

Appendix 5: Pictures of prime numbers and ideals for complex fields of class number 2

The pictures show the quadratic character and a picture of prime numbers, units and non-principal prime ideals for all complex quadratic fields of class number 2, namely

the fields of discriminant congruent 0 modulo 4:

$$\mathbb{Q}(\sqrt{-5}), \mathbb{Q}(\sqrt{-6}), \mathbb{Q}(\sqrt{-10}), \mathbb{Q}(\sqrt{-13}), \mathbb{Q}(\sqrt{-22}), \mathbb{Q}(\sqrt{-37}), \mathbb{Q}(\sqrt{-58})$$

and the fields of discriminant congruent 1 modulo 4:

$$\mathbb{Q}(\sqrt{-15}), \mathbb{Q}(\sqrt{-35}), \mathbb{Q}(\sqrt{-51}), \mathbb{Q}(\sqrt{-91}), \mathbb{Q}(\sqrt{-115}), \mathbb{Q}(\sqrt{-123}), \mathbb{Q}(\sqrt{-187}), \mathbb{Q}(\sqrt{-235}), \mathbb{Q}(\sqrt{-267}), \mathbb{Q}(\sqrt{-403}), \mathbb{Q}(\sqrt{-427}).$$

At the top, each picture mentions the field, $\mathbb{Q}(\sqrt{r})$, and displays its quadratic character (as far as space allows).

In the pictures, rational integers are placed on the x-axis and numbers of the form \sqrt{r} times rational integers on the y-axis.

When $d \equiv 0 \pmod{4}$, we use a square grid, otherwise a staggered grid, where the grid points form roughly equilateral triangles.

The pictures display the prime numbers, which generate the principal prime ideals, but not those irreducible numbers which are not prime.

Moreover, the non-principal prime ideals are displayed as follows.

The non-principal ideals are obtained by dividing principal ideals by a certain non-principal prime ideal, I , generated by its norm and some integer of $\mathbb{Q}(\sqrt{r})$. In the picture, the non-principal prime ideals then are represented by those numbers whose norm is equal to a prime norm times the norm of I . This norm of I is mentioned at the top of the picture.

