Why Daan and Sanne can't add

Kolloquiumsvortrag

Seminar für Mathematik und ihre Didaktik

Mathematisch-Naturwissenschaftliche Fakultät

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Jan van de Craats

Universiteit van Amsterdam

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In fact, since two years, we have a 'math war', with people blaming new educational methods for the deteriorating of arithmetical skills in primary schools.

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'Veel leerlingen hebben helemaal geen weet van ons rekenstelsel en hebben rekenen altijd gezien als gegoochel. Velen zijn ook van mening dat je rekenen ofwel kan ofwel niet kan. Van regels e.d. hebben ze nooit gehoord en toepassen is dan dus ook bijzonder moeilijk.'

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Repairing what went wrong

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At the end of secondary school (age 16-18) there will be an obligatory written exam in arithmetics.

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Driving force behind this process: didacticians, mostly connected to the Freudenthal Institute of Utrecht University. Their *credo*: 'realistic mathematics education'.

Three myths, five blunders

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- 'Smart calculation' (using all kinds of tricks, depending on the special numbers involved)

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- It is nearly impossible to prove that a student ‘understands’ a subject (e.g., long division)

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- Teachers always underestimate the number of (similar) exercises that are needed to master a subject.

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- Only very gifted students are able and willing to 'invent' solution methods.
- The others only get confused by 'smart' methods that only work in special cases.
- They end up in despair, hating math. This already occurs in grades 1 and 2.

Didactical blunders

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‘Columnwise’ addition and subtraction (from left to right)

Bij het **kolomsgewijs optellen en aftrekken** werk je van *links naar rechts* en kijk je steeds naar de betekenis van de cijfers in de kolommen. Je laat de getallen in hun waarde.
Voor het aftrekken werk je met **tekorten** in de kolommen, als dat nodig is.

Voorbeeld 1

Kolomsgewijs optellen

386	
673	
900	*(= 300 + 600)*
150	(= 80 + 70)
9	(= 6 + 3)
1059	

Kolomsgewijs aftrekken

803	
261	
600	*(= 800 – 200)*
–60	(0 – 60 = 60 tekort)
2	(= 3 – 1)
542	

Didactical blunders

‘Columnwise’ multiplication (writing out all partial products)

Voor het **kolomsgewijze vermenigvuldigen** ga je uit van de vier deelproducten van $(30 + 7) \times (30 + 8)$, beginnend met de grootste waarde (van links af). Daarna tel je weer op. Dit kan ook van rechts naar links.

Van links naar rechts

38		
37	$\times$	
900		(30×30)
240		(30×8)
210		(7×30)
56	+	(7×8)
1406		

Van rechts naar links

38		
37	$\times$	
56		(7×8)
210		(7×30)
240		(30×8)
900	$\times$	(30×30)
1406		

Didactical blunders

‘Taking chunks’(unsystematic repeated subtraction) instead of standardized long division

Delen door herhaald aftrekken

431 : 12	
120	10 × 12
311	
120	10 × 12
191	
120	10 × 12
71	
60	5 × 12
11	35 × 12, rest 11

431 : 12	
360	30 × 12
71	
60	5 × 12
11	
	35 × 12, rest 11

Didactical blunders

‘Taking chunks’ (continued)

b De ThiemeMeulenhoff-site is de laatste 39 dagen precies 33 384 keer bezocht. Hoeveel bezoeken zijn dat gemiddeld per dag?
Hierbij hoort de rekenzin 33 384 : 39 (Schatting vooraf 40 000 : 40 = 1 000.)

39	33 384	
	31 200	800
	2 184	
	1560	40
	624	
	390	10
	234	
	156	4
	78	
	78	2
over	0	856 bezoekers

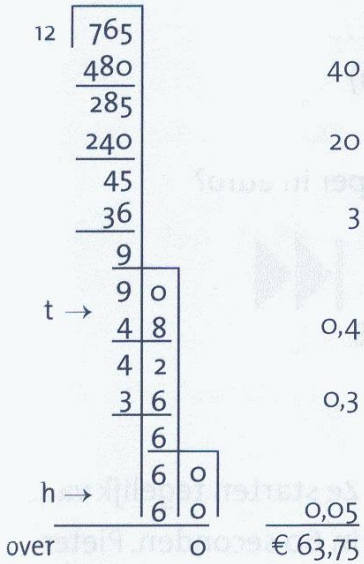
39	x
39	1 x
390	10 x
3 900	100 x
7 800	200 x
15 600	400 x
31 200	800 x

Natuurlijk zijn ook andere staarten mogelijk, bijvoorbeeld via de 5x- en 500x-happen. Maak weer je eigen keuze, want dit geeft het beste resultaat om een deling uit te rekenen en het vergroot je zelfvertrouwen.

Didactical blunders

‘Taking chunks’ (continued)

c (Bij dit voorbeeld wordt doorgegaan met delen tot op 2 decimalen achter de komma. De letters t, h, d en td die eventueel gebruikt kunnen worden, staan voor tiende, honderdste, duizendste en tienduizendste.)
Een studentenverzekering kost per jaar € 765,–. Hoeveel kost deze verzekering per maand?
Een jaar heeft 12 maanden, dus hierbij hoort de rekenzin 765 : 12.
(Schatting vooraf 600 : 12 = 50)

