# The ILLC (1).

**Institute for Logic, Language and Computation.**

- Early beginnings: *Instituut voor Grondslagenonderzoek en Filosofie der Exacte Wetenschappen, 1952. Instituut voor Taal, Logica en Informatie* (ITLI), 1986.
- Established in 1991.

# The ILLC (2).

**Mission Statement.** Many broad flows of information drive the modern technological world. It is a challenge for contemporary science to provide a deeper understanding, and where possible, enhance existing practice. Indeed, in the course of this century, information has become a crucial theme for scientific studies across many disciplines. Encoding, transmission and comprehension of information are the central topics of research at the ILLC. The broader context in which ILLC sees itself is that of an upcoming information science or ‘informatics’, which is concerned with information flow in natural and formal languages, as well as many other means of communication, including music and images of various kinds.

Research at ILLC aims at developing logical systems that can handle this rich variety of information, making use of insights across such disciplines as linguistics, computer science, cognitive science, and artificial intelligence. Additional methods are actively pursued as well, whenever relevant, ranging from statistics to argumentation theory. The ILLC aims at overcoming traditional borderlines between faculties and disciplines, and serves as a rallying point for information scientists across computer science, linguistics, philosophy, or social sciences. Moreover, the institute propagates exact logical standards of semantic clarity, algorithmic perspicuity, and increasingly also efficient computability.

The resulting view of information science transcends the boundaries of the university. ILLC is also committed to dissemination of its results into the broader world of general education, vocational training, and industrial research.

# The ILLC (3).

**Research Groups ("Projects").**

- *Logic & Language.*
- *Logic & Computation.*
- *Language & Computation.*

"Horizontal interest groups": Cognition, Games.

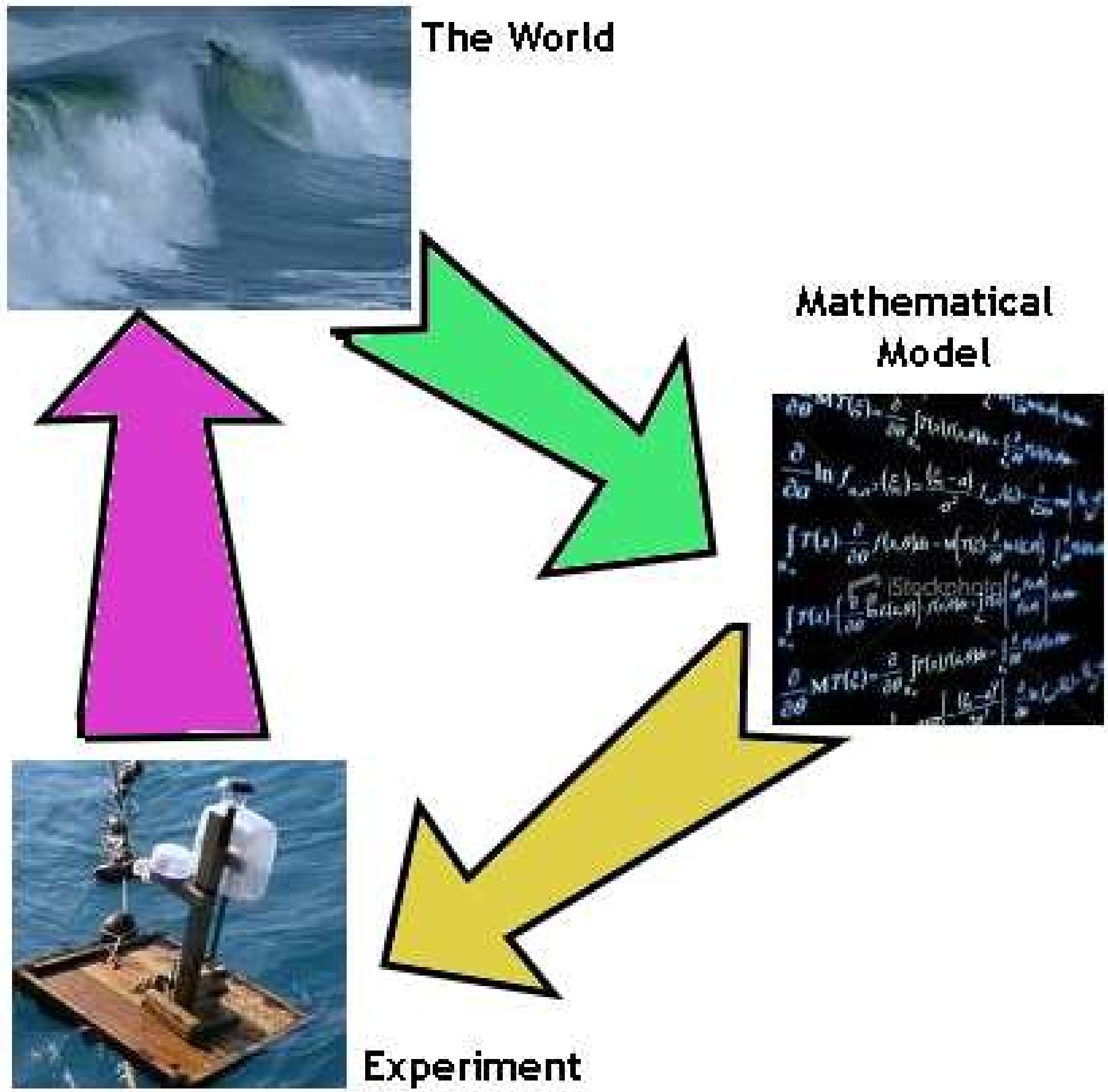
# Conceptual Modelling.

