

# Medieval theories of consequence

- A genuine **medieval invention**.
- Medieval theories of consequence present a **level of systematization** not to be found in previous investigations (with the possible exception of Stoic logic).
- Some see in them the forerunners of the ‘**propositional turn**’ in logic, as opposed to term logic.
- But the **logic of terms** also occupies a prominent place in 14<sup>th</sup> century theories of consequence.

- 14<sup>th</sup> century logicians were probably the first to attempt a **systematization** of the **propositional rules of inference** such as contraposition, *ex falso*, the behavior of conjunctions, disjunctions etc.
- Medieval logicians sought not only to establish the **validity** of such basic rules; they also made **inquiries on the very nature of logical consequence** and inference.
- In this sense, their investigations overlap not only with modern ‘**proof theory**’, but also with modern **philosophy of logic**.

# Historical development

- Controversy: how come treatises with titles such as *De consequentiis* began to appear at the very beginning of the 14<sup>th</sup> century?
- The very term *consequentia* was in use in exactly this sense since Boethius (5<sup>th</sup>-6<sup>th</sup> Century): the topic itself occupied a prominent position in earlier traditions (cf. Abelard).
- But no treatises or chapters with this name seem to have been written before the 14<sup>th</sup> Century.

# Possible sources

(See N. Green-Pedersen 1984, *The Tradition of the Topics in the Middle Ages*)

- Commentaries on the *Topics*.
- Treatises on *syncategoremata*, especially *si* (if ... then).
- Commentaries on the *Prior Analytics*.
- Hypothetical syllogisms.

# Development in the 14<sup>th</sup> Century

- Very early, rather ‘primitive’ treatises: two anonymous and Burley’s *De consequentiis*.
- Second stage – more elaborate treatises, but external validation for consequences (middles, reworked Topics): Ockham’s *Summa Logicae*, Burley’s *De Puritate Artis Logicae*.
- Continental tradition – sophisticated treatises, where formal consequences are defined substitutionally: Buridan, Albert of Saxony, Pseudo-Scotus, Marsilius of Inghen.
- British tradition – characterized by the definition of formal consequence in terms of containment of the consequent in the antecedent: Richard Billingham, Ralph Strode, Richard Lavenham.

# What is a *consequentia*?

- A conditional sentence? No.
- A consequence, i.e. a logical relation between contents?  
Yes/no.
- An inference, i.e. the act performed by somebody of drawing a conclusion from premises? Yes/no.

# General definition of consequence

- All medieval authors accept as a **necessary condition** for the validity of a consequence the incompatibility (modal) criterion:

**It is impossible for the antecedent to be true  
while the consequent is false.**

- Whether this is a **sufficient condition** is a matter of contention.
- Adjustments of the general formulation of this condition are often necessary.

# Buridan's definition of consequence

“A proposition is antecedent to another when it is related to it [the other] in such a way that it is impossible for things to be in whatever way the first signifies them to be without their being in whatever way the other signifies them to be, when these propositions are put forward together.” (Buridan 1976, 22)

- ‘No proposition is negative, therefore no donkey is running’.
- Buridan is committed to proposition-tokens.



# Formal vs. Material Consequences

- This distinction is probably one of the most important aspects of medieval theories of consequences.
- The terms were introduced by Ockham in the *Summa Logicae*, but his criterion to separate formal from material consequences did not move forward (based on weird ‘middles’).

- With Buridan (Pseudo-Scotus, Albert of Saxony), substitutional criterion:

“Formal consequence means that [the consequence] holds for all terms, retaining the form common to all.” (Buridan 1976, 22/23).

- In the British school, formal consequences are defined as those where the consequent is contained in the antecedent:

“A consequence is formal when the consequent necessary belongs to the understanding of the antecedent, as it is in the case of syllogistic consequence, and in many enthymematic consequences.” (Lavenham)

- These two notions of formal consequences disagree not only with respect to the intension of the concept of formal consequence, but also with respect to its extension.
- Enthymemes are formal consequences according to the British school, but not according to Buridan.