$Q(\sqrt{-5})$

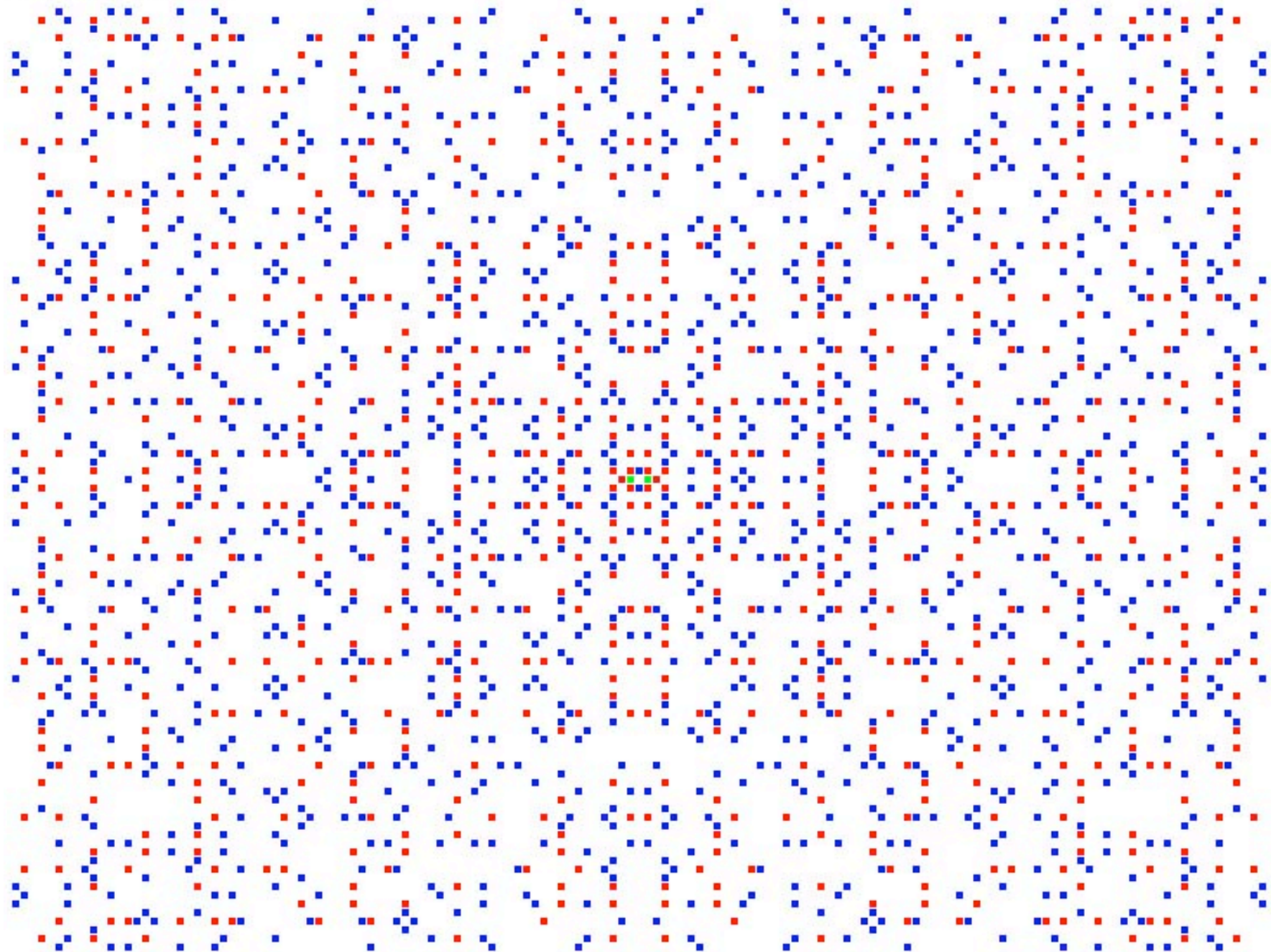
chi

prime numbers

units

prime ideals by norm 2

0+0+000+0+0-0-000-0-



$Q(\sqrt{-6})$

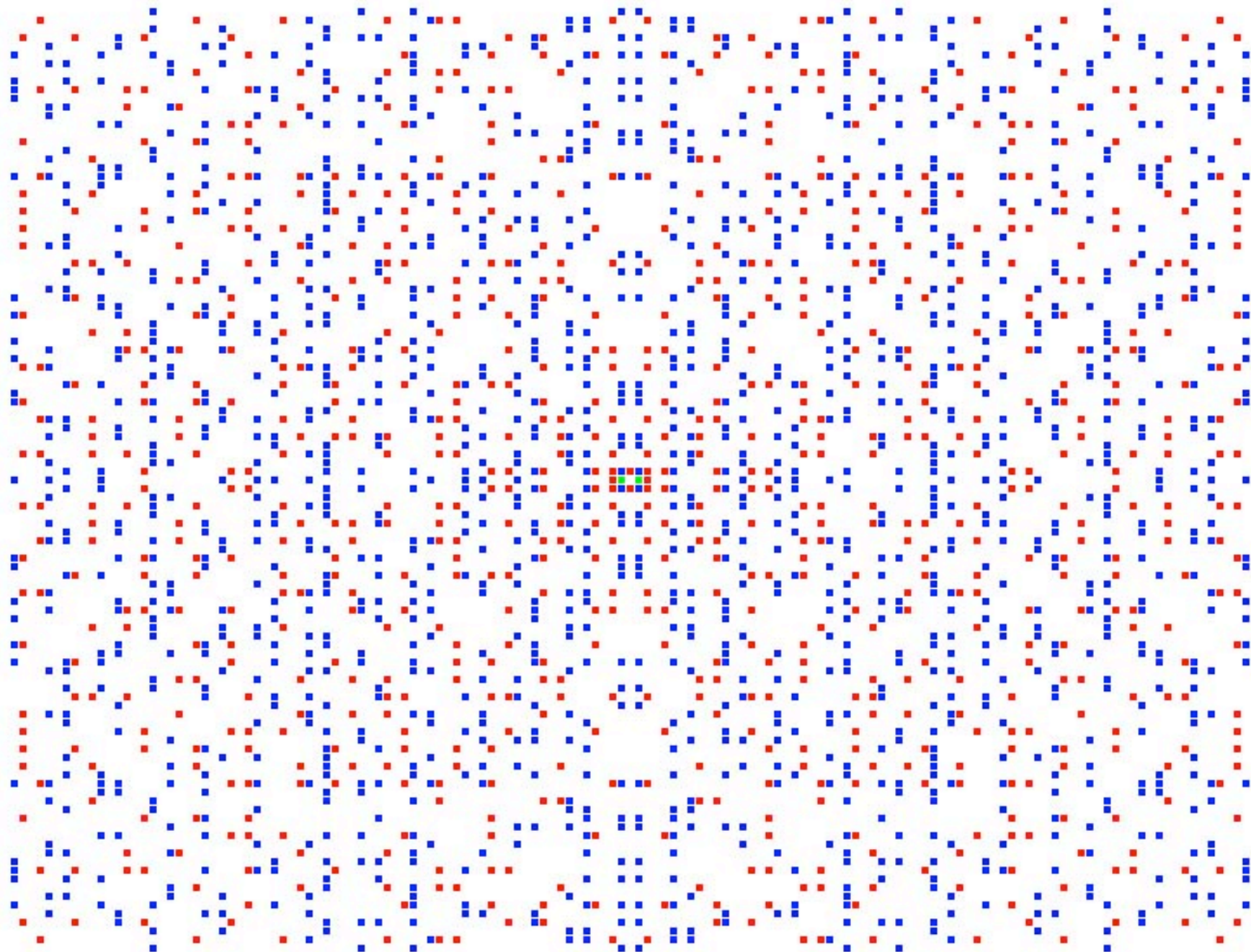
chi

prime numbers

units

prime ideals by norm 2

0+000+0+000+0-000-0-000-



$Q(\sqrt{-10})$

