Nonlinear valuation under credit gap risk, initial & variation margins, funding costs & multiple curves

14-th Winter School in Mathematical Finance, Jan 26 – 28, 2015

De Werelt, Lunteren, The Netherlands

Prof. Damiano Brigo
Chair, Mathematical Finance Section
Stochastic Analysis Group
Dept. of Mathematics, Imperial College London
and Director of the CAPCO Institute

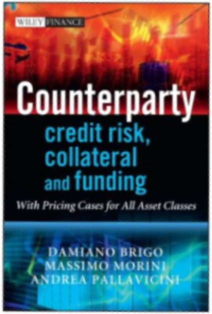
1. Financial Modeling Post 2007-08: Where now?
   - Derivatives models and history
   - Quick introduction to options and derivatives
   - 10 $\times$ planet GDP: Thales, Bachelier and de Finetti
   - The (Black) Scholes & Merton Nobel Award & LTCM
   - Quantitative Finance and the Crisis
   - The case of Collateralized Debt Obligations (CDO)
   - Real modeling problems with synthetic CDO
   - ... but especially Policies and Managerial problems
   - An example of 2006 partial solution for CDOs
   - Dynamics and structured losses: GPC and GPCL models
   - Mathematics and Statistics guilty?
   - Valuation Reloaded

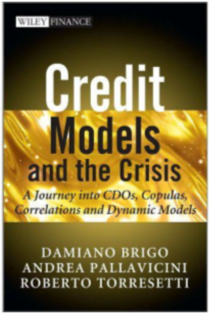
2. Credit Risk under collateralization
   - CVA and DVA for uncollateralized OTC deals
   - Problems with DVA and “perverse incentives”?
   - DVA Hedging?

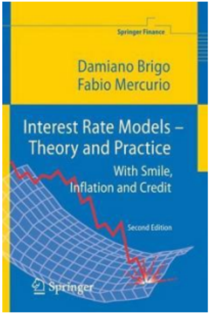
- Funding and Credit VA's in case of EFB policy
- V^0, CVA, DVA, LVA, FCA, FBA, CVA_F, DVA_F
- Double counting in the EFB case
- Funding and Credit VA's in case of RBB policy
- V^0, CVA, DVA, LVA, FCA, FBA=DVA, DVA_F
- Double counting in the RBB policy case

6. Funding Costs: Advanced modeling issues
   - Nonlinear effects: PDEs and BSDEs
   - Black Scholes benchmark case
   - Funding costs, aggregation and nonlinearities
   - Nonlinearity Valuation Adjustment
7. Hints at Multiple Interest Rate curves
8. CCPs: Initial margins, clearing members defaults, delays...
   - Numerical example of CCP costs
   - Numerical example of CCP vs SCSA costs
9. CVA and FVA Desks: Best Practice
10. References

Course based on 2001-15 Research and on Books







Many papers available for free in SSRN, arXiv, Repec, damianobrigo.it
See references at the end of the course

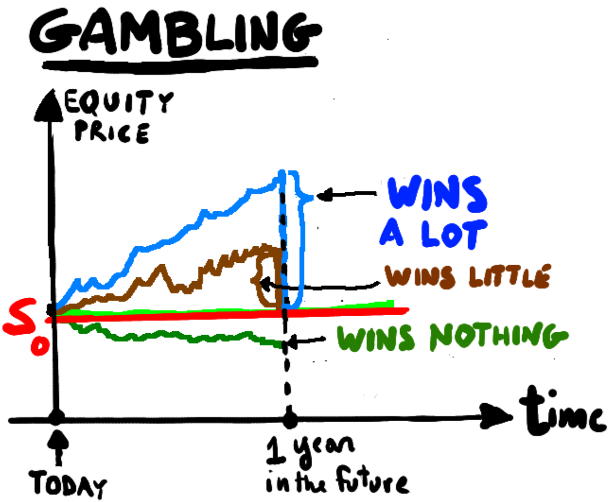


Figure: A one-year maturity Gamble on an equity stock. **Call Option**: $Y = (S_T - K)^+$, $T = 1$year, $K = S_0$.

Valuation of financial products, options and derivatives

An option is a contract built on an underlying asset, for example an equity stock S. Call Option: $(S_T - K)^+$.

To price this options we do this: we try to find a trading strategy in the underlying stock S and on a risk free bank account B that perfectly replicates the option at the final time T.

Replicates: Final value V of the strategy satisfies $V_T = (S_T - K)^+$.

The strategy is also self-financing: It does not require any cash injection (or allow for cash withdrawal).

The initial cost V_0 of setting up the strategy then leads to the price of the option.

This is obtained by a LINEAR (parabolic) PDE that is derived via: *The self financing condition + Ito's formula* (=The Chain rule for Differential Equations driven by Brownian noise).

Valuation of financial products, options and derivatives

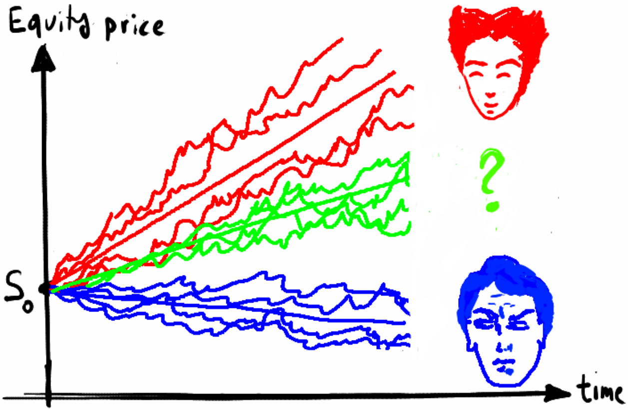
Then we have a theorem (Feynman Kac) that allows to interpret the solution of the PDE as a risk neutral expectation.

Namely: the price of the option is simply an expected value of the discounted payoff $D(t,T)(S_T - K)^+$, *but under a probability measure where the local return of S is the same as the risk free bank account B.*

WE DON'T NEED TO KNOW THE LOCAL RETURN OF S TO PRICE AN OPTION ON S's RETURN!!!

(... which is just as well since the local return is hard to estimate, if you can do that you become very rich...)

This contributed to the popularity of the derivatives markets.



One would think that Red Investor, perceiving a high local growth μ, should be willing to pay a higher price for the call option with respect to Blue Investor, who perceives a low μ. Instead, both have to pay the gamble according to the green scenarios, with local growth r. The volatility σ is a key input of the price, but not the local growth μ.

