Appendix 8: Pictures of prime numbers and ideals for real fields of class number 3

The pictures show the quadratic character and a picture of prime numbers, units and two mutually conjugate classes of non-principal prime ideals, one class red, and the other class green for some real quadratic fields of class number 3, namely

the fields of discriminant congruent 0 modulo 4:

Q(√79), Q(√142), Q(√223), Q(√254), Q(√326), Q(√359)

and the fields of discriminant congruent 1 modulo 4:

Q(√-229), Q(√257), Q(√321), Q(√469), Q(√473).

At the top, each picture mentions the field, ℚ(√r), and displays its quadratic character (as far as space allows).

In the pictures, rational integers are placed on the x-axis and numbers of the form √r times rational integers on the y-axis.

When d ≡ 0 modulo 4, we use a square grid, otherwise a staggered grid, where the grid points form roughly equilateral triangles.

The pictures display the prime numbers, which generate the principal prime ideals, but not those irreducible numbers which are not prime.

Moreover, the non-principal prime ideals are displayed as follows.

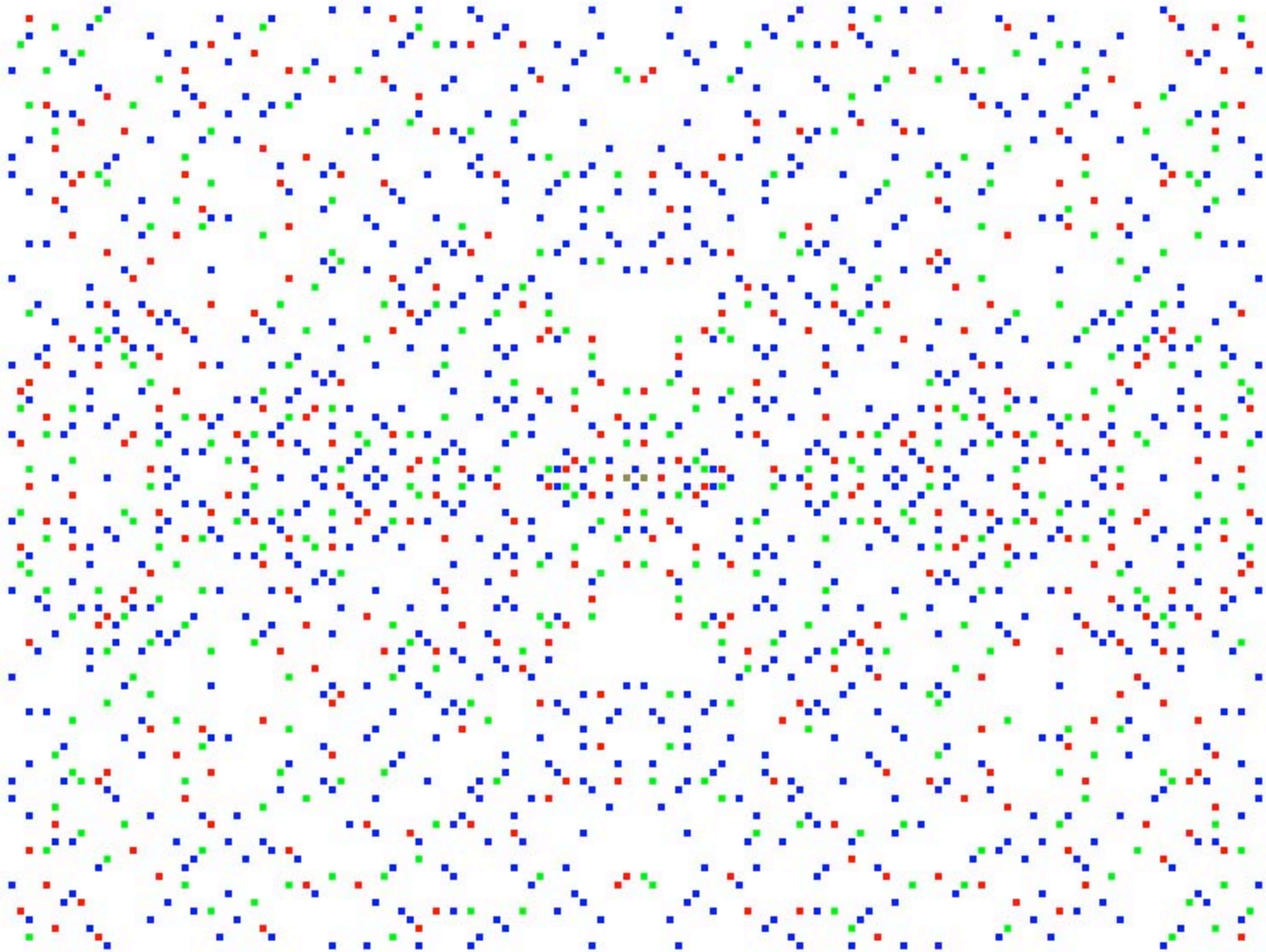
The non-principal ideals are obtained by dividing principal ideals by a certain non-principal prime ideal, I, or its conjugate, where I has the form

I := [norm, shift + (d mod 4 + √d) / 2],

i.e. I is generated by 'norm' being its norm, and the element 'shift + (d mod 4 + √d) / 2'.
In the picture, the non-principal prime ideals then are represented by those numbers whose norm is equal to a prime norm times the norm of I. This norm of I and shift are mentioned at the top of the picture, shift being needed to distinguish between the two mutually conjugate classes of non-principal ideals.