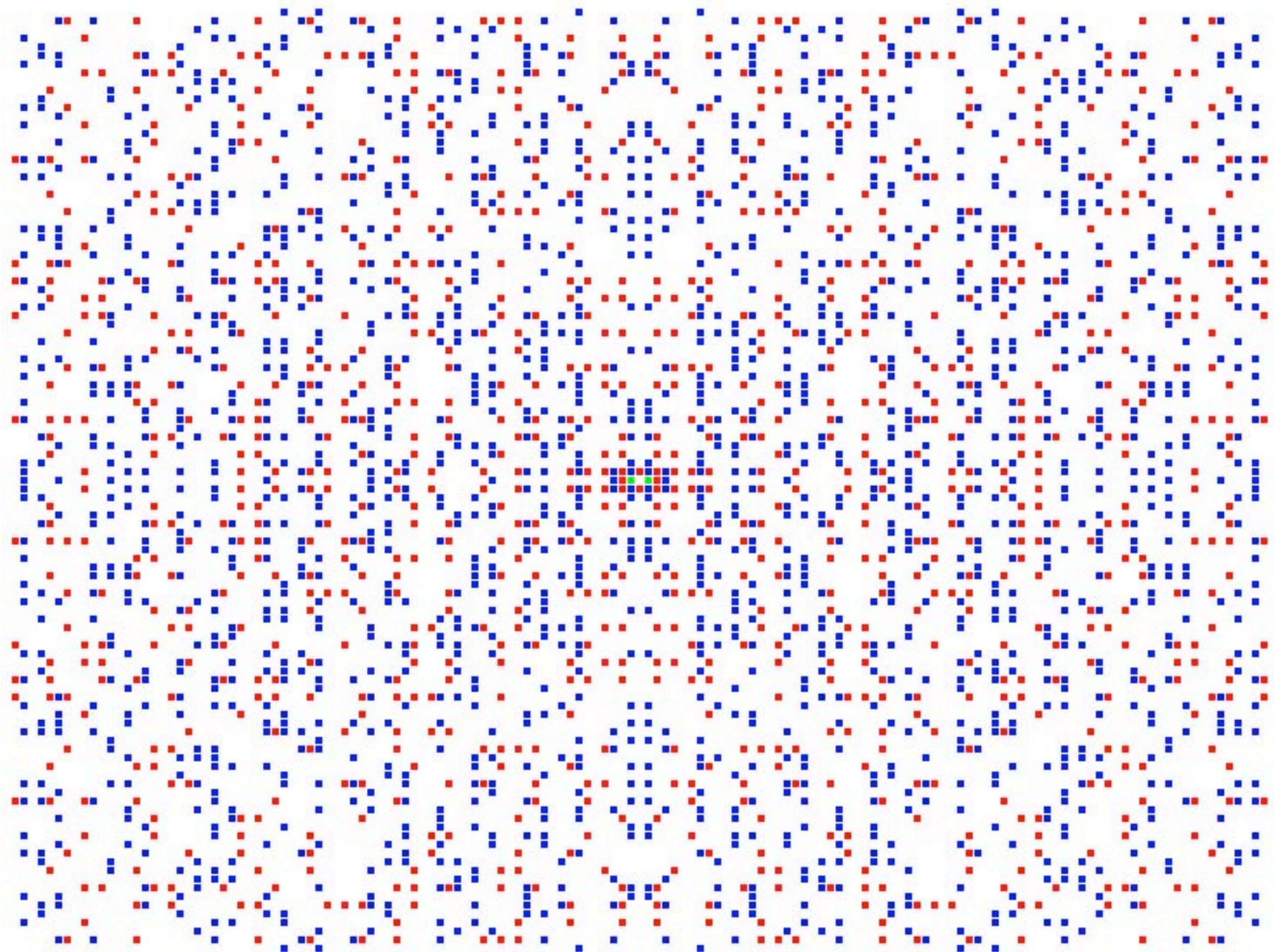
chi

prime numbers

units

prime ideals by norm 2

0+0-000+0+0+0+000-0+0-0+000-0-0-0-000+0-



$Q(\sqrt{-13})$

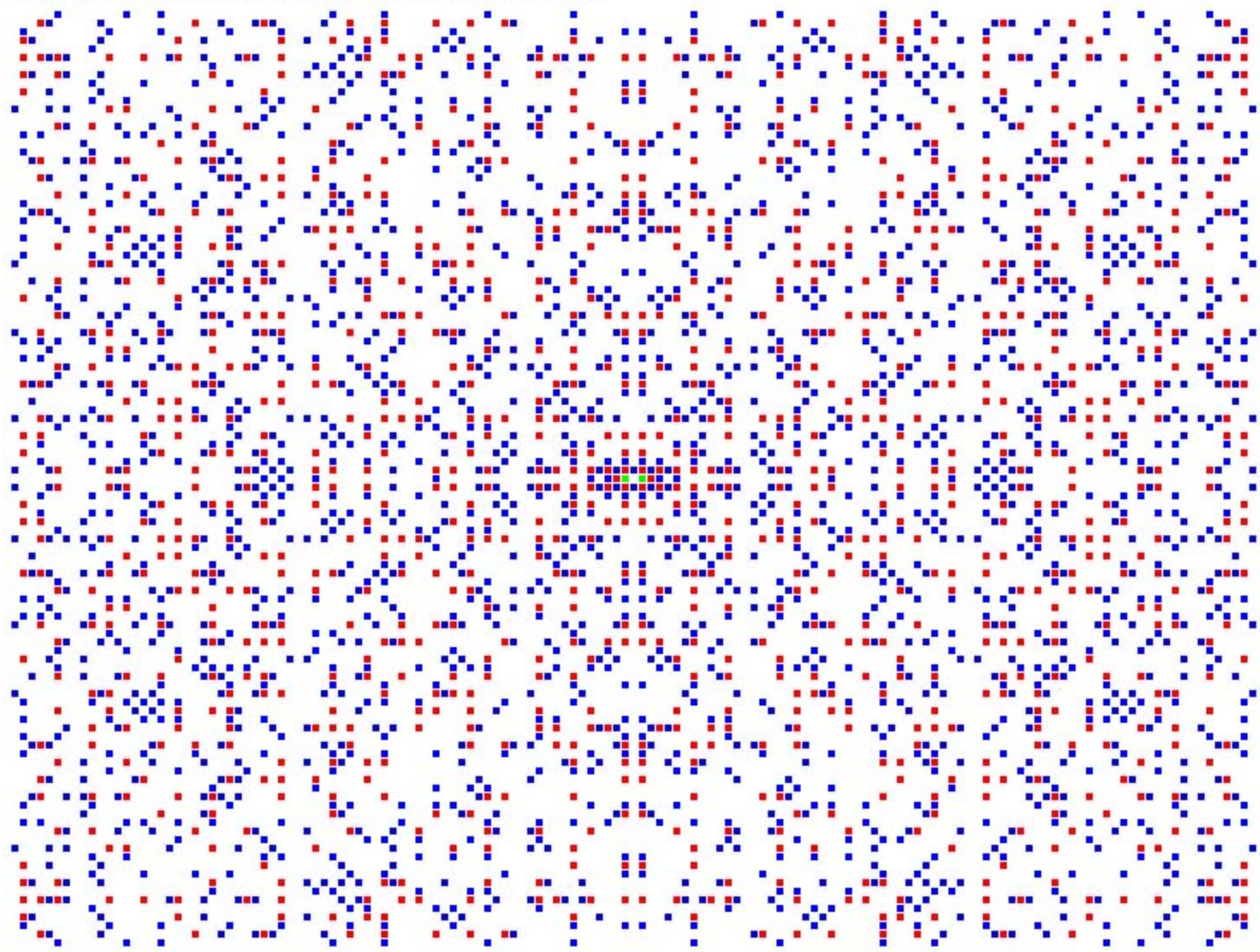
chi

prime numbers

units

prime ideals by norm 2

0+0-0-0+0+0+000+0+0+0-0-0+0-0+0+0-0-0-000-0-0-0+0+0-



$Q(\sqrt{-15})$

