1974, pp. 148-53, 160-62.

we get the repugnance 'if it's day, then it's not light' and so on. Boethius describes the relation between a consequence and the corresponding repugnance in terms of the truth-value of their consequents with respect to their antecedents. So 'it's true and it follows, if it's day, that it's light and it's false and repugnant, if it's day, that it's not light' <eg. *ITC* V, 1134B>.

Repugnance exists between the antecedent and consequent of a repugnant proposition only because the original conditional is true. There is absolutely no suggestion that the negative particle is inserted to produce a proposition which stands in a particular logical relationship to the original no matter what that might be. The aim is rather to obtain a proposition which contains the conditional connective and which in some way manifests a repugnance. Of course, according to Boethius' theory, no pair of propositions of the form 'if it's A, then it's B' and 'if it's A, then it's not B' can be true together. Again, he does not speak of truth and falsity but he nevertheless takes a consequence and the corresponding repugnance to be contrary and so expressed by propositions of the same kind. Both are conditionals.

Boethius rings all the changes on the varieties of simple conditional to obtain the corresponding repugnant propositions. For example: 'if it's a man, then it's not an animal', 'if it's not day, then it's not night' and 'if it's awake, then it's snoring'. Since he believes that a conditional of the form 'if it's not A, then it's B' holds only of immediate opposites, that is things such as day and night which can neither both be present nor both be absent at the same time, he also allows that to the consequence 'if it's not day, then it's night' there corresponds the repugnance 'if it's day, then it's night'.

As they stand repugnant propositions are useless in arguments since they are bound to be false. However, by preposing the negative particle to a repugnance with an affirmative antecedent and a negative consequent Boethius obtains a proposition which he claims is in all respects the same as the original consequence:

> For because it is understood to be consequent and true, if it's day, that it's light, it is repugnant and false, if it's day, that it not be light. Which denied it is once more in this way true: 'not if it's day, then it's not light'. Thus it is alike in all respect <*consimilis*> to the affirmation: 'if it's day, then it's light' since a doubled negation makes an affirmation. Similarly *argumenta* are obtained from propositions with repugnant parts if there are conjoined two negations, a negation and an affirmation or an affirmation and a negation.
> <*ITC* V, 1134B>

It is quite clear, I think, that Boethius is not claiming that by preposing a negative particle to a conditional proposition expressing a repugnance one thereby obtains a proposition true merely because the original is false. He says that the sense of 'not if it's day, then it's not light' is that if it's day, then it may not be that it's not light[35]. What becomes once more true is not the conditional but its consequent, if its antecedent is true. The result is 'in all respects the same' as the original because the double negation of a simple proposition returns the same simple proposition. Boethius, incidentally, says nothing at all about how it can do this. As Frege remarks, negation must be like a sword which on the second strike mysteriously joins together what was severed with first.

Some confirmation of all this comes from Boethius' treatment of the negation of the second repugnance obtainable from a consequence between immediate opposites. Starting with 'if it's not day, then it's night' we get the repugnance 'if it's day, then it's night'. Preposing negation we arrive at 'not if it's day, then it's night' from which with 'it's day' we may infer 'it's not night'. Presumably Boethius doesn't mention an argument from 'it's not day' to 'it's night' because there isn't one. The negated conditional is entirely similar to 'if it's day, then it's not night'. Preposing negation does not in general return the original consequence from the corresponding repugnance but serves rather simply to transform the quality of the consequent.

Boethius thus, as it were, reads the preposed negative particle as separated from the conditional by a comma rather than a colon. His remark that the negation applies to the whole proposition is not an observation about its scope but merely about the order of the terms.

As I mentioned at the beginning, Eleonore Stump has recently suggested that Boethius be sought out for his opinions on Stoic logic. She represents Boethius' version of the Third Indemonstrable propositionally and suggests that he might have understood 'it is not the case that if p, then not q' as 'expressing the negation of' the incompatibility of 'p' and 'not q'[36]. Stump apparently understands Boethius as stipulating that 'p' and 'not q' are incompatible just in case they cannot be true together. If that is so, however, then she seems bound to read 'it is not the case that if p, then not q' as

[35] *ITC*, V, 1134A.
[36] Stump, *op. cit.*, p. 16. Stump notes, however, that Boethius' words seem to suggest that he supposes that the two negations cancel out.

saying that 'p' and 'not q' can be true together! This is hardly compatible with Boethius' claim that 'not if it's A, then it's not B' is equivalent to 'if it's A, then it's B'.