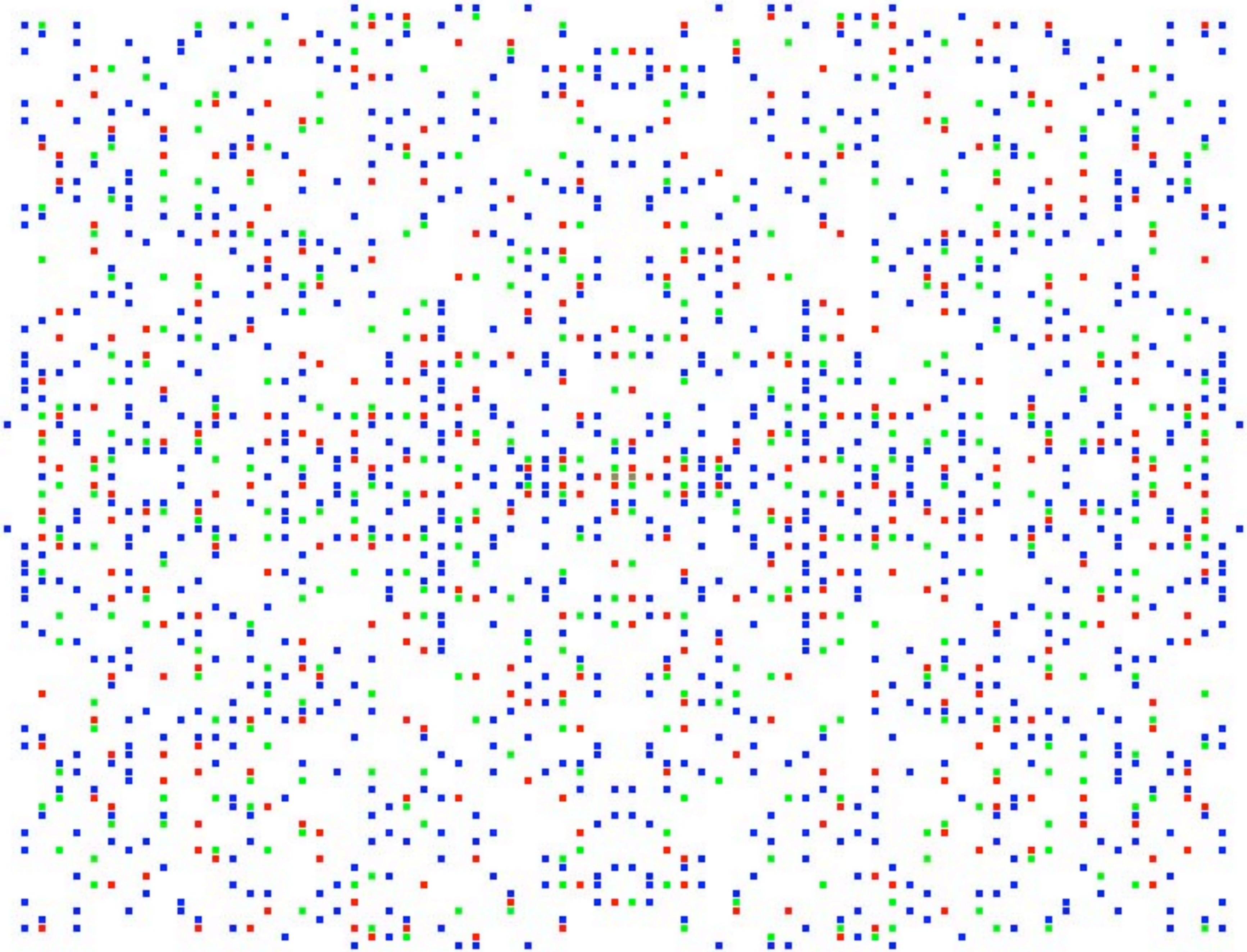
Q(√79)   chi   prime numbers   units   prime ideals by norm 3 shift 1