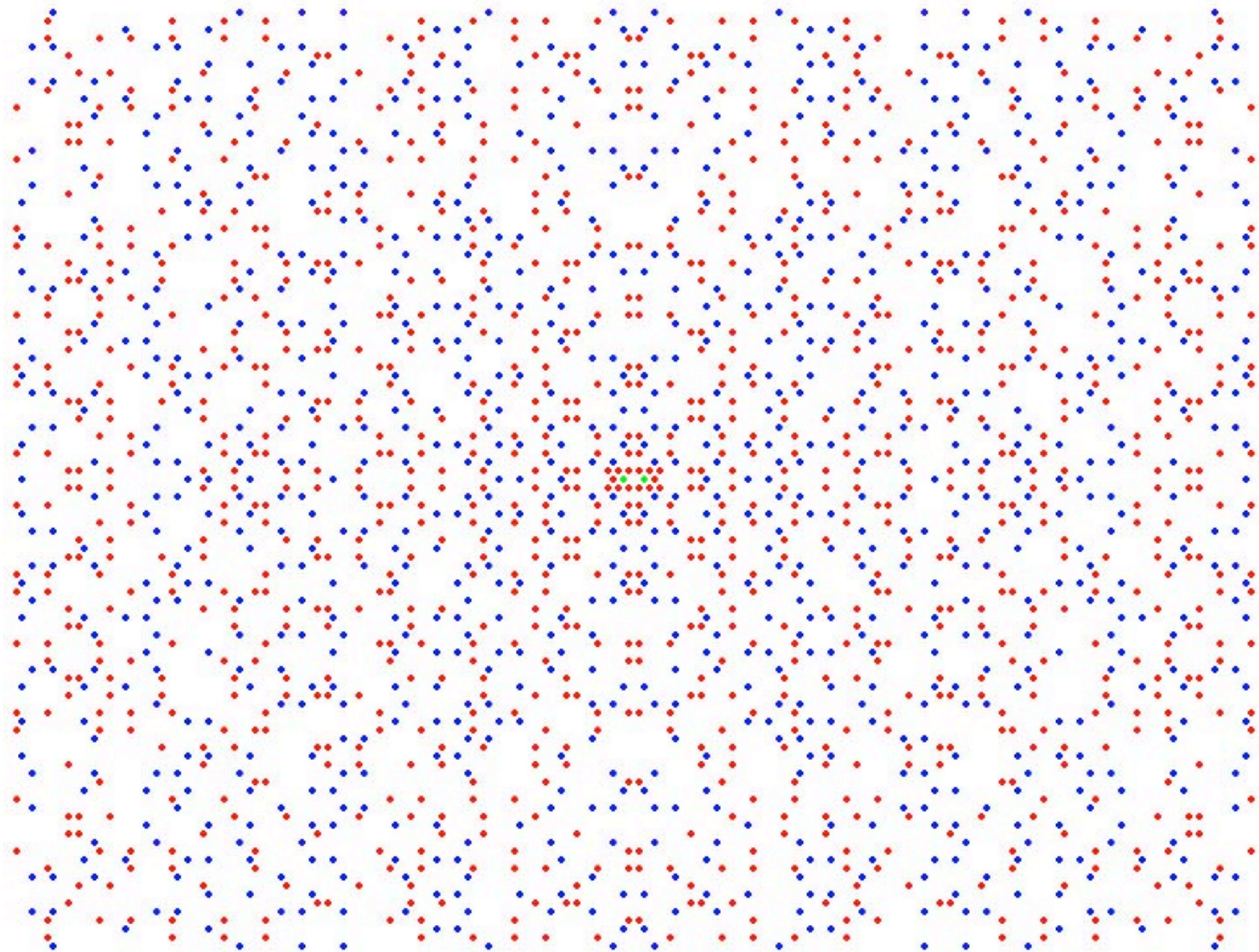
chi

prime numbers

units

prime ideals by norm 2

0++0+00--+00-0--



$Q(\sqrt{-22})$

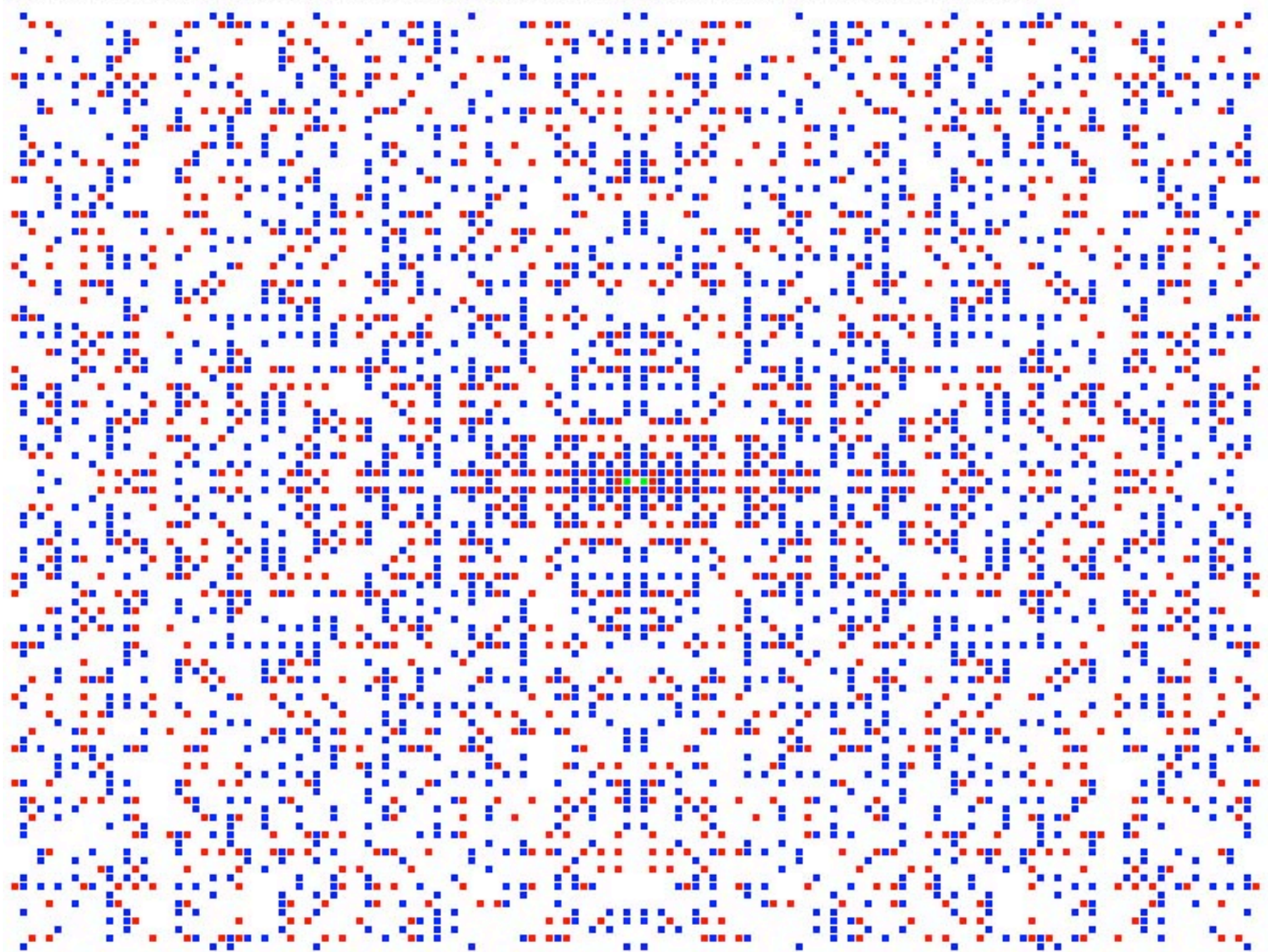
chi

prime numbers

units

prime ideals by norm 2

0+0-0-0-0+000+0+0-0+0+0+0+0+0-0+0+000+0-0-0-0+0-0+0+0+0-000-0-0+0-0-0-0+0-0-000-0+0+0+0-



