10th Winter School– Energy Markets
Lecture 2
Risk Premia in Energy Markets I

Professor Dr. Rüdiger Kiesel

Faculty of Economics
Chair of Energy Trading & Finance

Centre of Mathematics for Applications,
University of Oslo

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1 Spot-Forward Relationships
- Classical Theory
- Forward Curves and Market Risk Premium

2 The Bessembinder-Lemon Model
- The Spot Market
- Demand for forward positions
- Forward bias

Agenda

1 Spot-Forward Relationships
- Classical Theory
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2 The Bessembinder-Lemon Model

Spot-Forward Relationship: Classical theory

Under the no-arbitrage assumption we have the spot-forward relationship

$$F(t,T) = S(t)e^{(r-y)(T-t)} \tag{1}$$

where r is the interest rate at time t for maturity T and y is the convenience yield.

Spot-Forward Relationship: Classical theory

- In the stochastic model this means

$$F(t, T) = \mathbb{E}_{\mathbb{Q}}(S(T)|\mathcal{F}_t)$$

  where $\mathcal{F}_t$ is the accumulated available market information (in most models the information generated by the spot price).
- $\mathbb{Q}$ is the risk-neutral probability
  - discounted spot price is a $\mathbb{Q}$-martingale
  - ..or, the expected return under $\mathbb{Q}$ is r

Spot-Forward Relationship: Classical theory

We observe normal backwardation: Futures prices are below spot price

- Producers accept paying a premium for securing future production
- This may be caused by hedging pressure for long term investments
- Convenience yield larger than risk-free rate

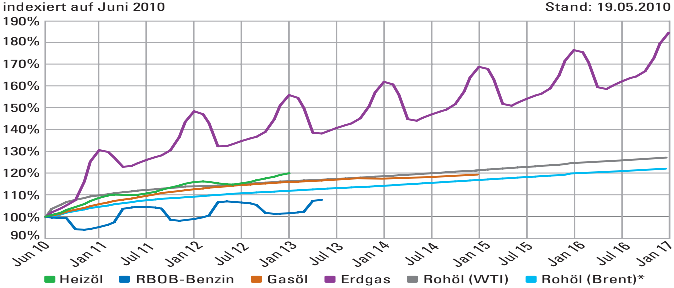
Spot-Forward Relationship: Classical theory

Most models give either normal backwardation or contango (Futures prices are above spot price)

- No stochastic change of sign (risk premium)
- True even for jump models

Forward Curves

Abb. 1: Contango bei fast allen Energieträgern

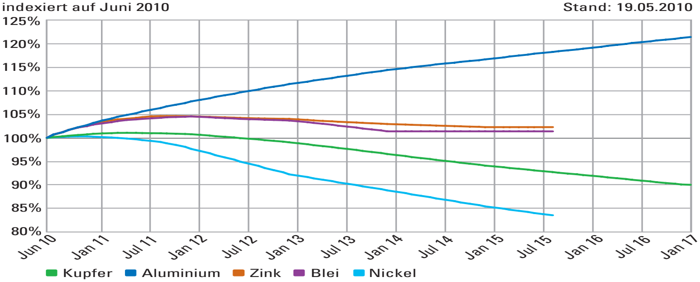


Die Energieträger Heizöl, Gasöl, Erdgas sowie die Rohölsorten Brent und WTI zeigen am kurzen Ende Forwardkurvenverläufe im Contango. *Quelle: Bloomberg L.P.*

* indexiert auf Juli 2010

Forward Curves

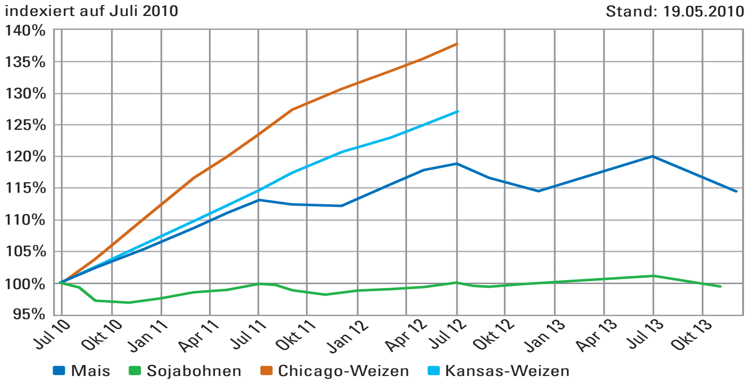
Abb. 2: Contango und Backwardation bei Industriemetallen



Am langen Ende zeigt nur die Aluminium-Forwardkurve den für Industriemetalle eher untypischen steigenden Verlauf, auch Contango genannt. *Quelle: Bloomberg L.P.*