Valuation of financial products, options and derivatives

However, all the above assumes a lot of things:

- Short selling is allowed
- Infinitely divisible shares
- No transaction costs
- No dividends in the stock
- No default risk of the parties in the deal
- No funding costs: Cash can be borrowed or lent at the risk free rate r
- Continous time and continuous trading/hedging
- Perfect market information
- ....

Derivatives

The market introduced options and more generally financial derivatives that may be much more complex than the above example.

Such derivatives often work on different sectors: Foreign Exchange Rates, Interest Rates, Default Events, Metheorological events, Energy, Mortality rates, etc.

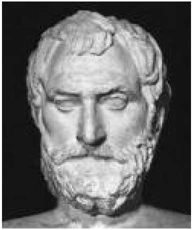
Derivatives can be bought to protect or hedge some risk, but also for speculation or ”gambling”.

However, the Black Scholes theory has often been extrapolated beyond its limits to accomodate these more general derivatives

Options and Derivatives

Derivatives outstanding notional as of June 2011 (BIS) is estimated at **708 trillions USD** (US GDP 2011: 15 Trillions; World GDP: 79 Trillions)

708000 billions, 7.08×10^{14} USD (staggering, despite double counting)

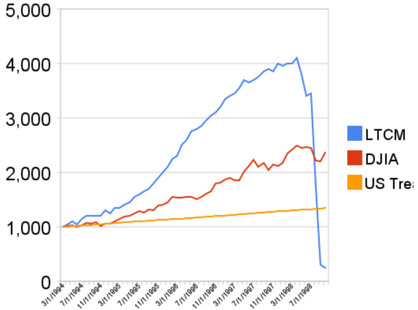


How did it start? It has always been there. Around 580 B.C., Thales purchased options on the future use of olive presses and made a fortune when the olives crop was as abundant as he had predicted, and presses were in high demand. (Thales is also considered to be the father of the sciences and of western philosophy, as you know).

Options and Derivatives valuation: precursors

- **Louis Bachelier** (1870 – 1946) (First to introduce Brownnian motion W_t in Finance, First in the modern study of Options);
- **Bruno de Finetti** (1906 – 1985) (Subjective interpret of probability; defines probabilities in a way that is very similar to current no arbitrage theories: coherent gambling, inequalities constraints, discrete setting, see also Frank Ramsey).

Modern theory follows Nobel awarded **Black, Scholes and Merton** (and then Harrison and Kreps etc) on the correct pricing of options.



Sometimes the timing of the Nobel committee is funny, and we are not talking about the peace Nobel prize. Warning: anedoctal

1997: Nobel award.

1998: the US Long-Term Capital Management hedge fund has to be bailed out after a huge loss. The fund had Merton and Scholes in their board and made high use of leverage (derivatives). This leads us to...

The Credit Crisis: Is this Mathematics fault?

Quantitative Analysts ("quants") and Academics guilty?

Over the past few years a number of articles has disputed the role of Mathematics in Finance, especially in relationship with Counterparty Credit Risk and Credit Derivatives (especially CDOs).
Quants have been accused to be unaware of models limitations and to have provided the market with a false sense of security.

- "The formula that killed Wall Street"[1]
- "The formula that fell Wall Street"[2]
- "Wall Street Math Wizards forgot a few variables"[3]
- "Misplaced reliance on sophisticated (mathematical) models"[4]
- BUT WHAT IS THIS FORMULA PRECISELY?

[1] Recipe for disaster. Wired Magazine, 17.03.

[2] The Financial Times, Jones, S. (2009). April 24 2009.

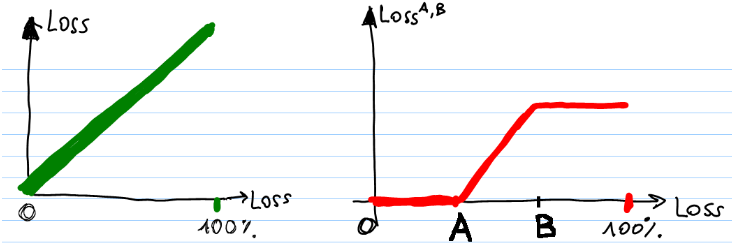
[3] Lohr (2009), New York Times of September 12.

[4] Turner, J.A. (2009). The Turner Review. 03/2009. FSA, UK. www.fsa.gov.uk/pubs/other/turner_review.pdf

CDOs: The standard synthetic case I

- Portfolio of names, say 125. Names may default, generating losses.
- A tranche is a portion of the loss between two percentages. The $3\% - 6\%$ tranche focuses on the losses between 3% (attachment point) and 6% (detachment point).
- The CDO protection seller agrees to pay to the buyer all notional default losses (minus the recoveries) in the portfolio whenever they occur due to one or more defaults, within 3% and 6% of the total pool loss.
- In exchange for this, the buyer pays the seller a periodic fee on the notional given by the portion of the tranche that is still “alive” in each relevant period.
- Valuation problem: What is the fair price of this “insurance”?

CDOs: The standard synthetic case II



- Pricing (marking to market) a tranche: taking expectation of the future tranche losses under the pricing measure.
- From nonlinearity, the tranche expectation will depend on the loss distribution: marginal distributions of the single names defaults **and** dependency among different names' defaults. Dependency is commonly called "correlation".
- Abuse of language: correlation is a complete description of dependence for jointly Gaussians, but more generally it is not.

Copulas

The complete description is either the whole multivariate distribution or the so-called “copula function” (marginal distributions have been standardized to uniform distributions).

CDO Valuation: The culprit.

One-factor Gaussian copula

$$\int_{-\infty}^{+\infty} \prod_{i=1}^{125} \Phi\left(\frac{\Phi^{-1}(1 - \exp(-\Lambda_i(T))) - \sqrt{\rho_i}m}{\sqrt{1-\rho_i}}\right) \varphi(m)dm.$$

“MEA COPULA!” From Nobel award to universal scapegoat

Introduced in Credit Risk modeling by David X. Li. Commentators went from suggesting a Nobel award to blaming Li for the whole Crisis.

The scapegoat

David Li, 2005, Wall Street Journal

[...] ”The most dangerous part,” Mr. Li himself says of the model, ”is when people believe everything coming out of it.” Investors who put too much trust in it or don’t understand all its subtleties may think they’ve eliminated their risks when they haven’t.

Indeed, these models are static. they ignore Credit Spread Volatilities, that in Credit can be 100%; this has further paradoxical consequences in copula models for wrong way risk, as we will see later on.