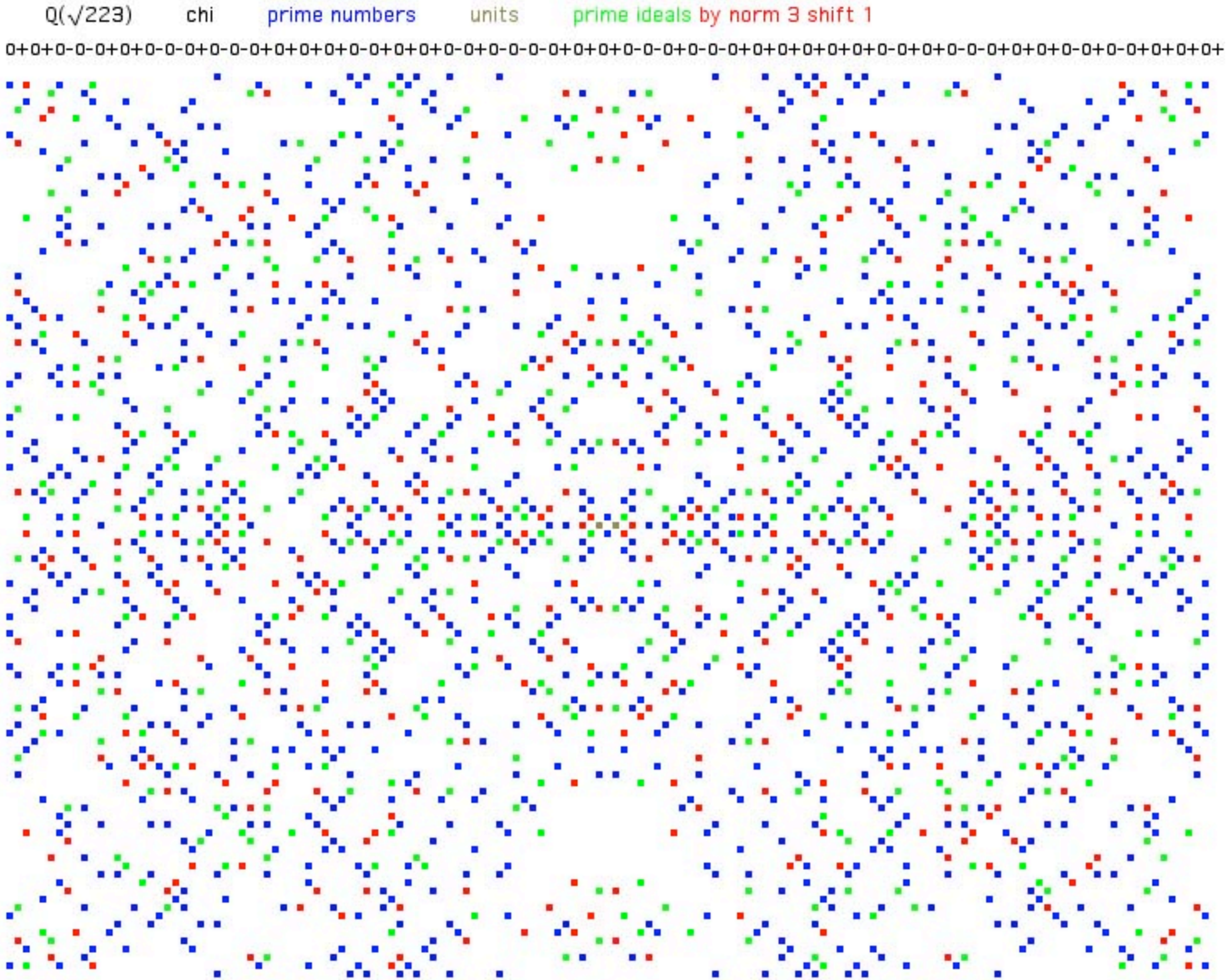
0+0+0+0+0+0-0+0+0-0-0+0-0+0+0-0-0-0+0-0+0-0+0+0+0+0-0-0-0-0+0-0+0+0-0-0+0+0+0-000+0-0-0-0+0+0-0-0+0-0+0+0+0-



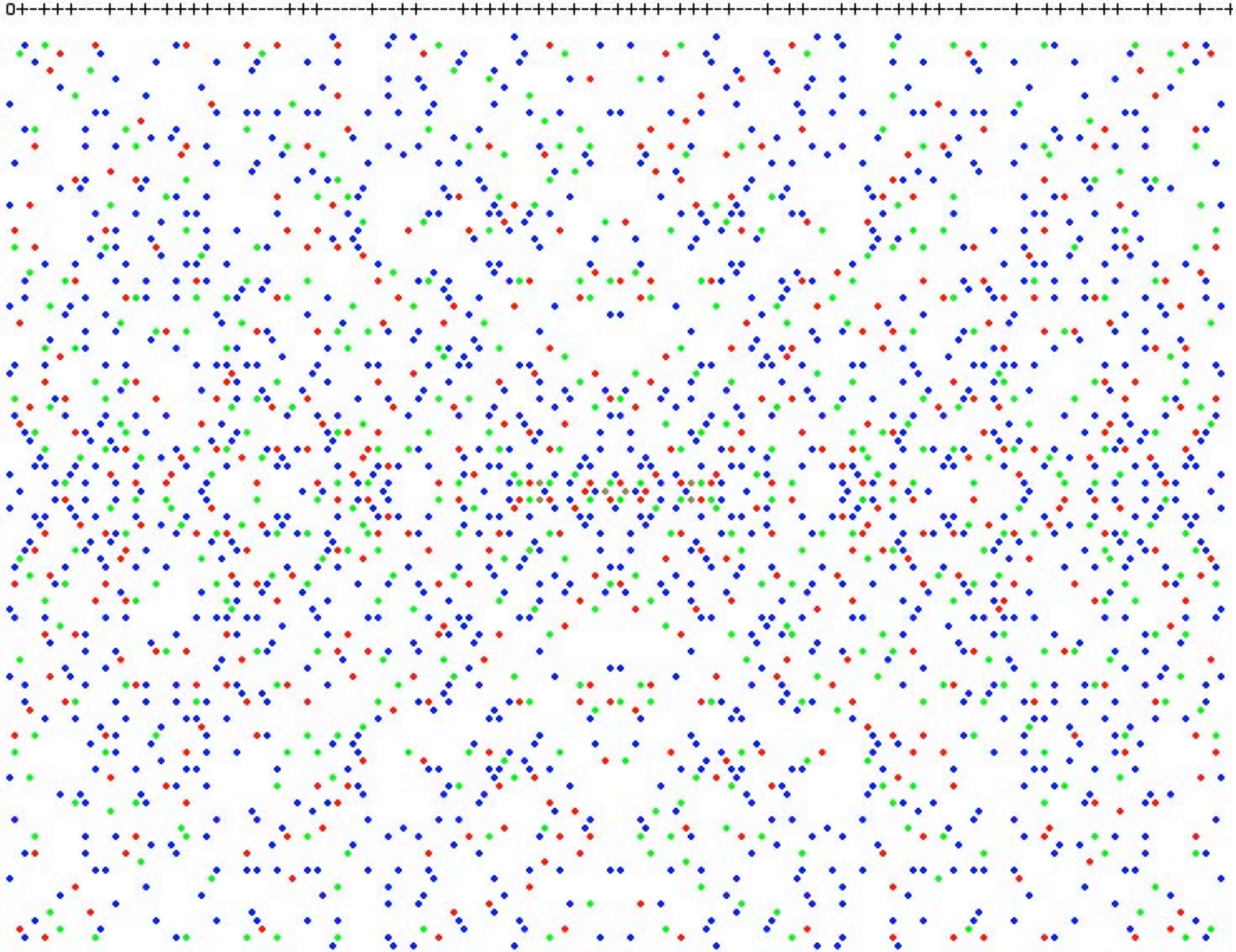
Q(√142) chi prime numbers units prime ideals by norm 3 shift 1