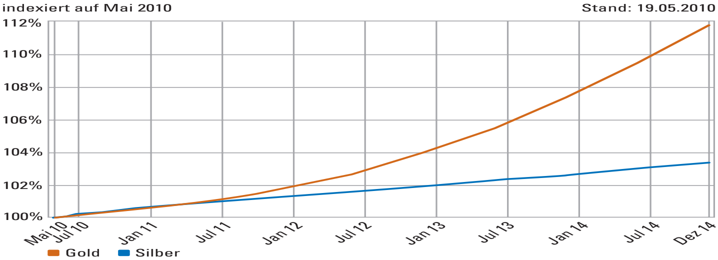
Forward Curves

Abb. 3: Unterschiedliches Bild beim Getreide



Forward Curves

Abb. 6: Contango bei Edelmetallen



Bei den beiden im S&P Goldman Sachs Commodity Index (S&P GSCI®) enthaltenen Edelmetallen Gold und Silber stellt der Contango den normalen Verlauf der Forwardkurve dar. *Quelle: Bloomberg L.P.*

KnowHow **06/2010**

Forward Curves

The shape of commodities forward curves for different delivery periods indicates the market players' (producers, retailers and speculators) 'attitudes' towards risk bearing in these markets.

Contrast to e.g. equity: Here, if interest rates and dividends are assumed deterministic, simple no-arbitrage arguments are employed to show that the arbitrage-free forward price will be the cost of borrowing net of collected dividends yielded by the equity.

Case of Electricity

- Storage of spot is not possible (only indirectly in water reservoirs)
- Delivery periods for Futures
- Buy-and-Hold strategy fails
- No foundation for "classical" spot forward relations

Forward Curves

In electricity markets one normally observes that,

- for 'long' dated forward contracts, markets are in backwardation (forward below spot)
- for 'shorter' maturities the markets are in contango (forward above spot).
- See e.g. Longstaff & Wang (2004, JF); Diko, Lawford & Limpens (2006, Studies in Nonlinear Dynamics & Econometrics)
- How to explain this behavior?

Market Risk Premium

The *market risk premium* or *forward bias* $\pi(t, T)$ relates forward and expected spot prices

It is defined as the difference, calculated at time t, between the forward $F(t, T)$ at time t with delivery at T and expected spot price:

$$\pi(t, T) = F(t, T) - \mathbb{E}^{\mathbb{P}}[S(T)|\mathcal{F}_t]. \tag{2}$$

Here $\mathbb{E}^{\mathbb{P}}$ is the expectation operator, under the historical measure $\mathbb{P}$, with information up until time t and $S(T)$ is the spot price at time T.

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Model specification

- One-period model
- Power companies are able to forecast demand in the immediate future with precision
- N_P identical producers; N_R identical retailers that buy power in the wholesale market and sell it to final consumers at fixed unit price
- P_R fixed unit price that consumers pay
- Q_{R_i} an exogenous random variable that denotes the realized demand for retailer i

The cost function

- Each producer i has cost function

$$TC_i = F + \frac{a}{c}(Q_{P_i})^c,$$

  where F are fixed costs, Q_{P_i} is the output of producer i, and $c \geq 2$.
- The cost function implies that the marginal production costs increase with output.
- If $c > 2$ marginal costs increase at an increasing rate with output.
- Moreover, the distribution of power prices will be positively skewed even when the distribution of power demand is symmetric.

Clearing prices

- First, assume that forward prices are gives
- Obtain optimal behaviour in the spot market
- Work back and find optimal positions in the forward market.

The wholesale spot market

- Producers sell to retailers who in turn distribute to power consumers
- P_W denotes the wholesale spot price, $Q_{P_i}^W$ quantity sold by producer i in the wholesale spot market, $Q_{P_i}^F$ quantity that producer i has agreed to deliver (purchase if negative) in the forward market at the fixed forward price P_F.
- The ex-post profit of producer i is given by

$$\pi_{P_i} = P_W Q_{P_i}^W + P_F Q_{P_i}^F - F - \frac{a}{c}(Q_{P_i})^c,$$

where each producer's physical production, Q_{P_i}, is the sum of its spot and forward sales $Q_{P_i}^W + Q_{P_i}^F$.