Tranches and Correlations

The dependence of the tranche on “correlation” is crucial. The market assumes a Gaussian Copula connecting the defaults of the 125 names, parametrized by a correlation matrix with 125*124/2 = 7750 entries. However, when looking at a tranche:

$$7750 \;\; \text{parameters} \longrightarrow \;\; 1 \;\; \text{parameter.}$$

The unique parameter is reverse-engineered to reproduce the price of the liquid tranche under examination. ”Implied correlation”. Once obtained it is used to value related products.

Problem with this implied ”compound correlation”

If at a given time the $3\% - 6\%$ tranche for a five year maturity has a given implied correlation, the $6\% - 9\%$ tranche for the same maturity will have a different one. The two tranches on the *same pool* are priced (and hedged!!!) with two inconsistent loss distributions

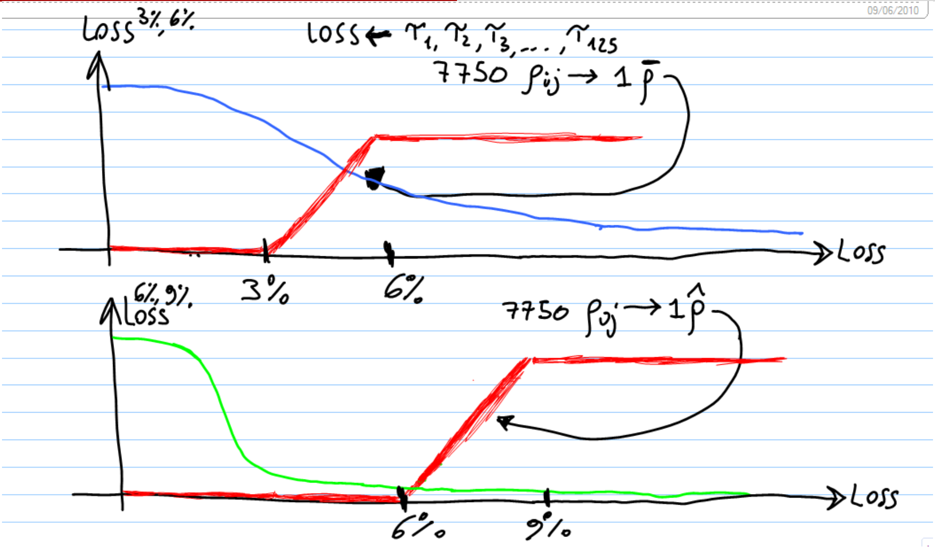


Figure: Compound correlation inconsistency

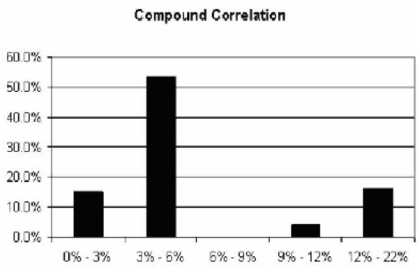


Figure: (After Edvard Munch's The Scream; Compound correlation DJ-iTraxx S5, 10y on 3 Aug 2005)

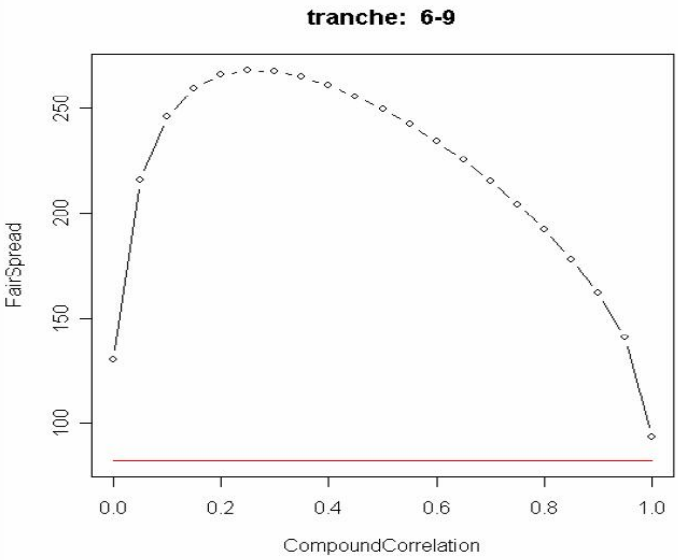


Figure: Non-invertibility compound correl DJ-iTraxx S5, 10y on 3 Aug 2005

Base correlation I

As a possible remedy for non-invertibility of compound correlation and other matters, the market introduced Base Correlation, which is still prevailing in the market.

Problems with base correlation

Base correlation is easier to interpolate but is inconsistent even at single tranche level, in that it prices the $3\% - 6\%$ tranche by decomposing it into the $0\% - 3\%$ tranche and $0\% - 6\%$ tranche and using two different correlations (and hence distributions) for those. This inconsistency shows up occasionally in negative losses (i.e. in defaulted names resurrecting).

[in the graph we use put-call parity to simplify]

Base correlation II

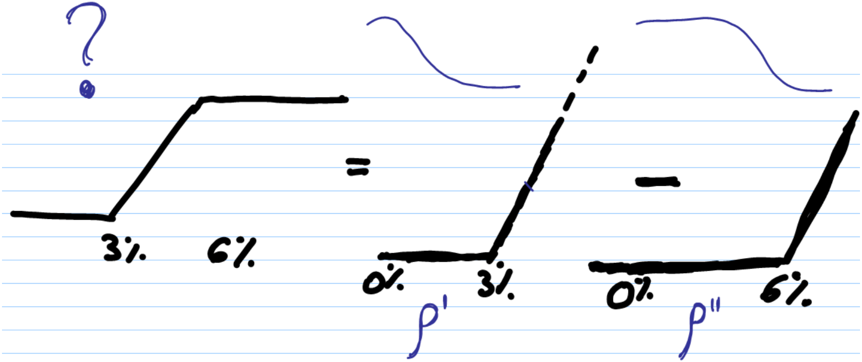
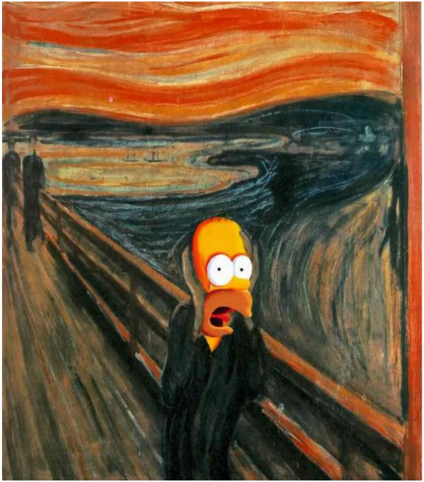