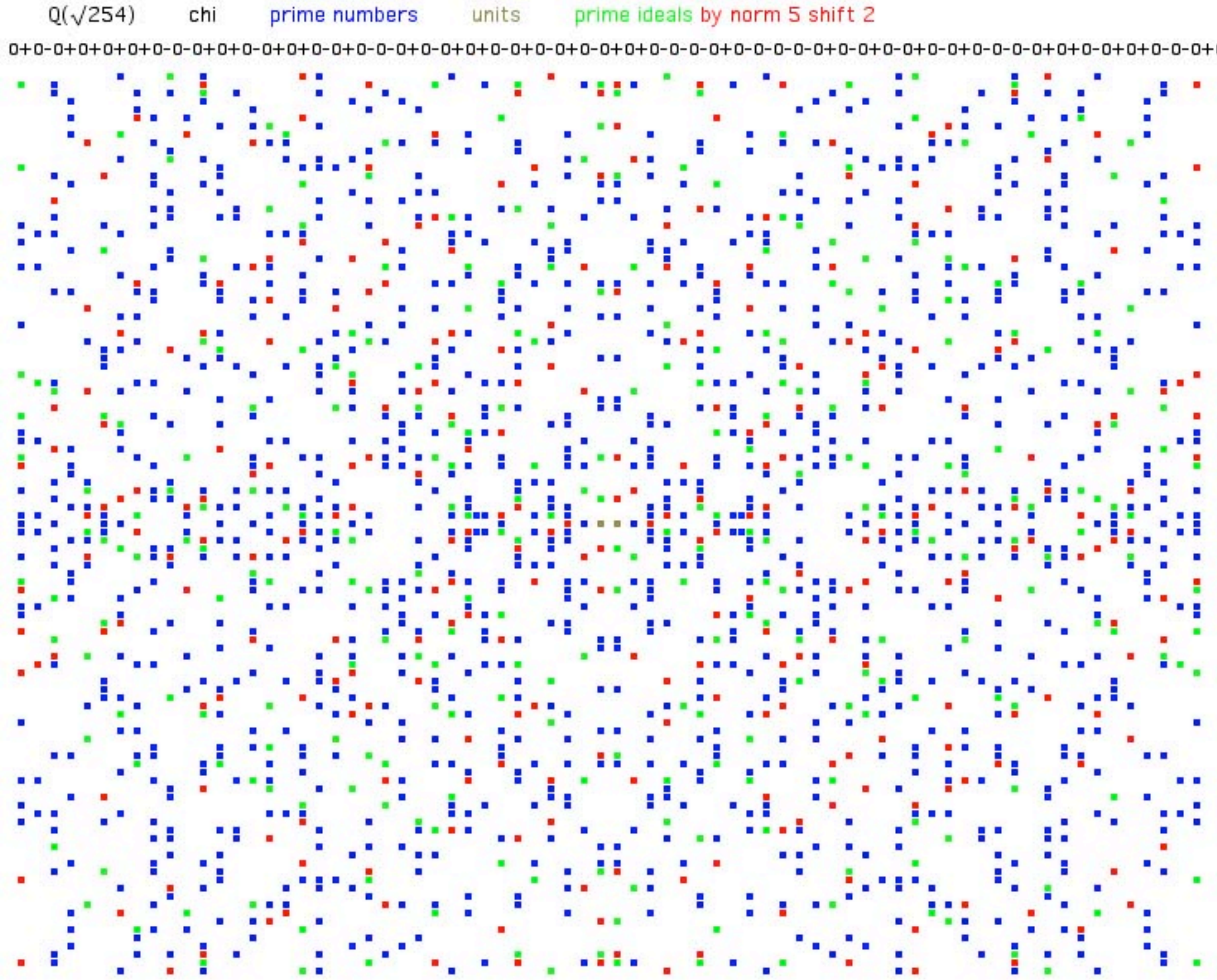
0+0+0-0+0+0-0+0-0-0+0+0+0+0+0-0+0-0-0-0+0-0+0-0+0+0-0+0+0+0-0+0+0-0-0+000+0+0-0-0+0+0+0-0+0+0+0-0-0-0-0-0-0-0-