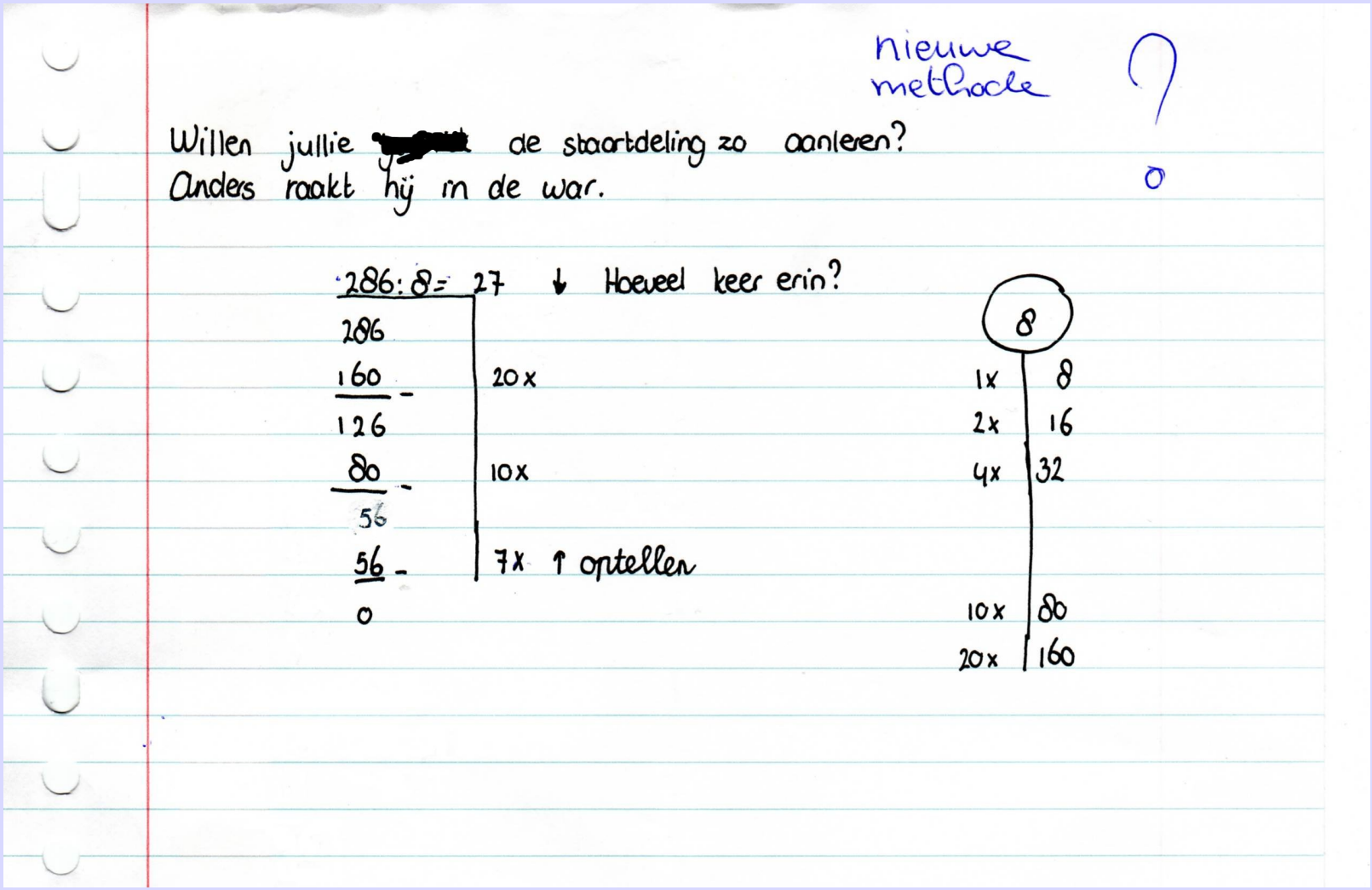


	12	x
40	12	1 x
20	24	2 x
3	120	10 x
0,4	480	40 x
0,3		

Een andere mogelijke schrijfwijze is:
9,0 in plaats van t → 90
4,8 in plaats van t → 48
0,60 in plaats van h → 60

Didactical blunders

‘Taking chunks’ (continued)



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The traditional algorithms for addition, subtraction, multiplication and division of integers, decimal numbers and fractions satisfy these requirements.

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Most students (and teachers!) think that calculating with big numbers (i.e., more than two digits) is very difficult! This is felt as a persistent calculating problem.

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There is no strategy to identify and remediate these problems. All children are obliged to take part in group discussions. Many of them do not understand what is going on.

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De leerkracht zie je worstelen. Hij geeft een som op en de leerlingen gaan door elkaar heen roepen wat voor oplossingsstrategieën er allemaal mogelijk zijn. Sommige leerlingen komen met zulke bizarre oplossingen, die leerkracht begrijpt niet eens wat er allemaal gezegd wordt. Slechts op een paar leerlingen kan hij ingaan.’

See also . . . (in Dutch)

The website of the Stichting Goed Rekenonderwijs:

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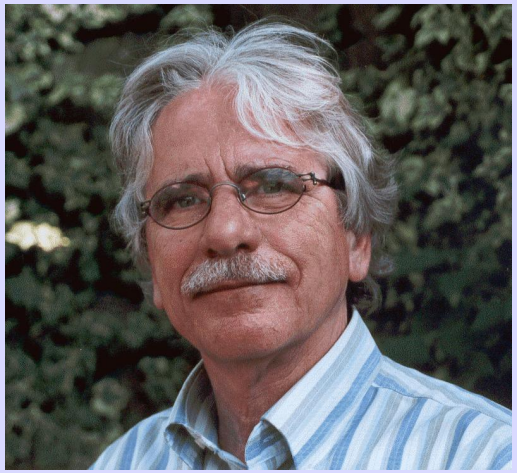

Vielen Dank!