Figure: Base correlation inconsistency

Base correlation III



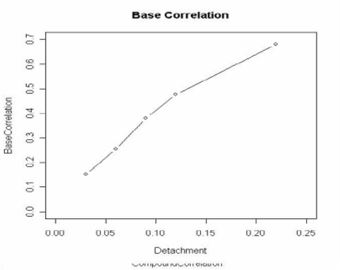


Figure: (Base correl DJ-iTraxx S5, 10y on 3 Aug 2005)

Base correlation

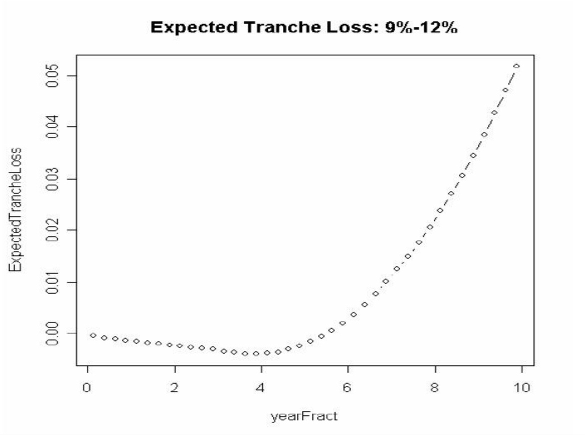
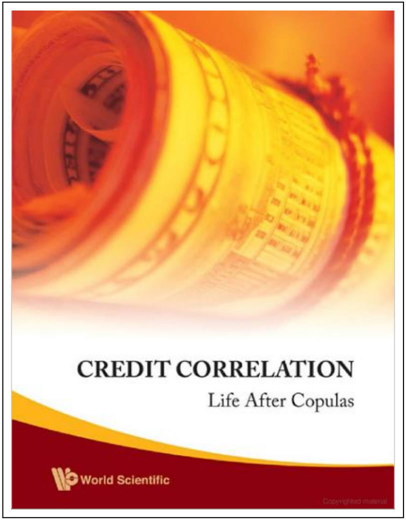


Figure: Expected tranche loss coming from Base correlation calibration, 3d August 2005, First published in 2006. The locally negative loss distribution means there are defaulted names RESURRECTING with positive probability

Some facts

Proceedings of a Conference held in London in 2006 by Merrill Lynch.
A number of proposals to improve the static copula models used (and abused) for credit derivatives have been presented. I was there.
Quants and Academics were well aware (and had been for years) of the models limitations and were trying to overcome them.



A few journalist have very short memory...

12 Sept 2005. Wall Street Journal

How a Formula [Base correlation + Gaussian Copula] Ignited Market That Burned Some Big Investors.

There are many other publications preceeding the crisis started in 2007. Such publications questioned the use of the Gaussian copula and the notion of implied and base correlation. For example, see our 2006 article

Implied Correlation: A paradigm to be handled with care, 2006, SSRN, `http://ssrn.com/abstract=946755`

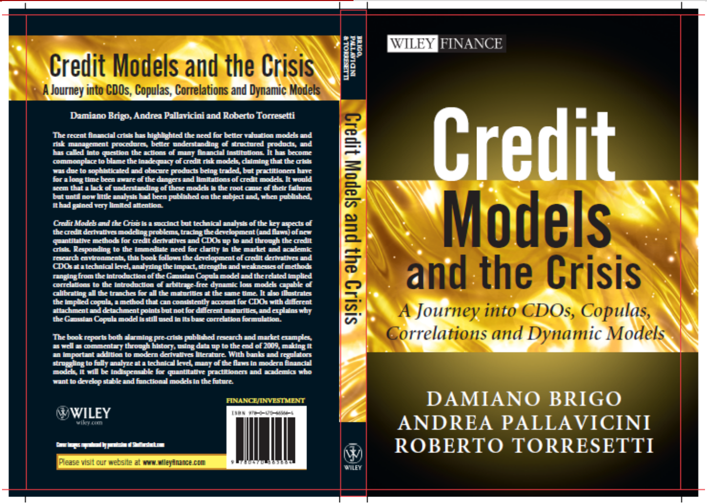


Figure: This book collects research published originally in 2006, warning against the flaws of the industry credit derivatives models. Related papers in the journals *Mathematical Finance, Risk Magazine, IJTAF*

Beyond copulas: GPL and GPCL Models (2006-on)

We model the total number of defaults in the pool by t as

$$Z_t := \sum_{j=1}^{n} \delta_j Z_j(t)$$

(for integers δ_j) where Z_j are independent Poissons. This is consistent with the Common Poisson Shock framework, where defaults are linked by a Marshall Olkin copula (Lindskog and McNeil).

$$\text{Example}: \quad n = 125, \;\; Z_t = 1\ Z_1(t) + 2\ Z_2(t) + \ldots + 125\ Z_{125}(t).$$

If Z_1 jumps there is just one default (idiosyncratic), if Z_{125} jumps there are 125 ones and the whole pool defaults one shot (total systemic risk), otherwise for other Z_i's we have intermediate situations (sectors).

The GPL and GPCL Models: Default clusters?

- Thrifts in the early 90s at the height of the loan and deposit crisis.
- Airliners after 2001.
- Autos and financials more recently. From the September, 7 2008 to the October, 8 2008, we witnessed seven credit events: Fannie Mae, Freddie Mac, Lehman Brothers, Washington Mutual, Landsbanki, Glitnir, Kaupthing.

S&P ratings and default clusters

Moreover, S&P issued a request for comments related to changes in the rating criteria of corporate CDO. Tranches rated 'AAA' should be able to withstand the default of the largest single industry in the pool with zero recoveries. Stressed but plausible scenario that a cluster of defaults in the objective measure exists.

The GPL and GPCL Models

Problem: infinite defaults. Solution 1: **GPL:** Modify the aggregated pool default counting process so that this does not exceed the number of names, by simply capping Z_t to n, regardless of cluster structures:

$$C_t := \min(Z_t, n)$$

Solution 2: **GPCL.** Force clusters to jump only once and deduce single names defaults consistently.
The first choice is ok at top level but it does not really go down towards single names. The second choice is a real top down model, but combinatorially more complex.