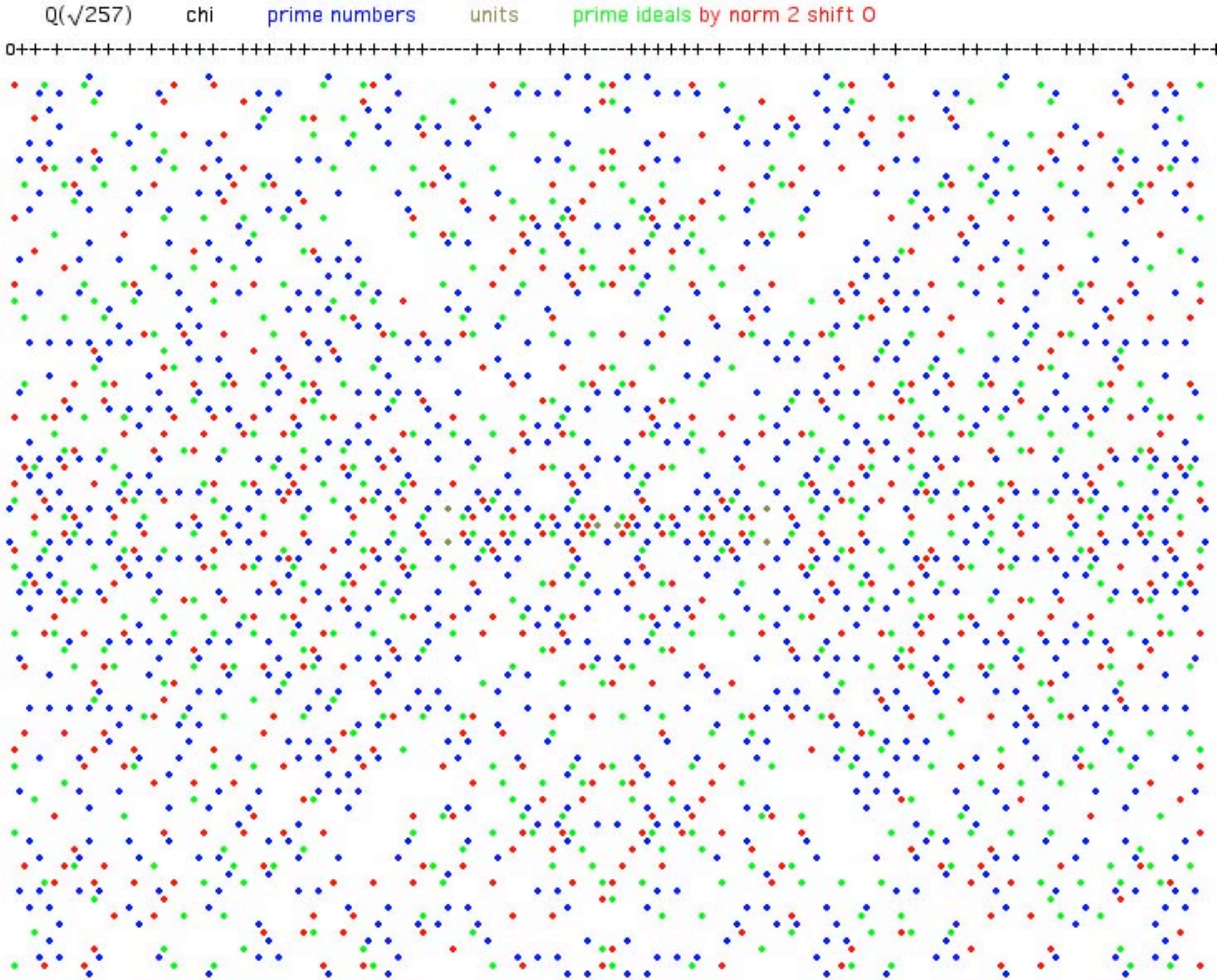


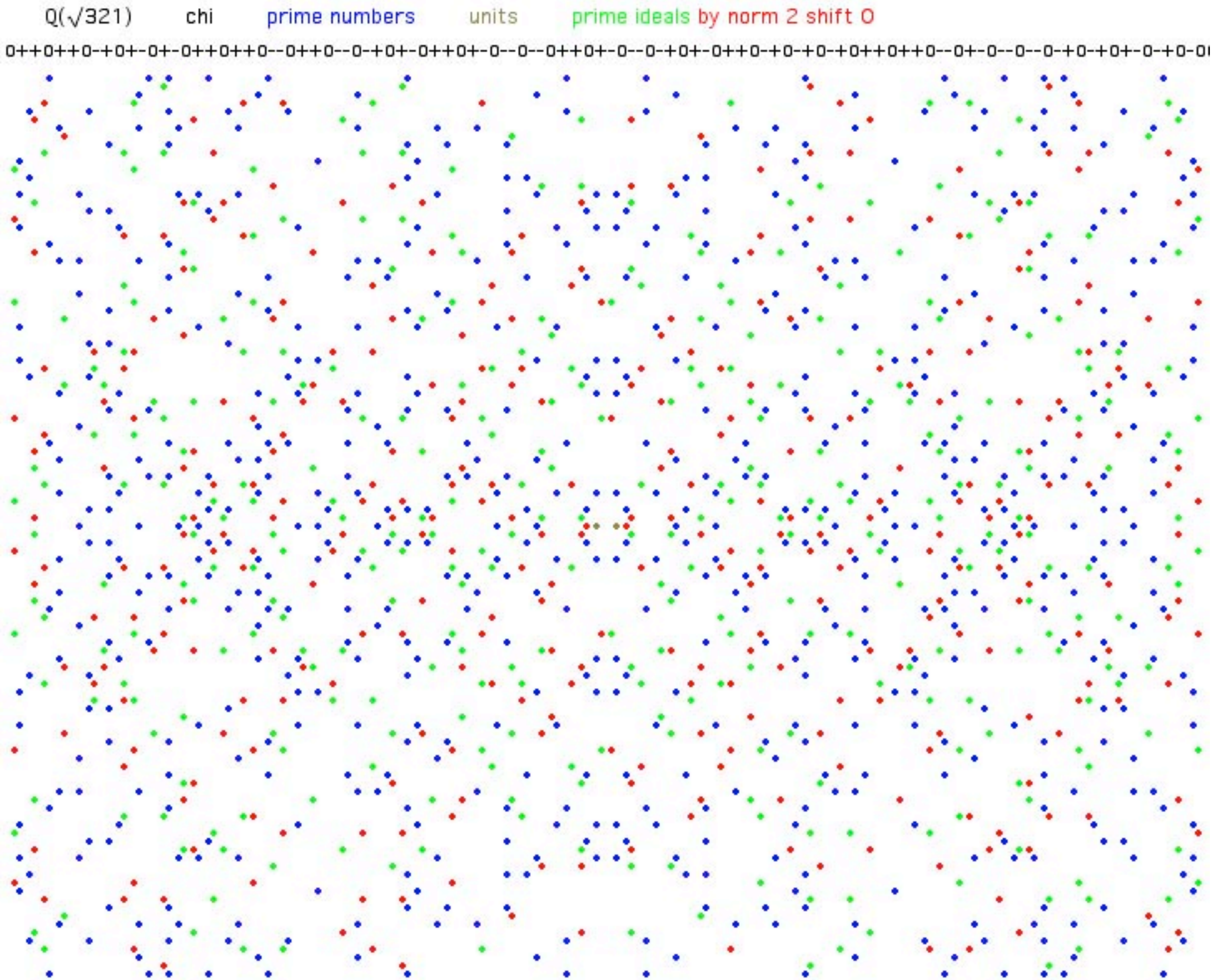

Q(√223)
chi
prime numbers
units
prime ideals by norm 3 shift 1
0+0+0-0-0+0+0-0-0+0-0-0+0+0+0+0-0+0+0+0-0+0-0-0-0+0+0+0-0-0+0-0-0+0+0+0+0+0+0-0+0+0-0-0+0+0+0-0+0-0+0+0+0+