## European Science Foundation:

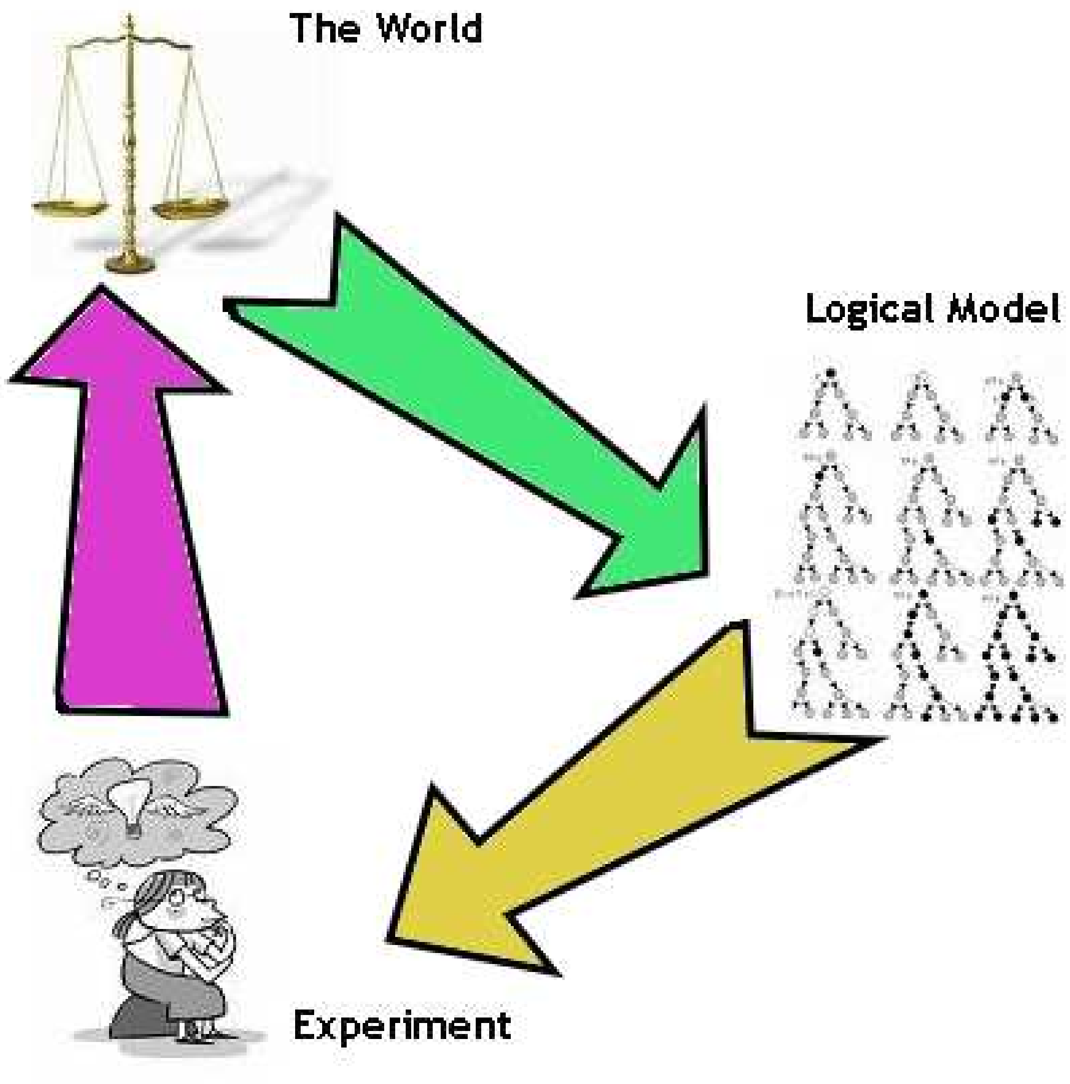
"Modelling intelligent interaction – Logic in the Humanities, Social and Computational sciences" (LogICCC)