It seems to me that Boethius is bound to be a particularly bad witness for Stoic logic and that this has nothing to do with his access to their works. You cannot understand what a propositional logic is if you do not have the concepts of propositional content and propositional operations. Boethius has neither.

At the beginning of *DHS* Boethius declares that he is the first to write in detail about hypothetical syllogisms. The Stoics, he tells us, produced nothing at all and among the Peripatetics only Theophrastus and Eudemus made even the barest beginnings. The first part of this claim is of course outrageously false though as we have seen Boethius may well have known nothing of the range of Stoic logic. The second part is curious and certainly some doubt is cast on Boethius' originality by the similarity of his account of the conditional to that given by Avicenna[37].

Although he does not prepose the negative particle to a compound proposition in *DHS* Boethius does speak of negative conditionals and seems to assume that 'if it's A, then it's B' and 'if it's A, then it's not B' divide truth and falsity between themselves. On the first point, the claim made by both the Kneales and Jonathan Barnes that the negative conditional is the propositional negation of the affirmative conditional is easily refuted. The Kneales maintain that Boethius says that the negative *of* 'if it's A, then it's

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[37] Boethius was almost certainly not a source for Avicenna. They agree for example on the distinction between *per se* and accidental conditionals though Avicenna makes a much clearer distinction between strict and conceptual inseparability. See Shehaby, *op. cit.*, esp. pp. 37-38. Unlike Boethius both al-Farabi and Avicenna seem to have the concept of propositional content and to use the conditional connective as a propositional operation. Avicenna clearly distinguishes the negation of a conditional from the negation of its consequent <*op. cit.* pp. 57-58>. At one point he criticises a work on hypothetical syllogisms which sounds in some ways similar to *DHS*. In particular its author thought that the quality of a conditional is determined by the quality of its consequent. The work was apparently attributed to Alexander of Aphrodisius but on the basis of what he considers to be its inept treatment of the conditional and hypothetical syllogisms Avicenna thinks that it could not be the work of that 'most excellent among later scholars' <*op. cit.* pp. 159-60>. Despite the similarities between their accounts of the conditional Boethius' and Avicenna's discussions of the hypothetical syllogism have little in common.

B' is 'if it's A, then it's not B'[38]. Barnes gives part of a passage which I will quote in a moment and on the basis of it concludes that Boethius holds that the contradictory negation of 'if P, then Q' is 'if P, then not Q' and that "he thus in effect assimilates 'if P, then not Q' to 'it is not the case that (if P, then Q)'. And that assimilation underlies and determines the form of his hypothetical syllogistic"[39]. This is wrong, I think, on three counts. Firstly the representation of Boethius' claims in terms of propositional operators is inappropriate. Secondly Boethius does not hold that the consequent negation of a conditional even in an appropriate form is its contradictory negation. Thirdly such an identification does not underlie his account of hypothetical syllogisms.

The remarks reported by the Kneales and Barnes are found at the end of a discussion of the logical relations between propositions. If he were going to introduce the negation of compound propositions anywhere it is surely here that Boethius would do so. He does not. He considers rather the appropriate way to negate simple propositions with various modal qualifications and calculates how many different kinds of conditional proposition there are from the number of kinds of simple proposition which may occur as antecedent or consequent. Finally he comes to the question of how one should oppose a conditional proposition. Continuing the quotation begun on page 287:

> Or if the conditional is negative, it is destroyed in the same way: as when we say 'if it's A, then it's not B' it isn't to be shown either that A is not or B is but rather when A is that the term B is able to be. Some hypothetical propositions are affirmative others negative ... affirmative as when we say 'if it's A, then it's B' <or> 'if it's not A, then it's B'; negative <as> 'if it's A, then it's not B' <or> 'if it's not A, then it's not B'. The consequent proposition is to be referred to in deciding whether a <conditional> proposition is affirmative or negative.
>
> <DHS I, ix, 5-7>