Q(√229)   chi   prime numbers   units   prime ideals by norm 3 shift 0



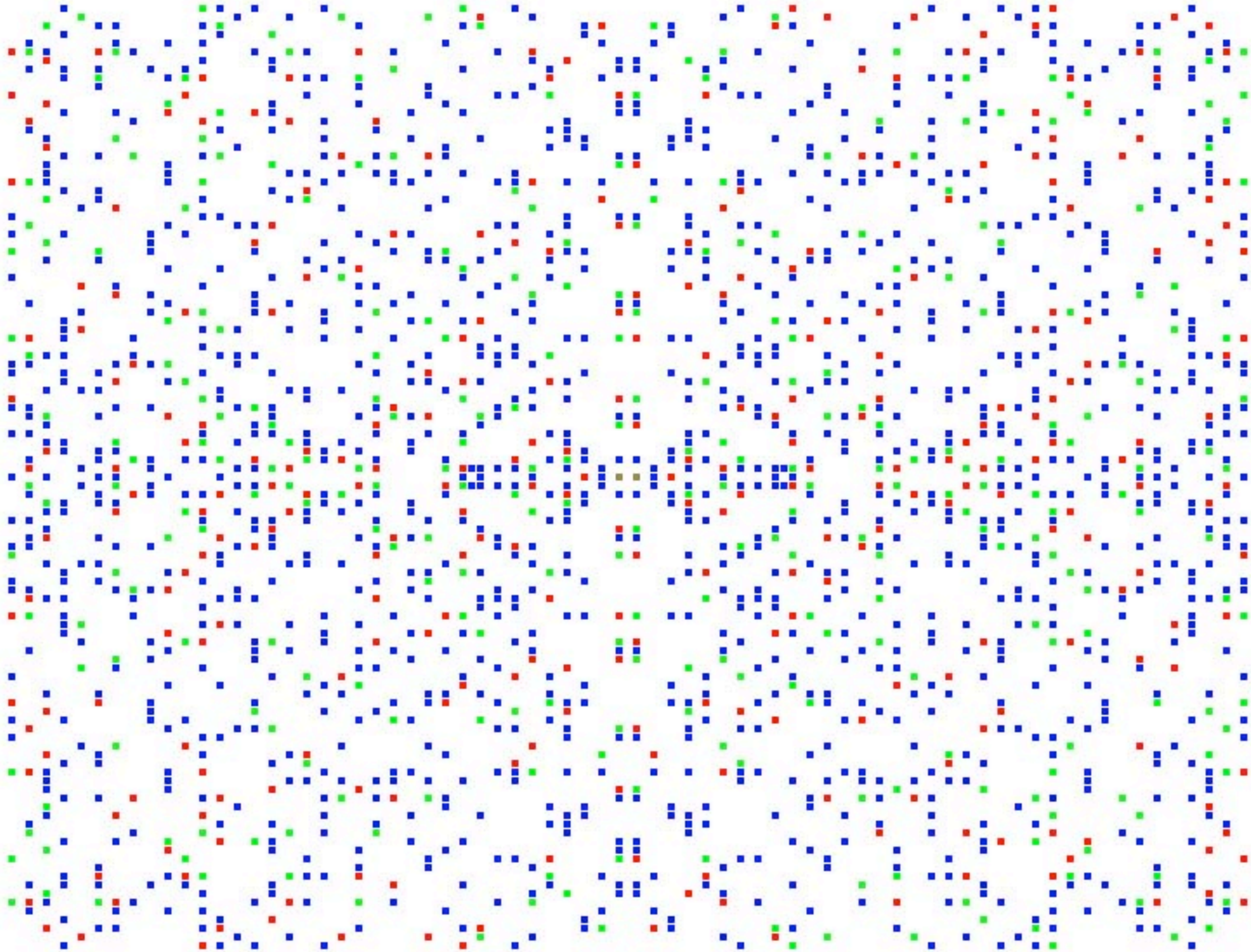

Q(√254) chi prime numbers units prime ideals by norm 5 shift 2
0+0-0+0+0+0+0-0-0+0+0-0+0+0-0+0-0-0+0-0+0+0-0+0-0+0-0+0-0+0+0-0-0-0-0+0-0-0-0-0+0-0+0-0+0-0+0-0-0-0-0+0+0-0-0+0+0-0-0-0+