**Every man is an animal.**

**Thus Socrates is an animal.**

What's missing?

# Simple vs. as-of-now consequences

- Absolute consequences hold always and necessarily, while as-of-now consequences hold at a specific time or under specific assumptions.
- Pseudo-Scotus: the absolute vs. as-of-now distinction applies only to material consequences and amounts to the modal value of the missing premise that can be added in order to turn the (enthymematic) consequence into a formal one:

- If the missing premise is a necessary truth, then it is a simple consequence; if it is a contingent truth, then it is an as-of-now consequence.

- Peter of Mantua presents the absolute vs. as-of-now distinction as primary: the formal vs. material distinction applies only to absolute consequences (as-of-now consequences are always material consequences).
- For him, an as-of-now consequence is a consequence in which the contradictory of the consequent can indeed be true at the same time as the antecedent, but not at the time indicated by the copula or verb in question.

# Rules of inference recognized by medieval authors

“Whatever follows from a consequent follows from the antecedent’, or alternatively, ‘whatever is antecedent to the consequent is antecedent to the consequent’ (Burley 2000, 4).

$$\text{Rule 2} \quad \frac{A \Rightarrow B \quad B \Rightarrow C}{A \Rightarrow C}$$

Two derived rules:

(2') "Whatever follows from a consequent and from its antecedent follows from the antecedent by itself" (Burley 2000, 6).

$$\text{Rule 2'} \quad \frac{A \Rightarrow B \quad A, B \Rightarrow C}{A \Rightarrow C}$$

(2'') “Whatever follows from a consequent with something added follows from the antecedent with the same thing added” (Burley 2000, 7).

$$\text{Rule 2''} \quad \frac{A \Rightarrow B \quad B, C \Rightarrow D}{A, C \Rightarrow D}$$



Derivations of the auxiliary rules:

Burley derives (2') from Rule 2 plus what he takes to be a logical fact, which the modern reader may recognize as a special case of right-weakening: 'every proposition implies itself together with its consequent'

$$\begin{array}{l} (2') \\ \text{WR} \\ \text{Rule 2} \end{array} \quad \begin{array}{c} A \Rightarrow B \\ \hline A \Rightarrow A, B \quad A, B \Rightarrow C \text{ (hyp.)} \\ \hline A \Rightarrow C \end{array}$$

Similarly, he derives (2'') from Rule 2 plus simultaneous applications of special cases of right-weakening and left-weakening: 'an antecedent together with something added implies the consequent with the same thing added'.

$$\begin{array}{l}
 (2'') \\
 \text{WR, LR} \\
 \text{Rule 2}
 \end{array}
 \begin{array}{c}
 A \Rightarrow B \\
 \hline
 A, C \Rightarrow B, C \quad B, C \Rightarrow D \text{ (hyp.)} \\
 \hline
 A, C \Rightarrow D
 \end{array}$$

## Other rules (from Pozzi 1978)

- ‘From the impossible anything follows.’
- ‘If the antecedent must be conceded, so must be the consequent.’
- ‘If the consequent must be denied, so must be the antecedent.’
- ‘From the contradictory of the consequent the contradictory of the antecedent follows’: contraposition.

- ‘Whatever follows from the contradictory of the antecedent follows from the contradictory of the consequent.’
- ‘Whatever is antecedent to the contradictory of the consequent is antecedent to the contradictory of the antecedent.’
- ‘From a conjunction to one of its parts constitutes a valid consequence.’
- ‘From one of its parts to the whole disjunction constitutes a valid consequence.’

- ‘From a conditional with its antecedent to its consequent constitutes a valid consequence.’ (Notice the distinction between consequence and conditional).

**Dissident voices:** some authors did not accept the *ex impossibili* and *ad necessarium* rules (Nicolaus Drukken of Dacia and Richard Ferrybridge).

What they seem to be proposing is that the criterion of containment of the consequent in the antecedent be used as a **necessary and sufficient** criterion for all valid consequences, and not only for the formal ones (as in Billinham, Strode *et al.*).