This seems to me to be entirely unambiguous. We show that a conditional sentence fails to express a consequence by showing that its antecedent can apply when its consequent does not. Boethius cannot give us a proposition which divides truth and falsity with a conditional because copulation is not for him proposition forming and, though we have not discussed this, he understands possibility *de re*. He could provide a proposition expressing inseparability which does not employ the conditional and he does so in *ITC*. Negated copulations are not mentioned in *DHS*, however, perhaps because

[38] William and Martha Kneale, *The Development of Logic*, Corrected edn., Oxford, 1975, p. 191.
[39] Barnes, 'Boethius and the Study of Logic', p. 83.

296

they are thought not to express the required explanatory connection.

Nothing in Boethius' remarks supports the Kneales' claim that one conditional is the 'negative of' another and Boethius never speaks in this way. What he offers is rather a classification of the quality of conditionals. Just the same sort of classification as that which designates 'A is B', 'every A is B' and 'some A is B' as affirmative and 'A is not B', 'not every A is B', 'no A is B' and 'some A is not B' as negative. Work has to be done to show that a particular pair divides truth and falsity.

The reason for classifying conditionals in this way is perhaps a desire that they should satisfy the standard description of a proposition as an expression signifying what is true or false and conform to Aristotle's principle that truth and falsity have to do with combination and separation. 'If it's A, then it's B' signifies a connection between being A and being B, 'if it's A, then it's not B' a separation of the one from the other. One way to destroy the substance of the first conditional would of course be to confirm that the second is true. They are contrary but there is no reason so far to suppose that Boethius regarded them as contradictory. The only use that he makes of the classification is to appeal to it in extending to compound conditionals his claim that there is consequence between a negative antecedent and an affirmative consequent if and only if they are immediately opposed.

One reason that has been given for supposing that Boethius thinks that the contradictory negation of a conditional is obtained by changing the quality of its consequent is his treatment of the figures and moods of the hypothetical syllogism in *DHS*. He has, as I said, been construed as accepting both $\models P \to (Q \to R)$, $\models Q \to -R/ \models -P$ and $\models (P \to -Q) \to R$, $\models -R/ \models P \to Q$ and so as committing himself to C.E.M. However, if anything is clear from his treatment of the hypothetical syllogism it is that it is not based on this principle.

I have argued that Boethius can have no recourse to propositional substitution and so he cannot set down single schemata for affirming the antecedent and denying the consequent. What we find in *DHS* amply confirms this. Seventy tedious pages are devoted to examining what he claims to be all the possible forms of hypothetical syllogism. Each variation on the quality of the component propositions in all the acceptable forms of conditional is considered in turn. In particular Boethius demonstrates many times over why we are not allowed to affirm the consequent. Thus he insists

that from 'if it's A, then it's B' and 'it's B' 'it's A' does not follow since, for example, something which is an animal may or may not be a man. C.E M. fails here and over and over again. We must seek a different explanation for the apparent peculiarities in the catalogue of hypothetical syllogisms.

The first thing to notice is Boethius' insistence that a conditional sentence can be a conditional proposition only if it contains a genuine condition. We have seen that there can be no vacuously true simple conditionals but the requirement of strict inseparability could obviously be met trivially by taking either a true conditional as a consequent or a false one as an antecedent. Boethius will not allow that the first sort of compound is a conditional 'since the apposed condition does not seem to bring about the necessity of the consequence' <*DHS* II, iv, 5>. Oddly, however, in his discussion of conditionals of the form 'if (if it's A, then it's B), it's C', he requires not that the antecedent not be false but that it not be true. Perhaps the worry here is not vacuity but rather that the existence of a consequence could never be a condition of the truth of a simple proposition. The major premisses of the first hypothetical syllogism of the second class, 'if it's A, then (if it's B, then it's C)', must thus be such that being B is separable from being C. That of the first hypothetical syllogism of the third class, 'if (if it's A, then it's B), then it's C', must be such that being A is separable from being B. Both requirements are imposed on the major premisses of the fifth class of syllogisms.