Calibration

The GPL model is calibrated to the market quotes observed on March 1 and 6, 2006. Deterministic discount rates are listed in Brigo, Pallavicini and Torresetti (2006). Tranche data and DJi-TRAXX fixings, along with bid-ask spreads, are (I=index,T=Tranche,Tl=Tranchelet)

	Att-Det	**March, 1 2006**		**March, 6 2006**		
		5y	7y	3y	5y	7y
I		35(1)	48(1)	20(1)	35(1)	48(1)
T	0-3	2600(50)	4788(50)	500(20)	2655(25)	4825(25)
	3-6	71.00(2.00)	210.00(5.00)	7.50(2.50)	67.50(1.00)	225.50(2.50)
	6-9	22.00(2.00)	49.00(2.00)	1.25(0.75)	22.00(1.00)	51.00(1.00)
	9-12	10.00(2.00)	29.00(2.00)	0.50(0.25)	10.50(1.00)	28.50(1.00)
	12-22	4.25(1.00)	11.00(1.00)	0.15(0.05)	4.50(0.50)	10.25(0.50)
Tl	0-1	6100(200)	7400(300)			
	1-2	1085(70)	5025(300)			
	2-3	393(45)	850(60)			

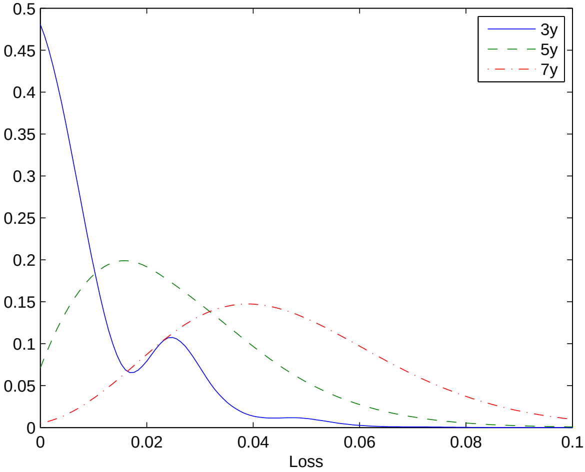
Calibration: All standard tranches up to seven years

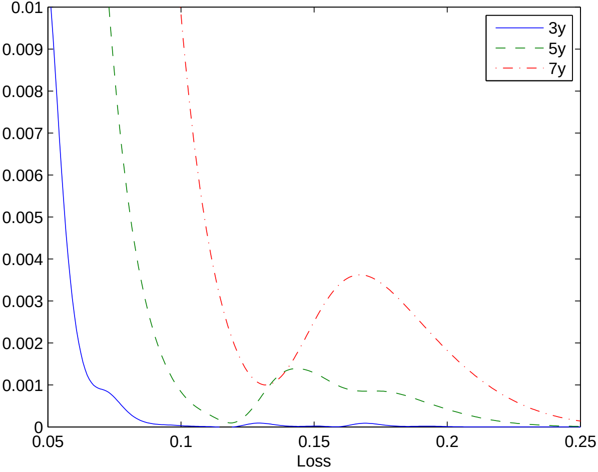
As a first calibration example we consider standard DJi-TRAXX tranches up to a maturity of 7y with constant recovery rate of 40%. The calibration procedure selects five Poisson processes. The 18 market quotes used by the calibration procedure are almost perfectly recovered. In particular all instruments are calibrated within the bid-ask spread (we show the ratio calibration error / bid ask spread).

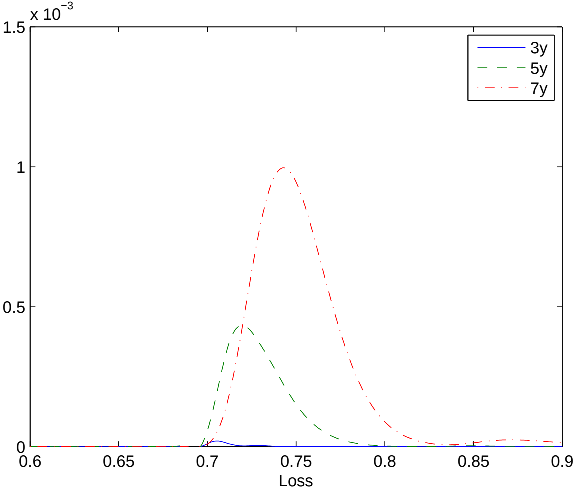
	Att-Det	**Maturities**		
		3y	5y	7y
Index		-0.4	-0.2	-0.9
Tranche	0-3	0.1	0.0	-0.7
	3-6	0.0	0.0	0.7
	6-9	0.0	0.0	-0.2
	9-12	0.0	0.0	0.0
	12-22	0.0	0.0	0.2

δ	$\Lambda(T)$		
	3y	5y	7y
1	0.535	2.366	4.930
3	0.197	0.266	0.267
16	0.000	0.007	0.024
21	0.000	0.003	0.003
88	0.000	0.002	0.007

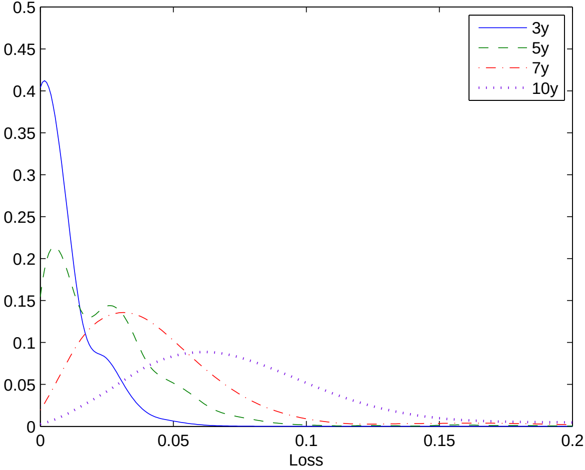
Loss distribution of the calibrated GPL model at different times