# References

## **Primary texts:**

Albert of Saxony 1988, *Perutilis logica*, in the incunabular edition of Venice 1522, with a Spanish translation by A. Muñoz García, Univ. del Zulia, Maracaibo.

John Buridan 1976, *Iohannis Buridani tractatus de consequentiis*. (ed. H. Hubien). Series *Philosophes Médiévaux* Vol. XVI. Louvain, Université de Louvain.

Pseudo-Scotus 2001, 'Questions on Aristotle's *Prior Analytics*: Question X'. In Yrjönsuuri 2001 (ed.), 225-234.

Richard Billingham 2003, *Richard Billingham "De Consequentibus" mit Toledo-Kommentar* (ed. S. Weber). Amsterdam, B.R. Grüner.

F. Schupp 1988, *Logical problems of the medieval theory of consequences, with the edition of the Liber consequentiarum*. Naples, Bibliopolis.

W. Seaton 1973, *An Edition and Translation of the "Tractatus de Consequentibus" by Ralph Strode, Fourteenth Century Logician and Friend of Geoffrey Chaucer*. Ph.D. Dissertation, University of California, Berkeley.

P.V. Spade 1974, "Five Logical Tracts by Richard Lavenham," in J. Reginald O'Donnell, ed., *Essays in Honour of Anton Charles Pegis*, Toronto: Pontifical Institute of Mediaeval Studies, 1974, pp. 70–124.

P.V. Spade 1976, "Robert Fland's *Consequentiae*: An Edition," *Mediaeval Studies* 38 (1976), pp. 54–84.

Walter Burley 1980, 'De Consequentibus' (ed. N.J. Green-Pederson). *Franciscan Studies* 40, 102-166.

Walter Burley 2000, *On the Purity of the Art of Logic – the shorter and the longer treatises* (transl. P.V. Spade). New Haven, Yale University Press.

William of Ockham 1974, *Opera Philosophica I*. St. Bonaventure, The Franciscan Institute.

### **Secondary texts:**

C. Dutilh Novaes 2005b, “Buridan’s *consequentia*: consequence and inference within a token-based semantics”. *History and Philosophy of Logic* 26(4), 2005, 277-297.

N.J. Green-Pedersen 1981, ‘Walter Burley, *De consequentiis* and the origin of the theory of consequence’. In H.A.G. Braakhuis, C.H. Kneepkens & L.M. de Rijk (eds.), *English Logic and Semantics*. Nijmegen, Ingenium, 279-301.



N.J. Green-Pedersen 1984, *The Tradition of the Topics in the Middle Ages*. München, Philosophia Verlag.

N.J. Green-Pedersen 1985, 'Early British Treatises on Consequences'. In P. Osmund Lewry O.P. (ed.), *The Rise of British Logic*. Papers in Mediaeval Studies 7. Toronto, Pontifical Institute of Mediaeval Studies, 285-307.

E. Karger 1993, 'A theory of immediate inferences contained in Buridan's logic', in K. Jacobi (ed.), *Argumentationstheorie*. Leiden, Brill, 407-429.

M. Kaufmann 1993, 'Nochmals; Ockhams consequentiae und die materiale implikation'. In Jacobi 1993, 223-232.

P. King 2001, 'Medieval proof theory'. In Yrjönsuuri 2001, 117-145.

C.J. Martin 1986, 'William's Machine'. *The Journal of Philosophy* **83** (10), 564-572.

L. Pozzi 1978, *Le consequentiae nella logica medievale*. Padova, Liviana.

S. Read 1993, 'Formal and material consequence, disjunctive syllogism and Gamma'. In Jacobi (ed.) 1993, 233-262.

S. Read 2001, 'Self-reference and validity revisited'. In Yrjönsuuri (ed.) 2001, 183-196.

M. Yrjönsuuri (ed.) 2001, *Medieval Formal Logic*. Dordrecht, Kluwer.