Compound conditionals are peculiar sorts of constructions and it is debatable whether those with a conditional antecedent make much sense in English. The problem with Boethius' conditionals, however, is not so much the sense that he supposes them to have but rather the rules of inference that he thinks proper for them. Since the consequent of a class 2 conditional is necessarily false it seems that an instance of *modus ponens* in which it is detached from its true antecedent will have a false conclusion and so be invalid. The reply, I suppose, has to be that what is detached is not the general conditional 'if it's A, then it's B' but rather the instance 'x is such that if it's A, then it's B'. Since the corresponding general conditional is false the particular conditional cannot express an explanatory connection but must rather indicate the inseparability in x of being A from being B. That is to say that while the major connective indicates the explanatory connection of a consequence of nature, the embedded conditional can stand only for accidental inseparability.

This all agrees with Abaelard's analysis of Boethius' hypothetical syllo-

gisms[40]. However, since Abaelard in effect construed them all as instances of the rules for eliminating propositional conditionals he could not allow what he took to be the misleading representation of the negation of a conditional as its consequent negation. To the contrary, I think that we can perhaps grant this once we understand the further restrictions that Boethius imposes on the general terms which may be substituted into his schemata. Thus in giving an account of the second figure:

> Let us run through the propositions in turn and consider their individual natures in this way: The first proposition with which it is asserted 'if it's A, then if it's B, it's C' has to be such that B may indeed be without A, if, however, A were to be, B would not be able not to be, on the other hand the same term B may also be when the term C is not, and so it is not necessary that with B posited C also is. It is only necessary to be C when the term B follows from the term A, as when A is man, B animate, C animal. For animate may be apart from man and apart from animal; if it's a man, however, it is necessary that it's animate, and since animate follows the being of man, it follows that that very animate <thing> is an animal.
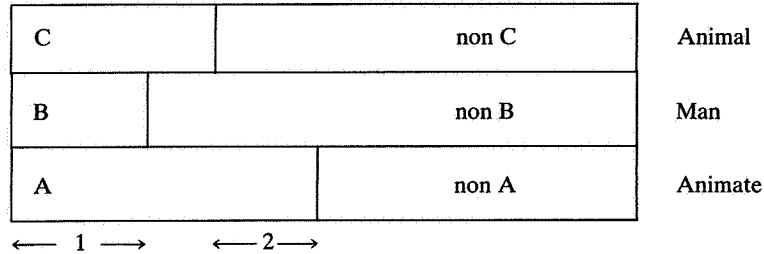> <DHS II, iv, 6>

So in terms of inseparability we have syllogism in this mood just in case (1) being B is separable from being A, (2) being A is inseparable from being B, (3) being B is separable from being C, (4) being C is inseparable from the inseparability of being B from being A.

It is a pity Boethius did not include diagrams in his work. The appropriate one in this case is:
The diagram shows the relative extension of the general terms A, B, and C. To guarantee the required inseparability the terms must be chosen appropriately. Since the principle connective is to mark an explanatory connection they must all belong to the same category. Boethius draws all his terms for second and third figure syllogisms from the category of substance. Here, A man, B animate, and C animal. The conditional holds because for any individual which is an A, that is to say which falls into region 1 of the

C	non C	Animal
A	non A	Man
B	non B	Animate

←— 1 —→   ←—2 —→
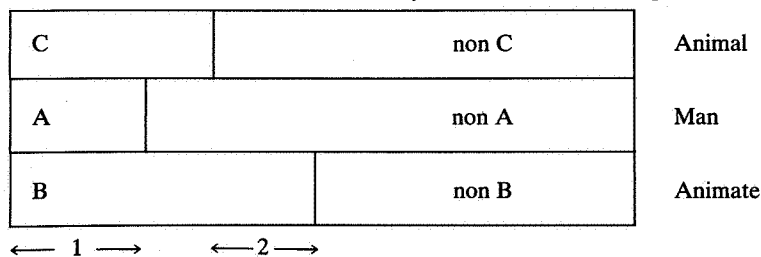
[40] Petrus Abaelardus, *Dialectica*, L. M. De Rijk (ed.), 2nd edn., IV.2, p. 505 sq.