One of the most crucial and striking features of humans and their societies, is the phenomenon of intelligent interaction. Many disciplines from the humanities to the physical sciences hold separate pieces of the puzzle posed by this pervasive but also elusive phenomenon. The EUROCORES programme "LogICCC – Modelling Intelligent Interaction" aims at a deeper understanding of intelligent interaction by letting logic in its modern guise act as a catalyst and a 'match maker' between these different disciplines. [...] [W]hat all participants in LogICCC projects have in common is their interest in understanding interaction, pursued with the common language and models provided by logic in its modern, pluriform, and outward-looking guise.

# Mathematical Modelling.



# Conceptual Modelling.



# Syntax of Conceptual Modelling (1).

A **language** consists of a set of variables $\mathfrak{V}$, a set of constant symbols, a set of function symbols, and a set of relation symbols. Functions and relations have **arities** that determine how many arguments they take.

Sometimes, we have **many-sorted** languages. Then we have several sets of variables $\mathfrak{V}_0$, ..., $\mathfrak{V}_n$ (called **sorts**), and we need to fix exactly what type of arguments function and relation symbols take.

*Example 1.* $\mathfrak{V}$ will be interpreted as human beings; we have three relations, two unary and one binary: $\mathrm{good}$, $\mathrm{evil}$, and $\mathrm{friend}$.

# Syntax of Conceptual Modelling (2).

*Example 1.* $\mathfrak{V}$ will be interpreted as human beings; we have three relations, two unary and one binary: $\mathrm{good}$, $\mathrm{evil}$, and $\mathrm{friend}$.

*Example 2.* $\mathfrak{V}_0$ will be interpreted as sheep and $\mathfrak{V}_1$ as human beings; we have two unary relations on $\mathfrak{V}_0$, $\mathrm{white}$ and $\mathrm{black}$, and a relation between $\mathfrak{V}_1$ and $\mathfrak{V}_0$, $\mathrm{owner}$.

**Putting a logic on top of the language.**

- Propositional logic symbols: $\wedge$, $\vee$, $\rightarrow$, $\neg$.
- Predicate logic symbols: $\exists$, $\forall$, $=$. (In many-sorted languages, we need to distinguish types of quantifi ers.)
- Modal logic symbols: $\Box$, $\Diamond$, $\mathbf{K}$.

# Syntax of Conceptual Modelling (3).

*Example 1.* $\mathfrak{V}$ will be interpreted as human beings; we have three relations, two unary and one binary: $\mathrm{good}$, $\mathrm{evil}$, and $\mathrm{friend}$.

Add predicate logic, and we are able to express “Evil people only have evil friends”:

$$\forall x(\mathrm{evil}(x) \rightarrow \forall y(\mathrm{friend}(x, y) \rightarrow \mathrm{evil}(y)))$$

# Syntax of Conceptual Modelling (3).

*Example 1.* $\mathfrak{V}$ will be interpreted as human beings; we have three relations, two unary and one binary: $\mathrm{good}$, $\mathrm{evil}$, and $\mathrm{friend}$.

*Example 2.* $\mathfrak{V}_0$ will be interpreted as sheep and $\mathfrak{V}_1$ as human beings; we have two unary relations on $\mathfrak{V}_0$, $\mathrm{white}$ and $\mathrm{black}$, and a relation between $\mathfrak{V}_1$ and $\mathfrak{V}_0$, $\mathrm{owner}$.

Add predicate logic, and we are able to express “No shepherd has only white sheep”.

(we write $X, Y, ...$ for elements of $\mathfrak{V}_1$ and $x, y, ...$ for elements of $\mathfrak{V}_0$)

$$\forall X(\exists x(\mathrm{owner}(X,x) \wedge \mathrm{black}(x)))$$

$$\forall X(\exists x(\mathrm{owner}(X,x) \rightarrow \exists y(\mathrm{owner}(X,y) \wedge \mathrm{black}(y))))$$

# Semantics of Conceptual Modelling (1).

To a language and a logic, we now assign a **universe** $U$ of objects. In the case of many-sorted languages, we fi x a universe $U_i$ for every sort. For every constant symbol $c$, we need to fi x an element $\mathbf{c} \in U$, for every $n$-ary function symbol $f$, we need to fi x an $n$-ary function $\mathbf{f} : U^n \to U$, and for every $m$-ary relation symbol $R$, we need to fi x an $m$-ary relation $\mathbf{R} \subseteq U^m$.
An **assignment** is a function $I : \mathfrak{V} \to U$, and we use the ordinary model-theoretic defi nitions:

- $U, \mathbf{R}, I \models R(v)$ if and only if $\mathbf{R}(I(v))$ holds, and
- $U, \mathbf{R} \models \varphi$ if and only if for all assignments $I$, we have $U, \mathbf{R}, I \models \varphi$.

