Exercises Portfolio Theory 2007-2008

1. Consider a financial market as in Example 1.8 and assume $N = 3$.

   (a) Characterize explicitly the set $\mathcal{P}$ as a set of vectors (p_1^*, p_2^*, p_3^*).

   (b) Consider a call option $C = (S - K)^+$ for some $K > 0$. The set $\Pi(C)$ will turn out to be an interval. What are the upper and lower limits?

2. Consider a *non-redundant* market (see Definition 1.13). Prove the implication (1.8) as well as the statement just below it (Remark 1.14).

3. Let C be a contingent claim in an arbitrage free market with discounted net gains vector Y. Let

   $$\pi^* := \inf\{m \in [0, \infty] \mid \exists \xi \in \mathbb{R}^d : m + \xi \cdot Y \geq \frac{C}{1+r} \; a.s.\}.$$

   Show that π^* is the lowest price of all portfolios $\bar{\xi}$ that are such that $\bar{\xi} \cdot \bar{S} \geq C$ a.s.

4. Prove the following statements.
   (i) Any (finite) market can be reduced to a non-redundant market.
   (ii) In a non-redundant market the implication $\xi \cdot Y = 0$ $\mathbb{P}$-a.s. $\Longrightarrow \xi = 0$ holds. Conversely, if this implication holds and the market is arbitrage free, then the market is also non-redundant.

5. Consider a market as in Example 1.8 with 1 riskless and d risky assets. Find a relation between n and d such that the market is non-redundant and complete.