The wholesale spot market

- Retailers buy in the real-time wholesale market the difference between realised retail demand and their forward positions
- $Q^F_{R_j}$ quantity sold (purchased if negative) forward by retailer j, P_R fixed retail price per unit
- The ex-post profit for each retailer is

$$\pi_{P_j} = P_R Q_{R_j} + P_F Q^F_{R_j} - P_W(Q_{R_j} + Q^F_{R_j}),$$

- The profit maximising quantity for producer i is (FOC wrt $Q^W_{P_i}$)

$$Q^W_{P_i} = \left(\frac{P_W}{a}\right)^x - Q^F_{P_i}$$

with $x = 1/(c-1)$

The wholesale spot market

- The equilibrium total retail demand is equal to total production and forward contracts are in zero net supply
- Hence we must have that summing over all producers production must equal total demand from retailers

$$N_P \left(\frac{P_W}{a} \right)^x = \sum_{j=1}^{N_R} Q_{R_j}$$

The wholesale spot market

- Therefore the market-clearing wholesale price is

$$P_W = a\left(\frac{Q^D}{N_P}\right)^{c-1},$$

where $Q^D = \sum_{j=1}^{N_R} Q_{R_j}$ is total system demand. We see that when $c > 2$ an increase in demand has a disproportionate effect on power prices.

- Each producers sale in the wholesale market is

$$Q_{P_i}^W = \frac{Q^D}{N_P} - Q_{P_i}^F.$$

Demand for forward positions

- Producers profit (with no forwards) is

$$\rho_{P_i} = P_W \frac{Q^D}{N_P} - F - \frac{a}{c}\left(\frac{Q^D}{N_P}\right)^c.$$

- Retailers profit (with no forwards) is

$$\rho_{R_j} = P_R Q_{R_j} - P_W Q_{R_j}.$$

Mean-Variance Analysis for optimal forward position

Assume that market players

$$\max_{Q^F_{\{P_i,R_j\}}} \mathbb{E}[\pi_{\{P_i,R_j\}}] - \frac{A}{2}\mathbb{V}\text{ar}[\pi_{\{P_i,R_j\}}]$$

where, for example, producers have the profit function

$$\pi_{P_i} = \rho_{P_i} + P^F Q^F - P_W Q^F.$$

FOCs imply

$$Q^F_{\{P_i,R_j\}} = \frac{P^F - \mathbb{E}[P_W]}{A\mathbb{V}\text{ar}[P_W]} + \frac{\mathbb{C}\text{ov}[\rho_{\{P_i,R_j\}}, P_W]}{\mathbb{V}\text{ar}[P_W]}.$$

Mean-Variance Analysis for optimal forward position

- The optimal forward position contains two components
  - The first term reflects the position taken in response to the bias $P^F - \mathbb{E}[P_W]$
  - The second term is the quantity sold or bought forward to minimize the variance of profits
- Forward hedging can reduce risk precisely because the covariance term is generally non-zero.

The equilibrium forward price

- Using the market clearing condition one can show that

$$P_F = \mathbb{E}[P_W] - \frac{N_P}{Nca^x}\left[cP_R \mathbb{C}\mathrm{ov}[P_W^x, P_W] - \mathbb{C}\mathrm{ov}[P_W^{x+1}, P_W]\right],$$

  where $N = (N_R + N_P)/A$ reflects the number of firms in the industry and the degree to which they are concerned with risk.
- The forward price will be less than the expected wholesale price, if the first term in brackets, which reflects retail risk, is larger than the second term, which reflects production cost risk.

Forward bias

We can approximate P^x and P^{x+1} using a Taylor series to see

$$P_F = \mathbb{E}[P_W] + \alpha \mathbb{V}\mathrm{ar}[P_W] + \gamma \mathbb{S}[P_W]$$

where

$$\alpha = \frac{N_p(x+1)}{Nca^x}\left([\mathbb{E}[P_W]]^x - P_R[\mathbb{E}[P_W]]^{x-1}\right)$$

and

$$\gamma = \frac{N_p(x+1)}{2Nca^x}\left(x[\mathbb{E}[P_W]]^{x-1} - (x-1)P_R[\mathbb{E}[P_W]]^{x-2}\right)$$

We see that $\alpha < 0$, since $\mathbb{E}[P_W] < P_R$.

Forward bias

- It must be the case that $\mathbb{E}[P_W] < P_R$
- If the distribution of spot prices is not skewed, $P_F < \mathbb{E}[P_W]$
  - The downward bias in the forward price in the zero-skewness case reflects retailer's net hedging demand, who want to sell in the forward market.
  - The profits of power retailers are positively exposed, on average, because more retail power is sold when P_W is high
  - To reduce risk, retailers want to sell forwards.
- Typically we have skewness $(c > 2)$,
  - So $\gamma > 0$ and the forward price increases with increasing skewness
  - This reflects the fact that the industry wants to hedge against price spikes