# Semantics of Conceptual Modelling (2).

*Example 1.* $\mathfrak{V}$ will be interpreted as human beings; we have three relations, two unary and one binary: $\mathrm{good}$, $\mathrm{evil}$, and $\mathrm{friend}$.

Let $U$ be the set of humans, and let $\mathbf{good}(u)$ if $u$ is a good person, $\mathbf{evil}(u)$ if $u$ is an evil person, and $\mathbf{friend}(u, v)$ if $u$ is a friend of $v$.
Let $U := \mathbb{N}$, and let $\mathbf{good}(n)$ if $n = 4k + 1$, $\mathbf{evil}(u)$ if $n = 4k$, and $\mathbf{friend}(u, v)$ if $u|v$.

$$\forall x(\mathrm{evil} \to \forall y(\mathrm{friend}(x, y) \to \mathrm{evil}(y)))$$

True!

# Semantics of Conceptual Modelling (3).

*Example 2.* $\mathfrak{V}_0$ will be interpreted as sheep and $\mathfrak{V}_1$ as human beings; we have two unary relations on $\mathfrak{V}_0$, $\mathrm{white}$ and $\mathrm{black}$, and a relation between $\mathfrak{V}_1$ and $\mathfrak{V}_0$, $\mathrm{owner}$.

Let $U_0 := \mathbb{N}$, let $\mathbf{black}(u)$ if and only if $u$ is a prime number and $\mathbf{white}(u)$ otherwise. Let $U_1 := \mathbb{N}$ as well (!). Let $\mathbf{owner}(n, m)$ if $m > 2n$ and $m$ is not a multiple of $n$.

$$\forall X(\exists x(\mathrm{owner}(X, x) \wedge \mathrm{black}(x)))$$

$$\forall X(\exists x(\mathrm{owner}(X, x) \rightarrow \exists y(\mathrm{black}(X, y) \wedge \mathrm{white}(y))))$$

True!

# The axiomatic method (1).

Use our intuitions to derive **axioms** for our logic and then derive consequences from these axioms.

*Example 1.*

- $\forall x(\mathrm{good}(x) \rightarrow \neg\mathrm{evil}(x))$
- $\forall x(\mathrm{evil}(x) \rightarrow \neg\mathrm{good}(x))$
- $\forall x, y(\mathrm{friend}(x, y) \rightarrow \mathrm{friend}(y, x))$

*Example 2.*

- $\forall x(\mathrm{white}(x) \leftrightarrow \neg\mathrm{black}(x))$
- $\forall X, Y, x((\mathrm{owner}(X, x) \wedge X \neq Y) \rightarrow \neg\mathrm{owner}(Y, x))$

# The axiomatic method (2).

We call a structure $\langle U, \mathbf{good}, \mathbf{evil}, \mathbf{friend}\rangle$ a **moral model** if it satisfi es the following axioms.

- $\forall x(\text{good}(x) \to \neg\text{evil}(x))$
- $\forall x(\text{evil}(x) \to \neg\text{good}(x))$
- $\forall x, y(\text{friend}(x, y) \to \text{friend}(y, x))$

We say that a moral model has the **dichotomy property** if every person is either good or evil ($\forall x(\text{good}(x) \leftrightarrow \neg\text{evil}(x))$). We say that **evil is isolated** if evil people only have evil friends

$$\forall x(\text{evil}(x) \to \forall y(\text{friend}(x, y) \to \text{evil}(y))),$$

and say **good is isolated** if good people only have good friends.

**Theorem.** In every moral model with the dichotomy property in which evil is isolated, good is also isolated.

# Vagueness (1).

*Phenomenon.* We have a natural language predicate “bald” that seems to be invariant under “having one hair less or more”. Suppose we have a human being with full hair (say, 100,000 hairs), then a chain of implications seems to show that this person is bald.

*Model.* Take a two-sorted language with people $U$ and natural numbers $\mathbb{N}$. We have a unary relation for people $\mathbf{bald}$, a function $h : U \to \mathbb{N}$, a constant in the natural numbers $0$, a relation $\leq$ on $\mathbb{N}$ and a function (the “successor function”) $s : \mathbb{N} \to \mathbb{N}$.