$Q(\sqrt{-35})$

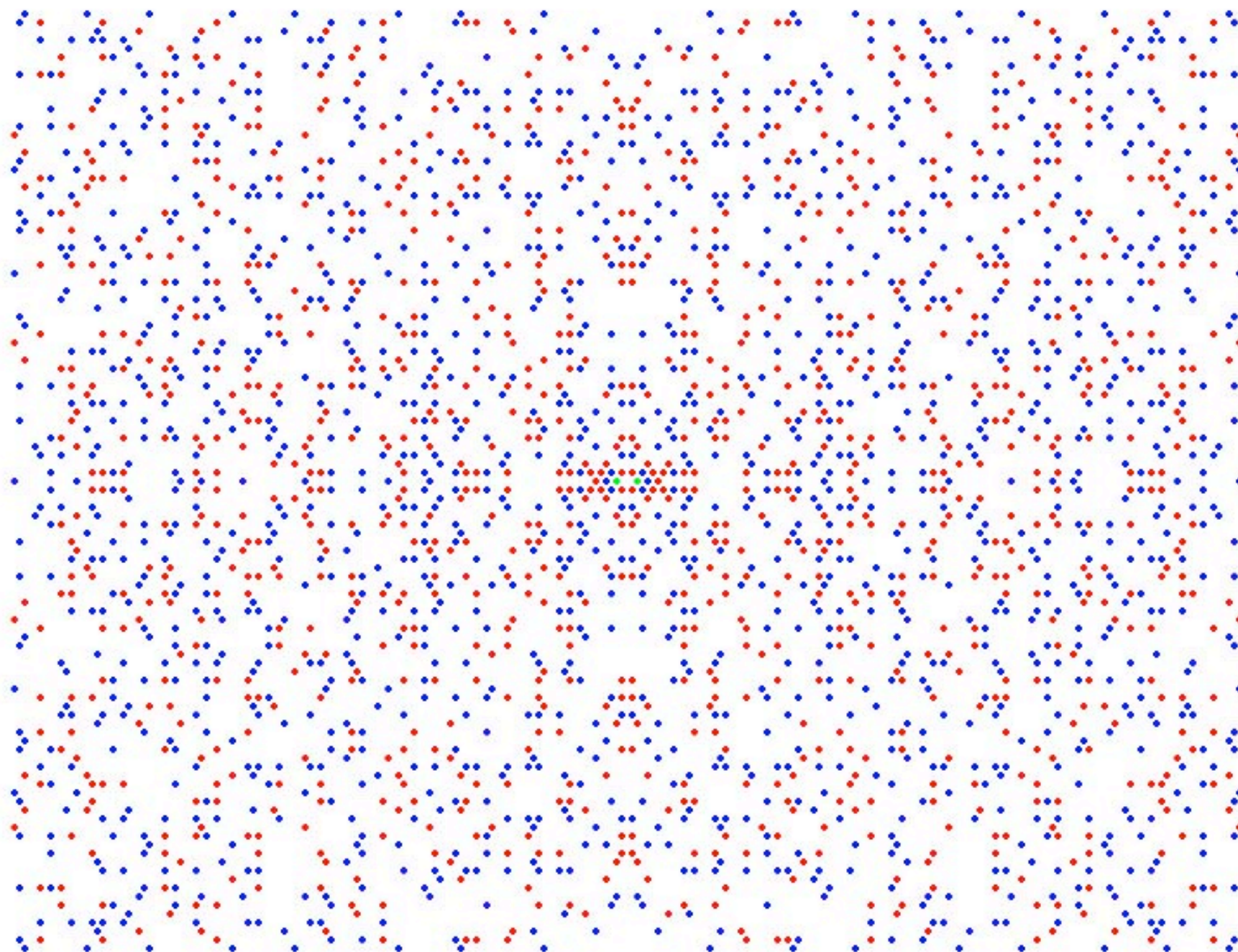
chi

prime numbers

units

prime ideals by norm 3

0++++0-0-+0++++00+++--00---0-+0+0---+



$Q(\sqrt{-37})$

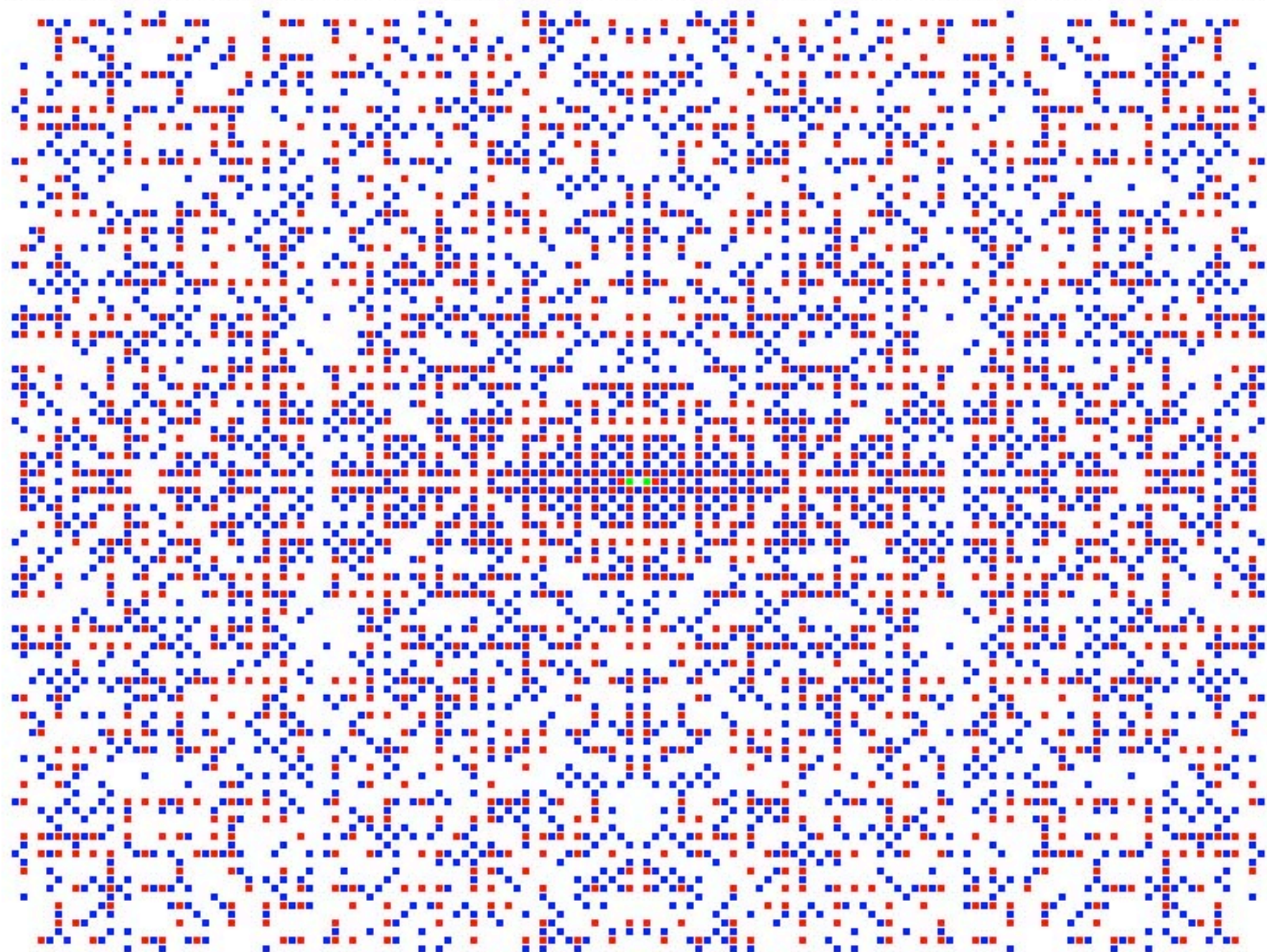
chi

prime numbers

units

prime ideals by norm 2

0+0-0-0-0+0-0-0+0-0+0+0+0+0-0-0+0+0+000+0+0+0-0-0+0+0+0+0-0+0-0-0+0-0-0+0-0+0+0+0-0+0+0-0+0-0-0+0+0-0-