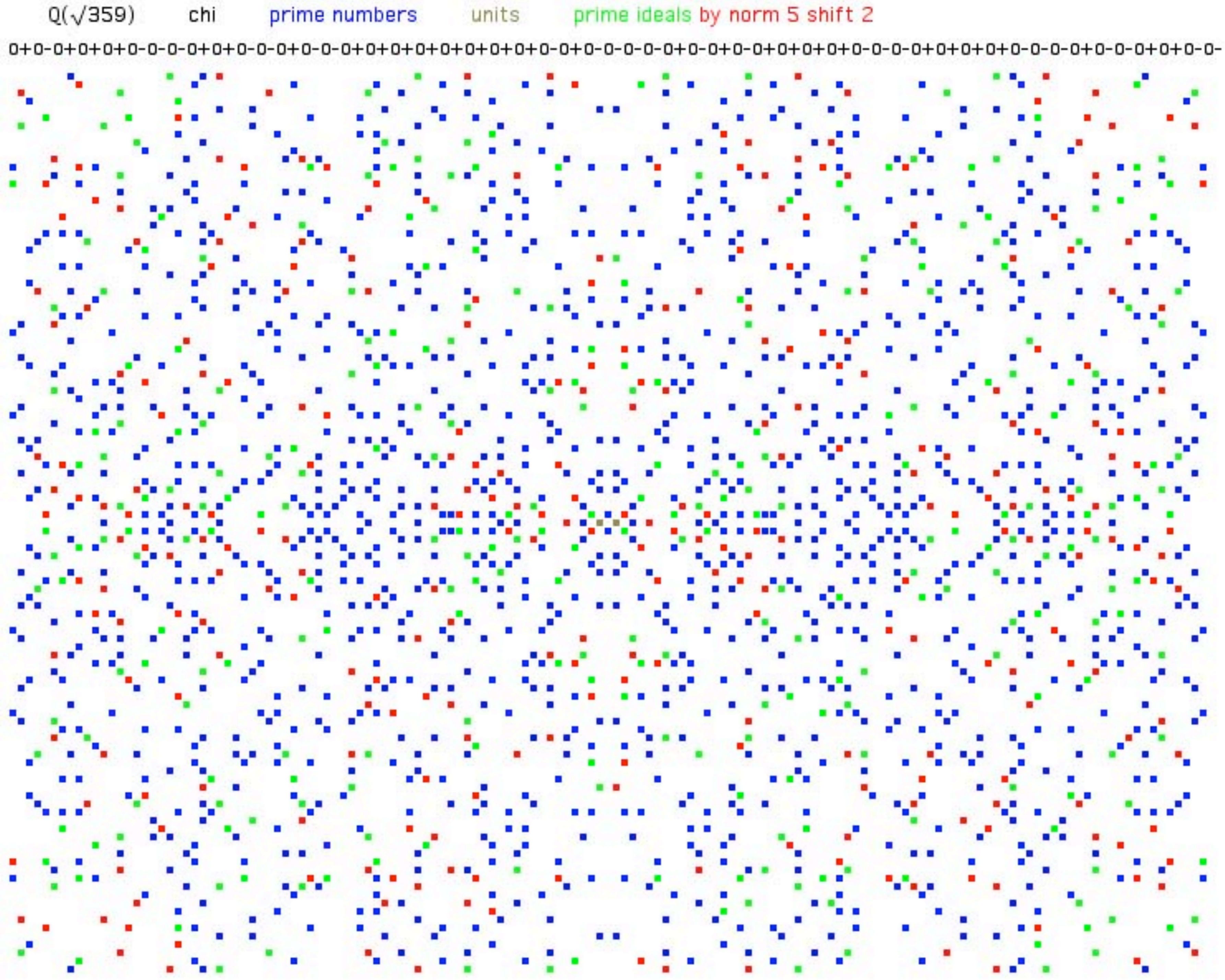

Q(√257)
chi
prime numbers
units
prime ideals by norm 2 shift 0

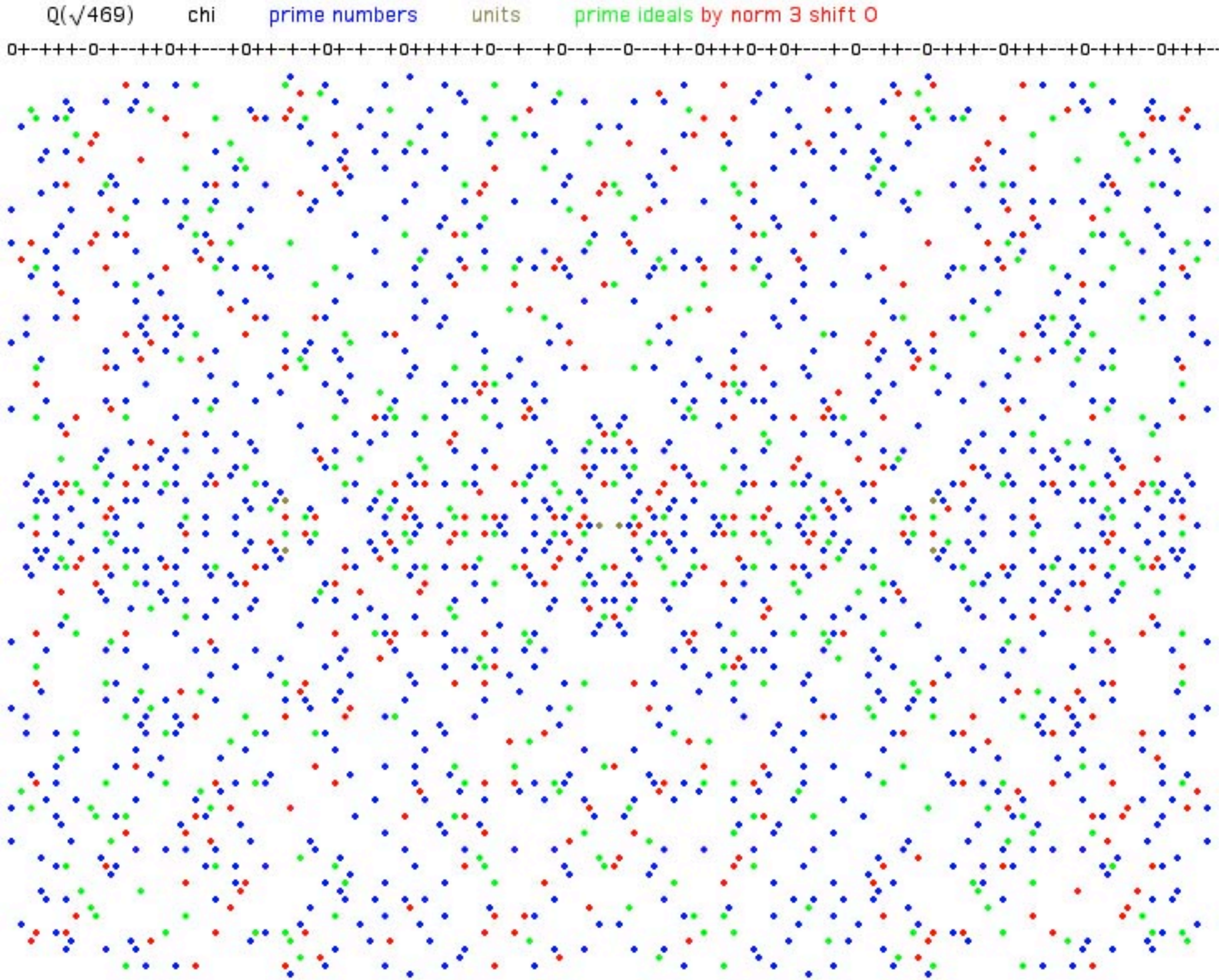

Q(√321)
chi
prime numbers
units
prime ideals by norm 2 shift 0
0++0++0-+0+-0+-0++0++0--0++0--0-+0+-0++0+-0--0--0++0+-0--0-+0+-0++0-+0-+0-+0++0++0--0+-0--0--0-+0-+0+-0-+0-0

Q(√326) chi prime numbers units prime ideals by norm 5 shift 1

0+0-0+0+0+0-0+0-0-0-0-0+0+0-0+0+0+0+0+0-0+0+0+0-0+0+0-0-0+0-0-0+0+0-0-0-0-0-0-0+0+0+0-0-0-0+0-0-0+0-0+0+0-0-0-




Q(√359) chi prime numbers units prime ideals by norm 5 shift 2
0+0-0+0+0+0-0-0-0+0+0-0-0+0-0-0+0+0+0+0+0+0+0+0-0+0-0-0-0-0+0-0+0-0+0+0+0+0-0-0-0+0+0+0+0-0-0-0+0-0-0+0+0-0-


Q(√469)
chi
prime numbers
units
prime ideals by norm 3 shift 0
0+-+++-0-+--++0++---+0++-+-+0+-+--+0++++-+0--+--+0--+---0---++-0+++0-+0+---+-0--++--0-+++--0+++--+0-+++--0+++--

Q(√473) chi prime numbers units prime ideals by norm 2 shift 0

0++-+--+++-0--+++-++--0+-+--+++++0--+-++---00-++-+++-+-0+-++++++++0+----++--+0+--+---+0-0---+-+--++0+-++-++---