*Axioms.*

- $\forall x, y(h(x) \leq h(y) \wedge \mathrm{bald}(y) \to \mathrm{bald}(x))$
- $\forall x, y(h(x) = s(h(y)) \wedge \mathrm{bald}(y) \to \mathrm{bald}(x))$
- $\forall x(h(x) = 0 \to \mathrm{bald}(x))$
- the usual (Peano) axioms for natural numbers

# Vagueness (2).

*Axioms.*

- $\forall x, y(h(x) \leq h(y) \wedge \mathrm{bald}(y) \rightarrow \mathrm{bald}(x))$
- $\forall x, y(h(x) = s(h(y)) \wedge \mathrm{bald}(y) \rightarrow \mathrm{bald}(x))$
- $\forall x(h(x) = 0 \rightarrow \mathrm{bald}(x))$
- the usual (Peano) axioms for natural numbers

Call a structure $\langle U, \mathbb{N}, \mathbf{bald}, h, \leq, 0, s\rangle$ a **hairy model** if it satisfi es the above axioms.

# Vagueness (3).

Add a third universe, $[0,1]$ of “degrees of truth”, so we now have a three-sorted language. We have a relation between people and degrees $\mathbf{bald}$, a function $h : U \to \mathbb{N}$, a constant in the natural numbers $0$, constants $\varepsilon$, $0$, and $1$ in the degrees, a relation $\leq$ on $\mathbb{N}$, a relation $\preceq$ on $[0,1]$, a function $+$ on $[0,1]$, and a function (the “successor function”) $s : \mathbb{N} \to \mathbb{N}$.

We interpret $\mathrm{bald}(x,d)$ as “$x$ is bald to degree $d$” where degree $1$ represents baldness and degree $0$ represents full hair.

*Axioms.*

- $\forall x, y, d(h(x) \leq h(y) \wedge \mathrm{bald}(y,d) \to \exists e \succeq d(\mathrm{bald}(x,e))$
- $\forall x, y(h(x) = s(h(y)) \wedge \mathrm{bald}(y,d) \to \exists e(e + \varepsilon \succeq d(\mathrm{bald}(x,e)))$
- $\forall x(h(x) = 0 \to \mathrm{bald}(x,1))$

# Vagueness (4).

*Axioms.*

- $\forall x, y, d(h(x) \leq h(y) \wedge \text{bald}(y, d) \rightarrow \exists e \succeq d(\text{bald}(x, e))$
- $\forall x, y(h(x) = s(h(y)) \wedge \text{bald}(y, d) \rightarrow \exists e(e + \varepsilon \succeq d(\text{bald}(x, e)))$
- $\forall x(h(x) = 0 \rightarrow \text{bald}(x, 1))$

Call a structure a **vaguely hairy model** if it satisfi es the above axioms.

**Theorem.** There are vaguely hairy models with bald and non-bald people.

# Omnipotence and benevolence (1).

**Theodicy.** How can we reconcile the existence of an omnipotent and benevolent entity with the existence of evil?

*Model.* We think of history as a sequence of discrete events or actions. Fix a tree $T = \langle V, E\rangle$. The vertices are interpreted as **states of affairs** and the edges as **actions** (changing one state of affairs into another). We add a set $A$ of **agents** and a relation $\mathrm{affect}$ between agents and states (i.e., $\mathrm{affect} \subseteq A \times V$). We interpret $\mathrm{affect}(a, v)$ as $a$ affects $v$. We have a unary relation $\mathrm{evil}$ on the set $E$ of actions, identifying evil actions.

Let $H$ be the set of maximal linearly ordered subsets of $T$, its branches or **histories**. For $h \in H$, let $V_h$ and $E_h$ be the subset of vertices and edges in $h$, respectively.

# Omnipotence and benevolence (2).

Let $H$ be the set of maximal linearly ordered subsets of $T$, its branches or **histories**. For $h \in H$, let $V_h$ and $E_h$ be the subset of vertices and edges in $h$, respectively.

If $h \in H$, we say that agent $a$ is **benevolent in** $h$ if no action affected by $a$ is evil, i.e.,

$$\forall v, w \in V_h(\text{affect}(a, v) \wedge \langle v, w \rangle \in E_h \rightarrow \neg\text{evil}(\langle v, w \rangle))$$

We say that $a$ is **omnipotent in** $h$ if $a$ affects all states, i.e.,

$$\forall v \in V_h(\text{affect}(a, v).$$

**Theorem.** If $h$ is any history that includes at least one evil action, then there is no agent that is omnipotent and benevolent in $h$.