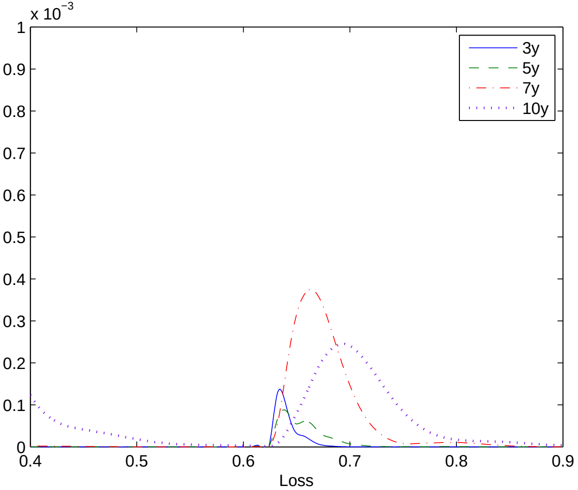

0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0.05
0.1
0.15
0.2
0.25
Loss
3y
5y
7y


x 10^-3
1.5
1
0.5
0
0.6
0.65
0.7
0.75
0.8
0.85
0.9
Loss
3y
5y
7y

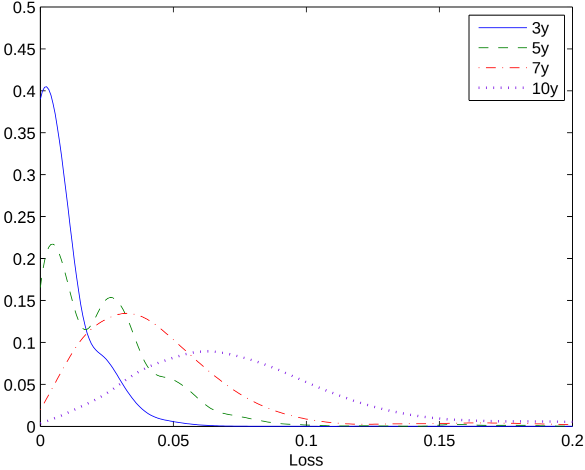
October 2 2006, GPL, Calibration up to 10y



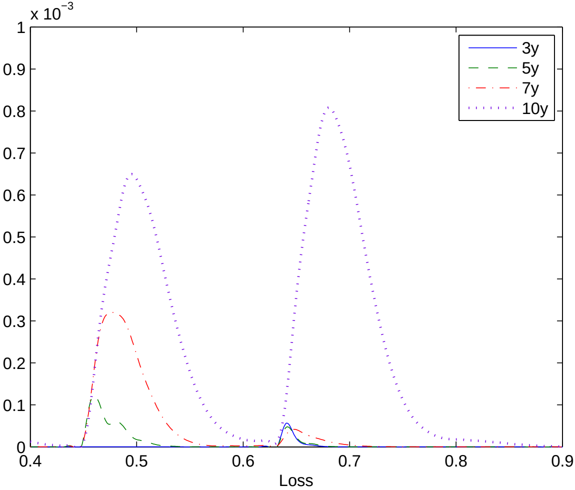
October 2 2006, GPL tail



October 2 2006, GPCL, Calibration up to 10y



October 2 2006, GPCL tail



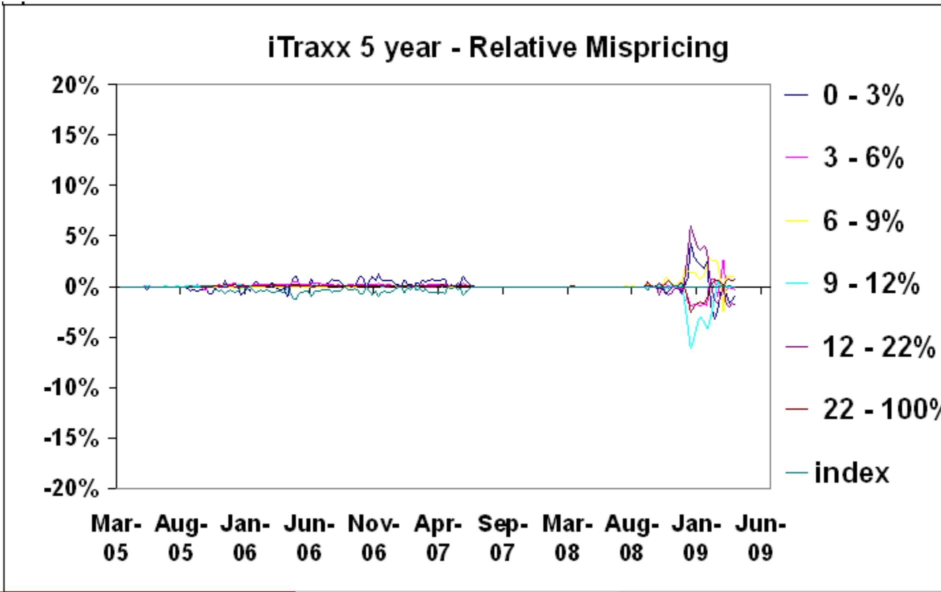
Calibration comments I

Sector / systemic calibration:

Notice the large portion of mass concentrated near the origin, the subsequent modes (default clusters) when moving along the loss distribution for increasing values, and the bumps in the far tail. Modes in the tail represent risk of default for large sectors. This is systemic risk as perceived by the dynamical model from the CDO quotes. With the crisis these probabilities have become larger, but they were already observable pre-crisis. Difficult to get this with parametric copula models, and impossible with flat correlations across names.

History of calibration in-crisis with a different parametrization (α's fixed a priori):