$Q(\sqrt{-51})$

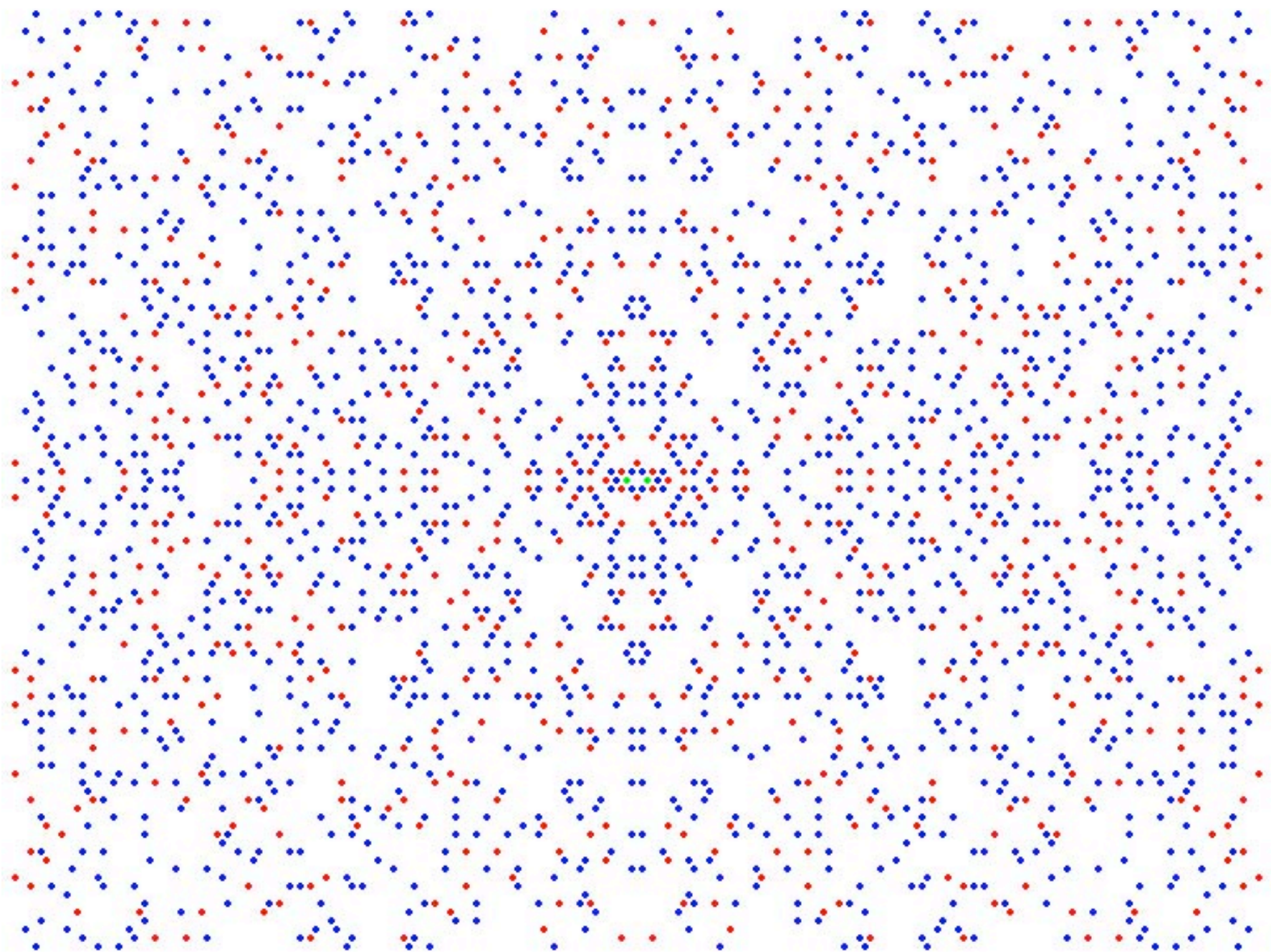
chi

prime numbers

units

prime ideals by norm 3

0+-0++0--0++0+00++0--0+-0--0--00-0--0++0--0+-



$Q(\sqrt{-58})$

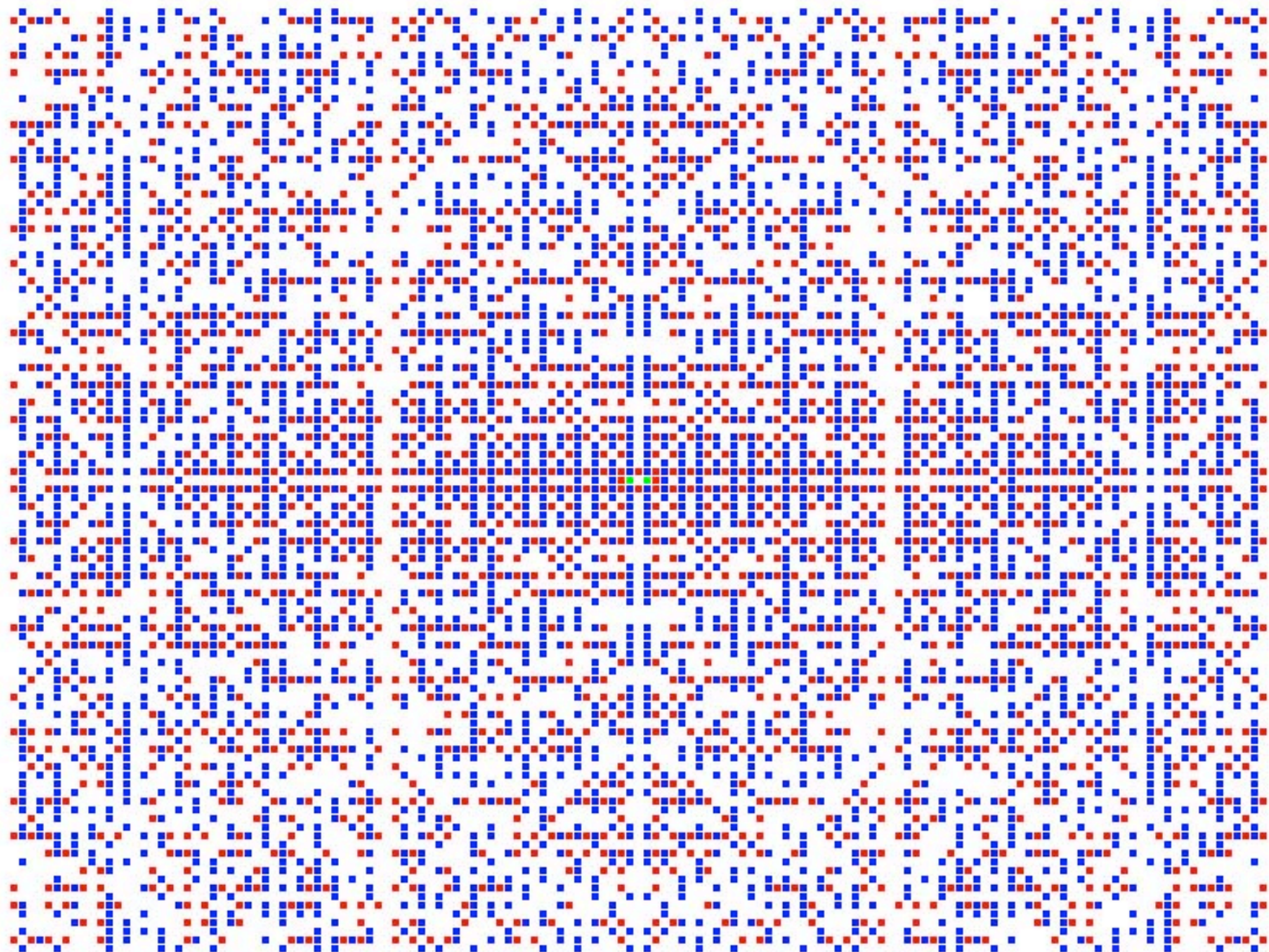
chi

prime numbers

units

prime ideals by norm 2

