Portfolio theory
Errata and additional exercises

Errata and changes

1. page 5, line -12: we take $\mathbb{P}^* \in \mathcal{P}$
2. page 7, line -11: due to the law of one price
3. page 9, example at the end: $p_1 = \frac{s_2-(1+r)\pi}{s_2-s_1}$, $p_2 = \frac{(1+r)\pi - s_1}{s_2-s_1}$, $C(\omega_i) = \xi_0(1+r) + \xi_1 S(\omega_i)$, $\xi_0 = \frac{c_1 s_2 - c_2 s_1}{(s_2-s_1)(1+r)}$ and $\pi^C = \xi_0 + \frac{\xi_1 \mathbb{E}^* S}{1+r}$
4. I changed Definition 1.13 in the sense that we always impose $C \geq 0$.
5. In Example 2.7, line 6: Note that $U(\alpha_{i+1}, 0) > U(\alpha_i, 1)$.
6. Proof of Theorem 2.12: The passage

   First we assume without loss of generality that u_0 and u_1 are not attained, otherwise we replace below $\mathcal{Z}$ with the non-empty set $\mathcal{Z} \setminus U_0^{-1}(\{u_0, u_1\})$. This set is dense in $((z^0, z^1))$, where z^0 and z^1 are such that $u_0 = U_0(z^0)$ and $u_1 = U_0(z^1)$.

   is confusing and will be rewritten.
7. Proof of Lemma 3.8, line 3: take $\mu \succ \lambda \succ \nu$.
8. In the proof of Theorem 3.9 we need the additional assumption that S is separable in order to apply Proposition A.11.
9. Proposition 5.8 : I rewrote part of the proof, hoping that it is clearer now. You can use the following text.

   (b) $\Rightarrow$ (a): Let f be continuous, bounded and increasing. We can obtain f (which is measurable) as the pointwise limit of an increasing sequence of simple functions f_n, that are increasing themselves. To see this, we assume for simplicity that $0 \leq f \leq 1$ and we follow the usual approximation scheme, known from measure theory.

   Let $n \in \mathbb{N}$ and define $E_{ni} = \{(i-1)2^{-n} < f \leq i2^{-n}\}$ for $i = 1, \ldots, 2^n$ and $E_{n0} = \{f = 0\}$. Put

   $$f_n = 2^{-n} \sum_{i=1}^{2^n} (i-1) \mathbf{1}_{E_{ni}} = 2^{-n} \sum_{i=2}^{2^n} (i-1) \mathbf{1}_{E_{ni}}.$$

   Then we know that $f_n \uparrow f$. Using that the E_{ni} are disjoint for each n, $\bigcup_{i \geq j+1} E_{ni} = \{f > j2^{-n}\}$ and $\{f > 1\} = \emptyset$, we rewrite

   $$f_n = 2^{-n} \sum_{i=2}^{2^n} (\sum_{j=1}^{i-1} 1) \mathbf{1}_{E_{ni}} = 2^{-n} \sum_{j=1}^{2^n-1} \sum_{i=j+1}^{2^n} \mathbf{1}_{E_{ni}} = 2^{-n} \sum_{j=1}^{2^n} \mathbf{1}_{\{f > j2^{-n}\}}.$$

   Note that the f_n are also increasing functions. Since f is continuous, the sets $\{f > j2^{-n}\}$ are open and since f is increasing, there are a_{nj} such that $\{f > j2^{-n}\} = (a_{nj}, \infty)$. Hence, $\int f_n \mathrm{d}\mu = 2^{-n} \sum_{j=1}^{2^n} \mu((a_{nj}, \infty)) = 2^{-n} \sum_{j=1}^{2^n} (1 - F_\mu(a_{nj}))$. The rest of the proof is unchanged.

10. Page 41, line 3: add to '$\xi \in \mathbb{R}^d \setminus \{0\}$' the text 'such that $\mathbb{P}(\xi \cdot Y = 0) < 1$'.

11. Check for yourself that the η_i and M in the proof of Theorem 6.5 are finite.

12. In the proof of Theorem 6.5: at the end of the 1st paragraph two $\mathbb{E}$'s are missing; non-redundancy is not needed for 'existence' (2nd paragraph).

13. In the proof of (b) $\Rightarrow$ (c) of Lemma 7.9, the integral over the set $(t, 1]$ should be over $(t, 1)$.

Additional exercises

1. Show by a direct argument (not referring to Theorem 2.6) that $[0,1]^2$ doesn't have a *countable* order dense subset for the lexicographic ordering.

2. Assume that $\succ$ is a continuous preference relation on a connected set $\mathcal{X}$, which is endowed with a topology that is first-countable (this allows you to work below with sequences). Let $\mathcal{Z}$ be a dense subset of $\mathcal{X}$. If $U : \mathcal{X} \to \mathbb{R}$ is continuous and its restriction to $\mathcal{Z}$ is a numerical representation of $\succ$, then U is also a numerical representation of $\succ$ on all of $\mathcal{X}$. To show this, one has to verify the implications (i) $x \succ y \Rightarrow U(x) > U(y)$ and (ii) $U(x) > U(y) \Rightarrow x \succ y$.
   Hints: To show (i) you complete the following steps. Show that there are $z, w \in \mathcal{Z}$ such that $x \succ z \succ w \succ y$. Choose then $z_n, w_n \in \mathcal{Z}$ such that $z_n \to x$ and $w_n \to y$ and finish the proof.
   For (ii) you show first that $U^{-1}(U(y), \infty) \cap U^{-1}(-\infty, U(x))$ is non-void and select $z, w \in \mathcal{Z}$ such that $U(x) > U(z) > U(w) > U(y)$. Use again convergent sequences.