Calibration comments II

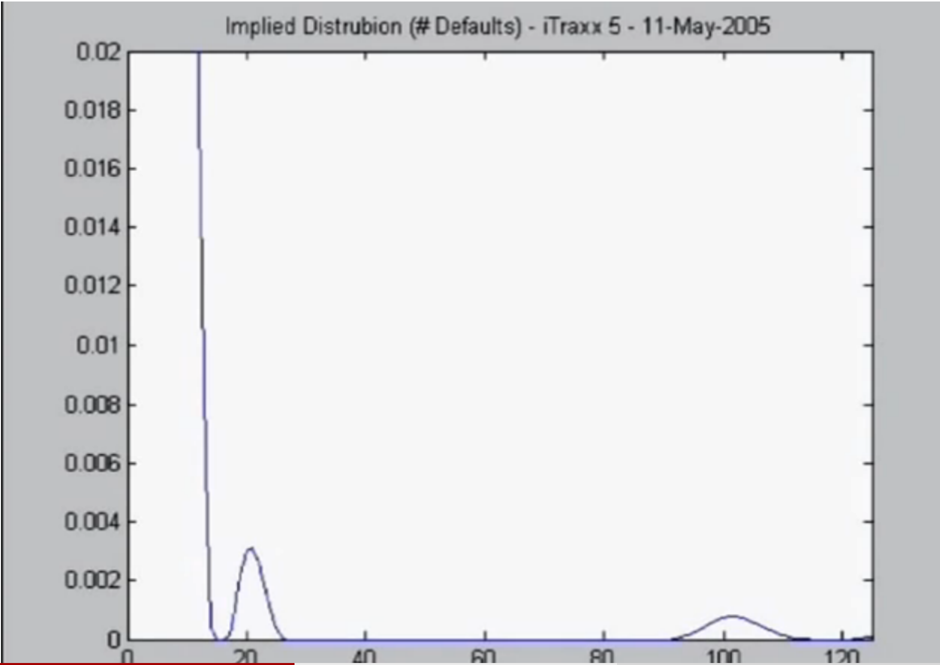


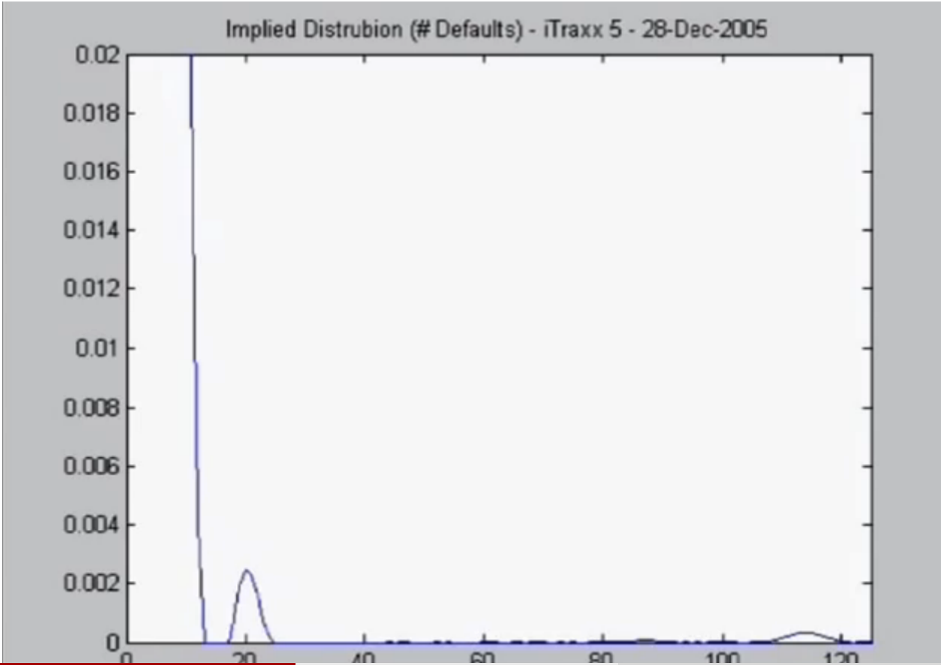
Loss distribution in GPL through the crisis

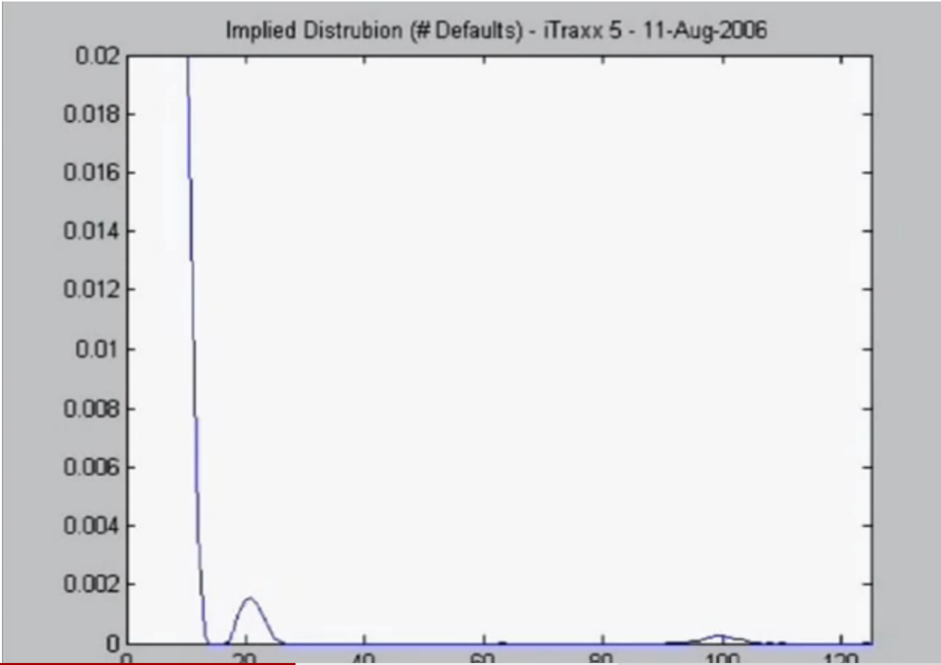
The loss distribution in the GPL model is arbitrage free, rich in structure and consistent with all market quotes, a feat impossible for implied correlation models.

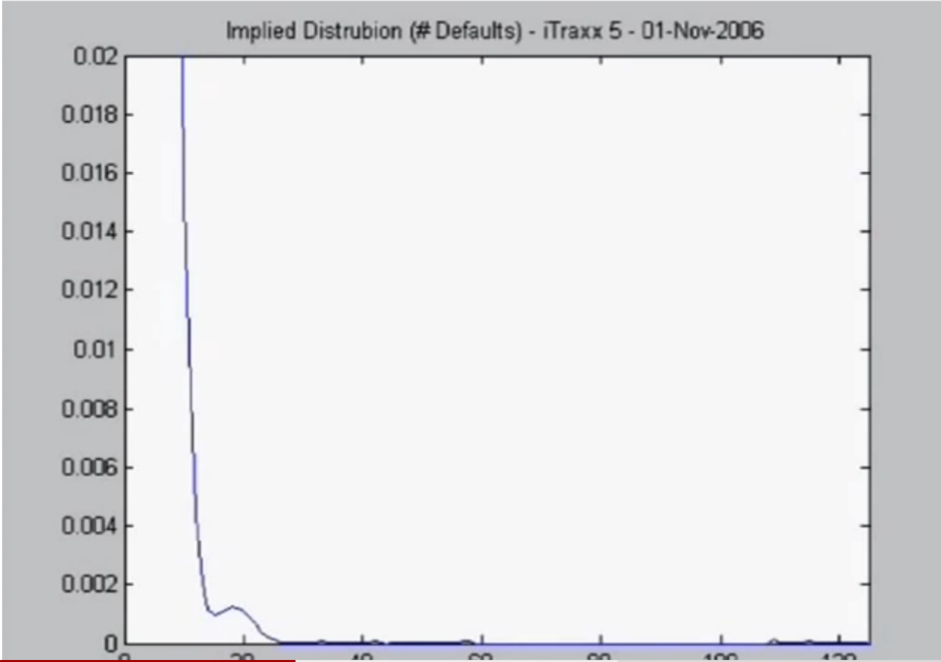
The following movie shows how structured the loss dynamics can be, as highlighted by the GPL model.