0+0-0-0-0+0-0-0+0-0-0+0-0-0+0-0-0-000+0+0+0+0+0-0-0-0+0+0+0-0+0+0+0-0+0+0+0-0-0-0+0+0+0+0+000-0+0-0+0-0-0+0-0-0+



$Q(\sqrt{-91})$

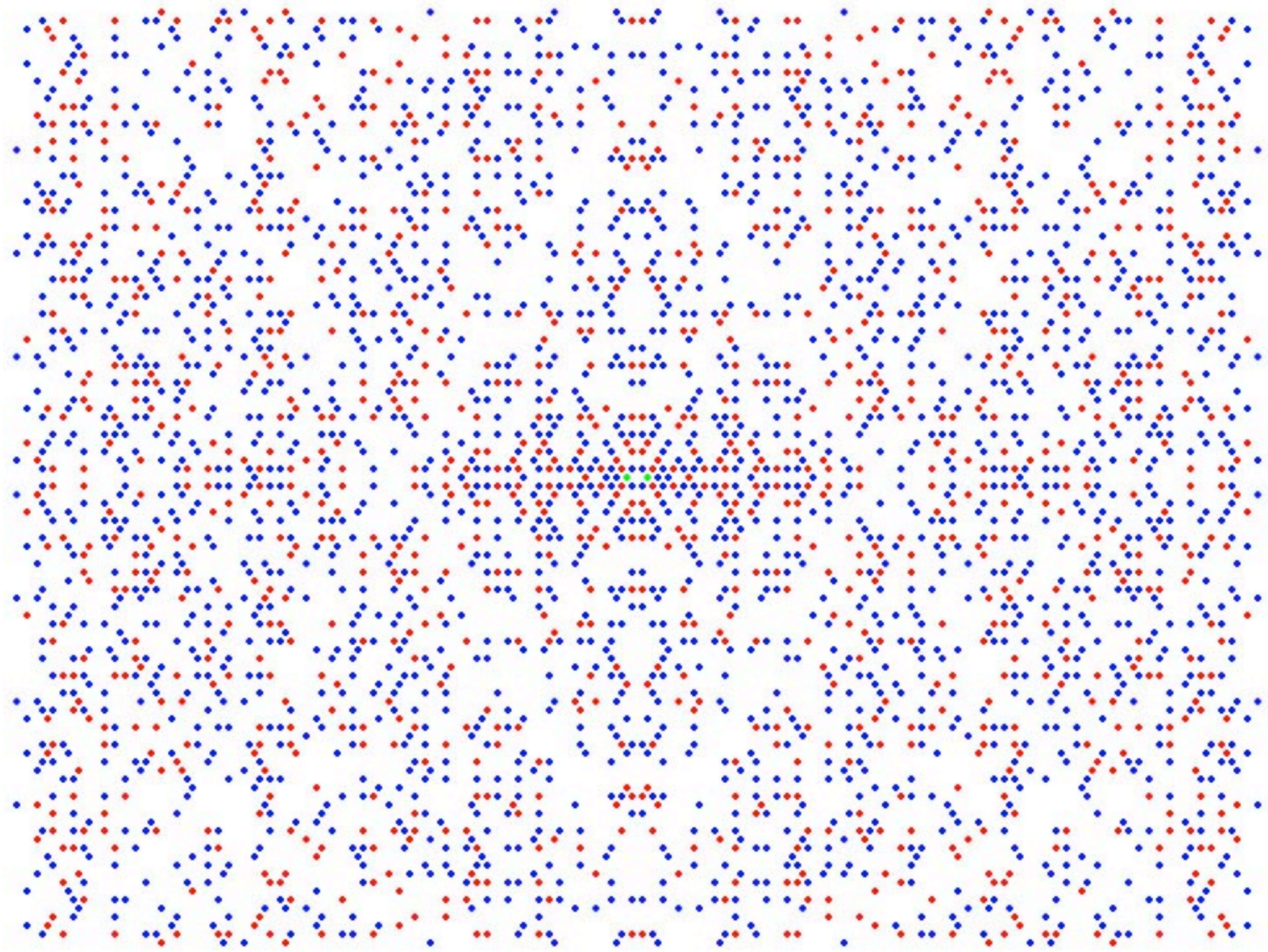
chi

prime numbers

units

prime ideals by norm 5

0+---++0+---00+---++0++++0-0++++++0+---0+0+---++0+0++0+---0+0---0+---++00++++0+---++