C		non C	Animal
B		non B	Man
A		non A	Animate

←— 1 —→   ←—2—→

diagram, being B is inseparable 'in' it from being C. Thus since Socrates is a man, being an animal is inseparable 'in' him from being animate. The major premisses of the remaining syllogisms of figure 2 are obtained by ringing all the changes on the quality of the terms. The constraints remain the same save that for a conditional with a negative antecedent and an affirmative consequent Boethius uses two terms which necessarily coincide in extension. In those cases affirmation of the antecedent and denial of the consequent is allowed in virtue of the *complexio propositionum* and affirmation of the consequent and denial of the antecedent in virtue, he says, of the *natura rerum*.

Given that he accepts the second class of hypothetical syllogisms only for terms related in this way, we can see a justification for concluding 'it's not A' from 'if it's A, then (if it's B, then it's C)' and 'if it's B, then it's not C'. If something is such that its being B is inseparable from its not being C, it must lie in region 2 and so it cannot be A.

Boethius imposes exactly the same kinds of restriction upon the major premisses of the syllogisms of the class 3. The appropriate diagram for its first mood is:
If something is such that being animate is inseparable in it from being a man, then it falls into region 1 of the diagram and so it must be an animal. However, if something is not an animal it certainly does not follow that being animate is inseparable in it from not being a man. It might well not be animate at all. Either Boethius is mistaken or has yet another restriction in mind. Here is how he summarizes the third class of syllogisms:

> In all of the syllogisms described above this is the principle <*ratio*> at work: if the term B is assumed just as it is posited in the major premiss, then the term C is concluded in just the same way as it was set down in the major premiss. But if the term C is assumed in the contrary manner to that in which it was posited in the major proposition, in a contrary manner the term B will be demonstrated in the conclusion.

<*DHS*, II, viii, 5>

300

No mention here of the term A and Boethius seems to assume its applicability throughout the argument. He reads the compound conditional, as it were, as 'if something, which is A, is B, then it is C'. This is just what he needs since if we agree to restrict our attention to As and know that being C is inseparable from the inseparability of being A from being B, then if we discover that one of our A's is not a C we can conclude that it is necessarily not a B. The argument is valid and it doesn't rely on C.E.M. We could take into account the restriction to As by adding an appropriate premiss to the syllogism and suppose that Boethius does not do this because he holds that a syllogism has only two premisses. There may, however, be more to compound conditionals than he has told us. There is certainly something peculiar about his account of the fifth figure. Here he gives as examples conditionals like 'if (if it's a man, it's a physician), then (if it's animate, it's skilled)'. Being a man is surely, however, never inseparable from being a physician and so a syllogism with this leading premiss can have no application. On the other hand it is easy to draw up a diagram for four terms from a single category and to read off a conditional satisfying Boethius conditions which might yield a sound argument. For example 'if (if it's mortal, then it's a man), then (if it's a body, it's an animal)'. The fifth figure of the syllogism provides further evidence if needed that Boethius is not at all concerned with what we would regard as paradigms of logical relations. On a number of occasions in *DHS* he proves the principle of contraposition: 'if (if it's A, then it's B), then (if it's not B, then it's not A). According to his account of such propositions, however, if the antecedent, and so its consequent, are true the conditional could not appear as the major premiss of a hypothetical syllogism <*DHS* IX, 1> !

Though Boethius' theory of the compound hypothetical syllogism is a little odd and its hard to conceive what use such arguments could be, it seems clear that it does not rest on C.E.M. There is one place, however, where, following Aristotle, he does seems to appeal directly to the principle.

In his discussion of the relations between modalities in *de Interpretatione*, 13, Aristotle claims that from '- necessary to be -' there must follow '- possible to be -' for otherwise the negation, '- not possible to be -' would have to follow[41]. He goes on to argue that:

[41] *2IDI* 13, pp. 432-446.

if - necessary to be -, then - possible to be -

if - possible to be -, then - not impossible to be -

if - not impossible to be -, then not necessary to be -

∴ if - necessary to be -, then - not necessary to be -

The conclusion is, as Boethius says, inappropriate, or embarrassing (*inconveniens*). It is avoided in *de Interpretatione* by giving up the assumption that '- not necessary to be -' follows from '- possible to be-'. What follows rather is '- not necessary not to be -'.

