UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2007/2008; 1st Semester
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Homework Set # 8 *Deadline:* November 7th, 2007

Exercise 27 (6 points).

In this exercise, we consider the systems of *positio* as described by Walter Burley and Roger Swyneshed. If a *positum* φ^* is given and φ_k (for $0 \leq k \leq n$) are proposed sentences of the **Opponent**, we let Φ_k^{Burley} be the set of **"currently accepted truths"** according to Burley's system on the basis of the sequence $\langle \varphi^*, \varphi_0, \ldots, \varphi_n \rangle$.
Prove the following properties of the two systems:

(1) If the *positum* φ^* is consistent, then for all $k \leq n$, the set Φ_k^{Burley} is a consistent set (3 points).
(2) If the *positum* φ^* is consistent and $k < \ell \leq n$ with $\varphi_k = \varphi_\ell$, then the **Respondent** in a Swyneshed-style *positio* will give the same answer in steps k and ℓ of the *obligatio* (3 points).

Exercise 28 (7 points).

We are considering a system reminiscent of Leibniz' attempts to arithmetize language. In the lecture, we introduced a system based on the divisor structure of the natural numbers, but this system was too simple as it didn't allow proper discussion of negative statements. Therefore, we add a number that should take care of the negative statements to the system.

(The approximate idea is: If 2 is *animal*, 3 is *rationalis* and 7 is *asinarius* ("donkey-like"), then $\langle 6, 7 \rangle$ would represent *homo* (to preclude the option of constructing a *homo asinarius*) and $\langle 14, 3 \rangle$ would represent *asinus* (to preclude the option of constructing an *asinus rationalis*.)

Formally: Call a pair $X := \langle \mathrm{p}_X, \mathrm{n}_X \rangle$ a **pseudo-Leibniz predicate** (**PLP**) if p_X and n_X are both positive natural numbers ≥ 2. We write $n|m$ for "n divides m" (*i.e.*, there is a $k \geq 1$ such that $nk = m$) and $n \perp m$ for "n and m are coprime" (*i.e.*, if $k|n$ and $k|m$, then $k = 1$). We define the following semantics for categorical propositions using PLPs:

$$\begin{aligned} X\mathrm{a}Y &:\equiv \mathrm{p}_X|\mathrm{p}_Y \;\&\; \mathrm{p}_Y \perp \mathrm{n}_X \\ X\mathrm{i}Y &:\equiv \exists k \geq 1(\mathrm{p}_X|k \cdot \mathrm{p}_Y \;\&\; k \cdot \mathrm{p}_Y \perp \mathrm{n}_X) \\ X\mathrm{e}Y &:\equiv \forall k \geq 1(\neg(\mathrm{p}_X|k \cdot \mathrm{p}_Y) \;\vee\; \neg(k \cdot \mathrm{p}_Y \perp \mathrm{n}_X)) \end{aligned}$$

In this semantics, **Barbara** can be expressed as:

$$\forall X, Y, Z\,((\mathrm{p}_X|\mathrm{p}_Y \;\&\; \mathrm{p}_Y|\mathrm{p}_Z \;\&\; \mathrm{p}_Y \perp \mathrm{n}_X \;\&\; \mathrm{p}_Z \perp \mathrm{n}_Y) \to \mathrm{p}_X|\mathrm{p}_Z \;\&\; \mathrm{p}_Z \perp \mathrm{n}_X)\,.$$

(1) Define a semantics for $X\mathrm{o}Y$ such that this is contradictory to $X\mathrm{a}Y$ (2 point).
(2) Give an example of a PLP that shows that **Barbara** is not valid with this semantics (2 points).
(3) Prove that **Celarent** is valid with this semantics (3 points).

Exercise 29 (9 points).
Read the text "Opposing and Responding" by Peter King (a link can be found on the course webpage). This text is the written version of a comment on Spade's talk at an APA meeting in 1993. There seems to be no written version of Spade's talk (even though some of the claims of Spade are most probably reflected in his "Why Don't Mediæval Logicians Ever Tell Us What They're Doing?" that you read a while ago). You will therefore have to reconstruct what Spade said from King's response.

(1) Reconstruct what Spade's talk was about, giving a hypothetical description of the talk in *one sentence*. Give a brief argument for your hypothetical description (at most half a page; 3 points).
(2) Why, according to King, do the "virtual mountain of examples" of *obligationes* not provide us with evidence that obligational disputations actually took place? (Answer in at most three sentences; 2 points)
(3) Give three ways that the historical evidence distinguishes between quodlibetal disputations and *obligationes*? (3 points)
(4) What is King's explanation for the "the very puzzling fact with which Spade began"? (Give in your own words; 1 point)