$Q(\sqrt{-123})$

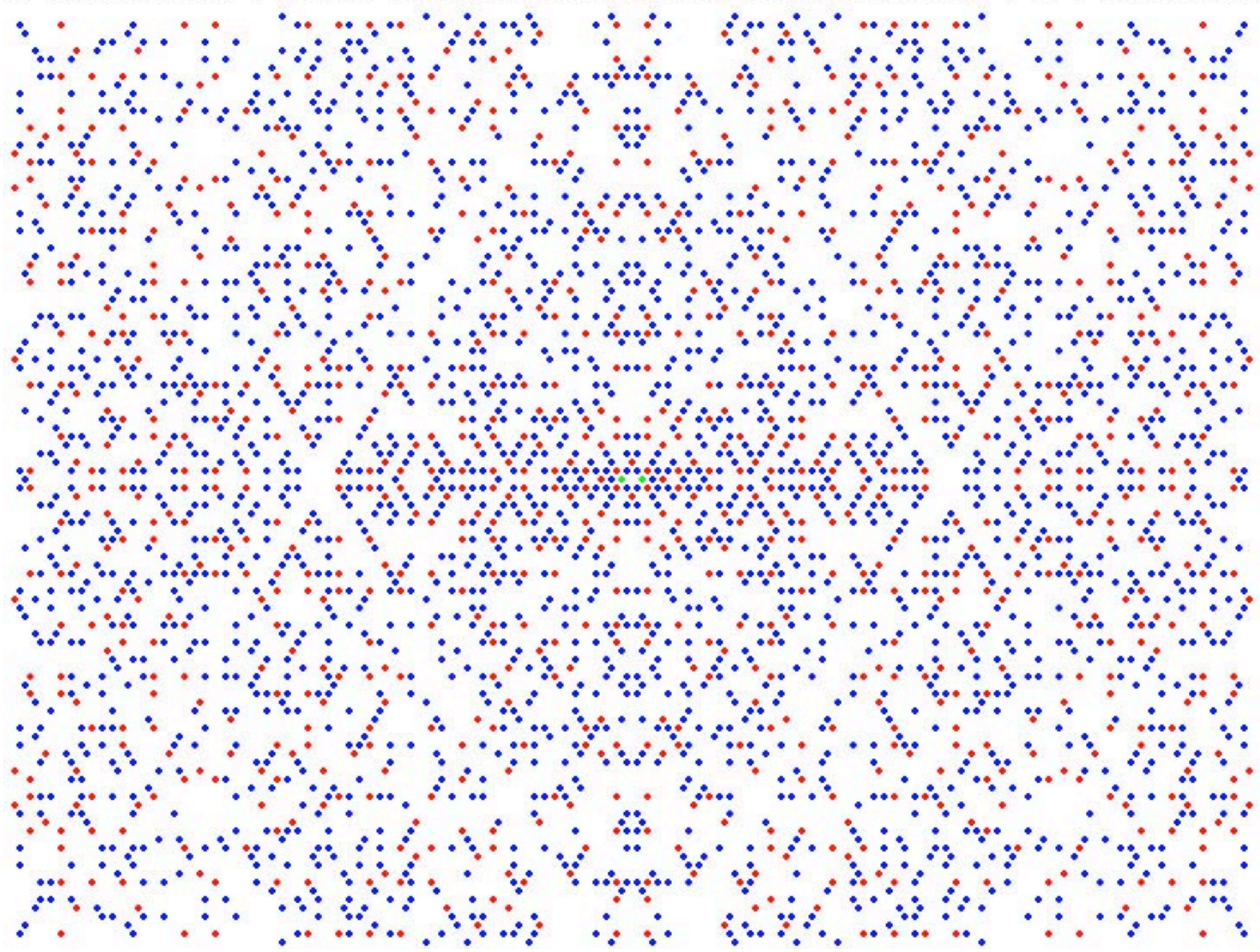
chi

prime numbers

units

prime ideals by norm 3

0+-0+-0--0++0-+0++0--0--0++0-+0+-0-+0++0+00++0++0+-0-+0-+0--0-+0++0-+0-+0--0--00-0--0-+0-+0-+0--0++0++0--0--



$\mathbb{Q}(\sqrt{-187})$

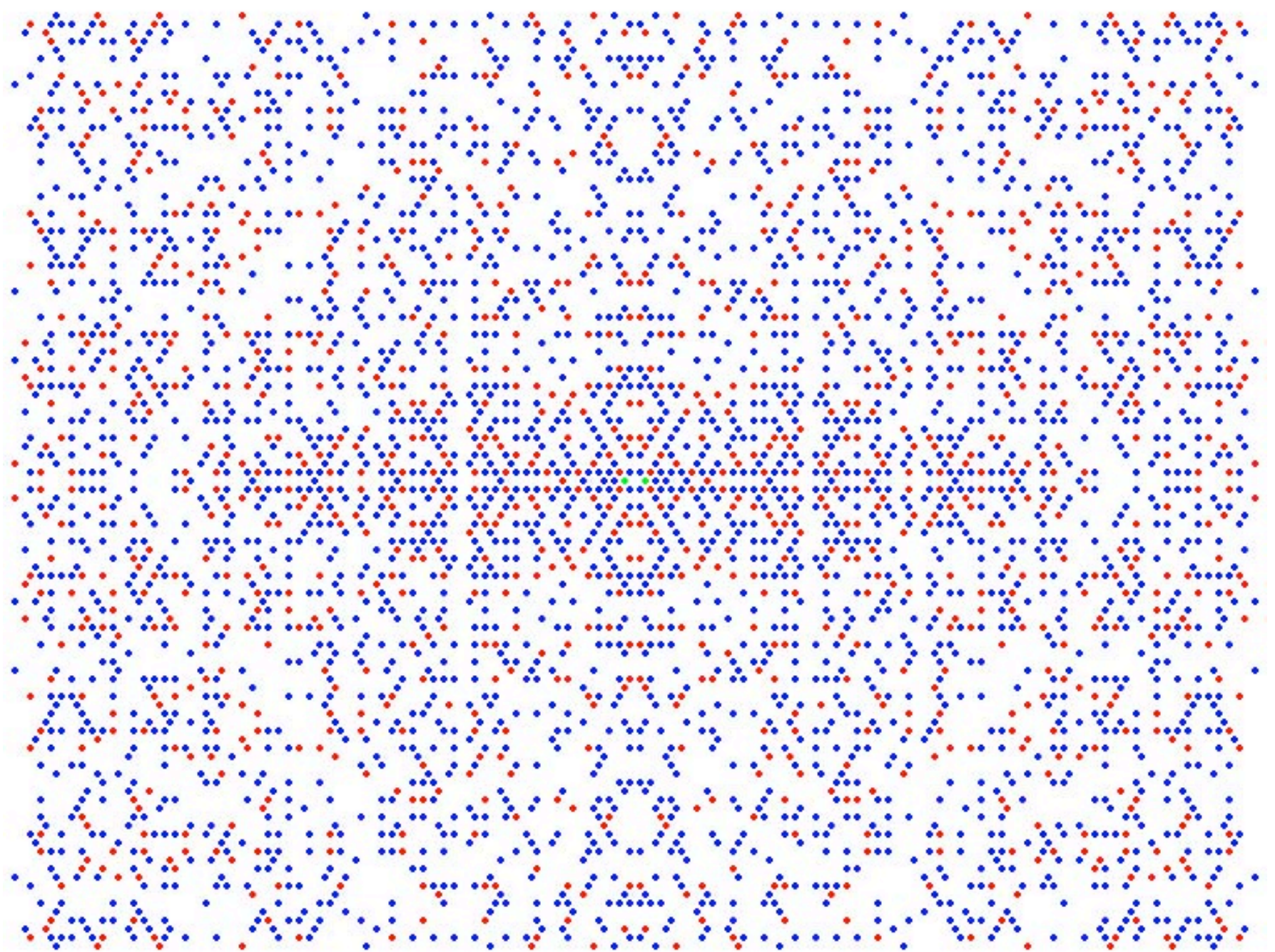
chi

prime numbers

units

prime ideals by norm 7

0+---+---+---+0---+---+0---0+---+---+---+---+00-+---+---+---+0-+---+---+0-+---+0+---+---+---+---+0+0+---+---+---+0-+---+---+0-+---+---+---+0-+0+---+---+---+0-+0+---+---+



$Q(\sqrt{-267})$

chi

prime numbers

units

prime ideals by norm 3

0+-0+-0--0+-0+-0+-0--0++0++0+-0--0++0+-0++0--0--0+-0--0++0+-0+-0++0+-0--0++0+-0+-0++0+00++0++0+-0++0+-0+