Despite the appearance I think that what we have here is not an appeal to C.E.M. but rather a *reductio* relying straightforwardly upon the Law of Excluded Middle for categoricals. The argument goes as follows: Suppose A is necessarily B. Like everything else, A is either possibly B or else not possibly B. Let us suppose that it is not possibly B. It is thus both necessarily B and not possibly B. But that is impossible. So if A is necessarily B, then it's possibly B. The inference holds independently of the choice of A and B. The argument seems to assume what it has to prove, the impossibility of being both necessarily B and not possibly B, but that is not the point. In general we get nowhere by supposing that A is B and noting that it either is C or is not C. Incidentally, the argument is also presented in this way, as a *reductio*, by Ammonius[42]. Since nothing else that Boethius says indicates a commitment to C.E.M. and much that he says is against it we surely should not attribute it to him when such a plausible alternative is available.

The embarrassing conclusion of Aristotle's argument recalls that of another argument the appearance of a version of which in *DHS* was absolutely crucial for the development of the theory of the conditional in the twelfth century. In the second book of the *Prior Analytics* <II.4, 57a13sq>. Aristotle famously argues that the same conclusion cannot follow from two pairs of premisses one of which consists of the negation of the others. The argument is famous because Aristotle seems to employ propositional variables, to rely on principles of propositional logic, and to come to grief because he ignores the distinction between the negation of a conjunction and a conjunction of negations[43].

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[42] Ammonius, *op. cit.*, pp. 236-238.
[43] See Peter Geach, 'Aristotle on Conjunctive Propositions' in *Logic Matters*, Berkeley, 1980.

Boethius does none of these things. He abruptly remarks in the middle of his discussion of the conditional that the time has come to explain Aristotle's claim that 'it is not necessary that the same is both if something is and is not'. He interprets this as the claim that 'if it's A, then it's B' and 'if it's not A, then it's B' cannot both be true. Here is how he proves it:

1. if it's A, then it's B	Assumption.
2. if it's not B, then it's not A	1., Contraposition.
3. if it's not A, then it's B	Hypothesis.
4. it's not B	Hypothesis.
5. it's not A	2, 4, MP.
6 it's B	3, 5, MP.

Thus if both 'if it's A, then it's B' and 'if it's not A, then it's B' were true being B would follow from not being B. This Boethius regards as an impossibility. Rightly so on his account of the conditional since 'B' is a simple general term.

An exactly similar argument can be constructed to show that the truth of 'if it's A, then it's B' is incompatible with that of 'if it's A then it's not B' provided that not being A cannot follow from being A. As I mentioned at the beginning Storrs McCall has dubbed the principle of propositional logic $\vdash (P \rightarrow Q) \rightarrow -(P \rightarrow -Q)$ Boethius' Thesis. I strongly object. For all the reasons I've given this principle has no place in Boethius' logic. Furthermore we know who stated it first and provided the proof. It was Peter Abaelard, an infinitely finer logician and someone whose name should be honoured wherever possible. Unfortunately the principle was to prove his logical undoing. Unlike Boethian logic, Abaelardian logic includes propositional operations on propositional contents. In particular propositional negation and propositional copulation. They combine with his principle that a proposition cannot entail its own negation to yield an explosive mixture[44]:

1. $(A \& -A) \rightarrow A$	Simplification.
2. $(A \& -A) \rightarrow -A$	Simplification.
3. $-A \rightarrow -(A \& -A)$	1., Contraposition.
4. $(A \& -A) \rightarrow -(A \& -A)$	2., 3., Transitivity.
5. $-\{(A \& -A) \rightarrow -(A \& -A)\}$	Abaelard's Principle.

Propositional logic has been discovered perhaps three times in the West. By

[44] See Martin, 'Embarrassing Arguments'.

the Stoics, by Abaelard and the logicians of the twelfth century, and by Frege. On each occasion there has been a furious debate over which propositional logic is the correct one. I hope that what I have said will help in the evaluation of the role of Boethius' works in the second of these debates and in understanding the achievements of those who had to rely on him as their major source.

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304