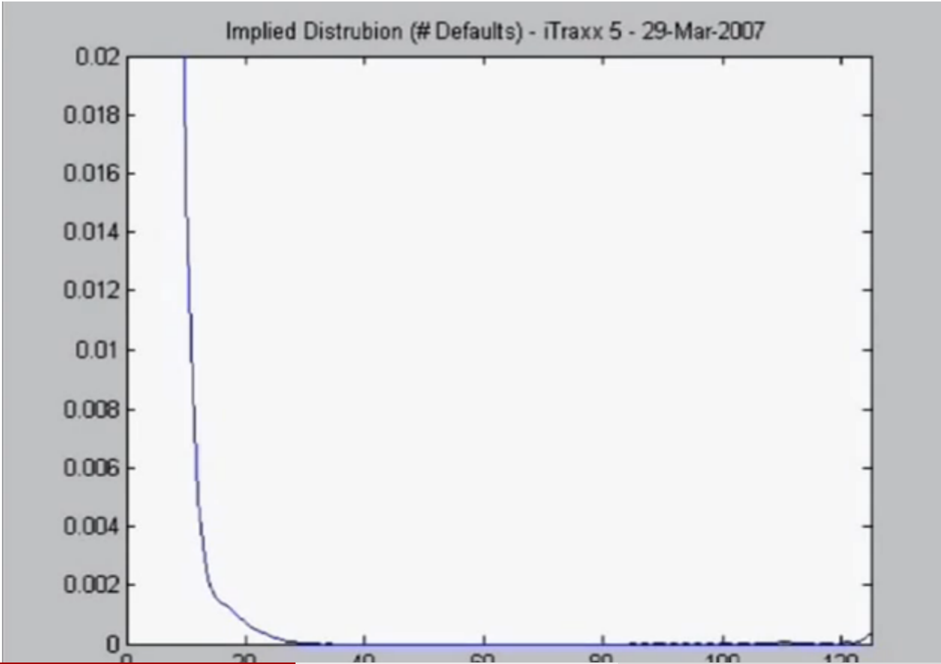
Animation showing how the loss distribution evolved in 2005+ is here
`http://www.youtube.com/watch?v=YZO-HeaGHkk&t=62m40s`

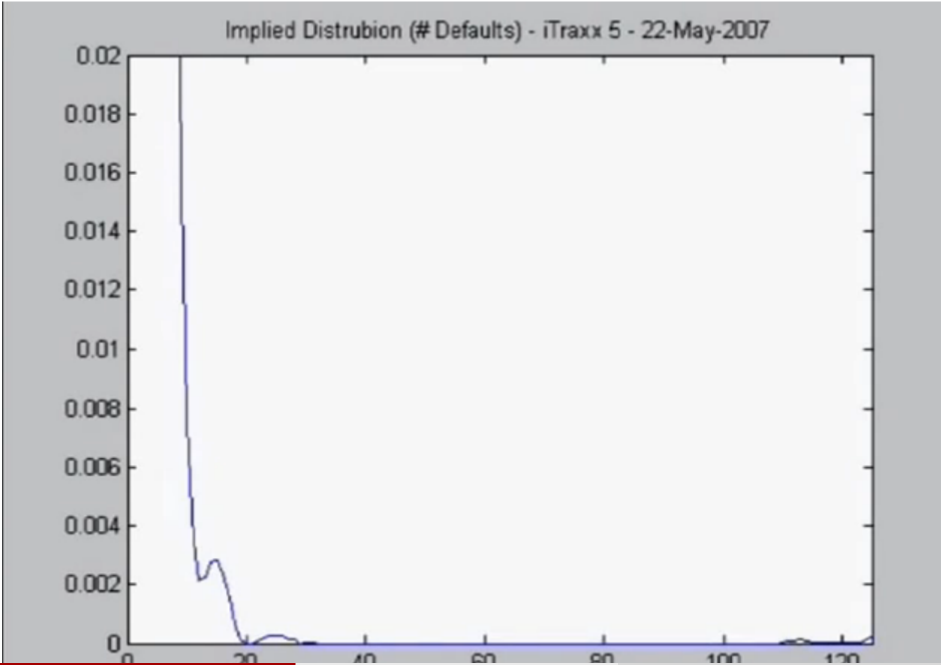

Implied Distrubion (# Defaults) - iTraxx 5 - 11-May-2005
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
20
40
60
80
100
120

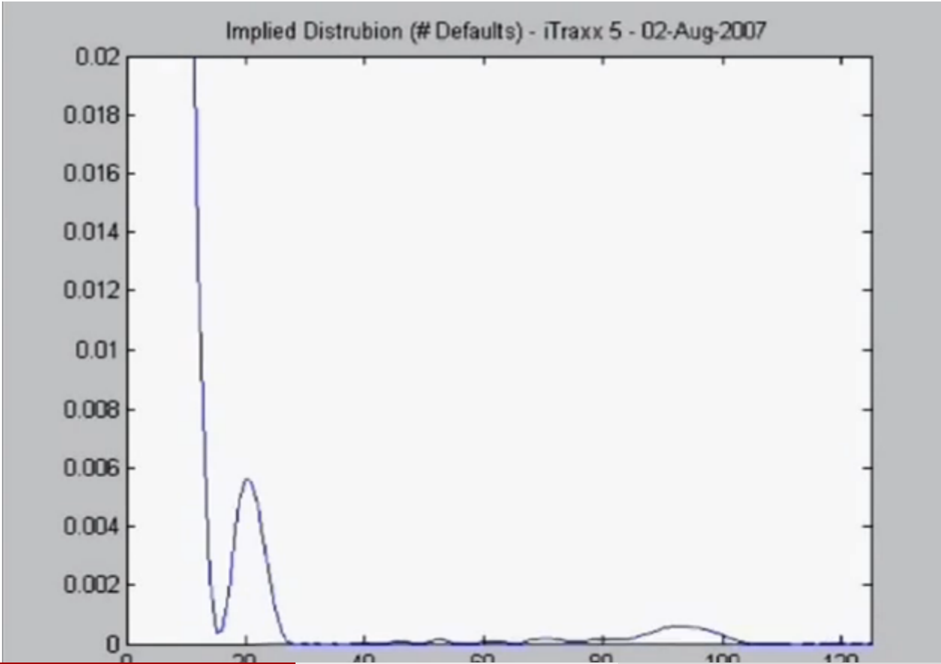

Implied Distrubion (# Defaults) - iTraxx 5 - 28-Dec-2005
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0

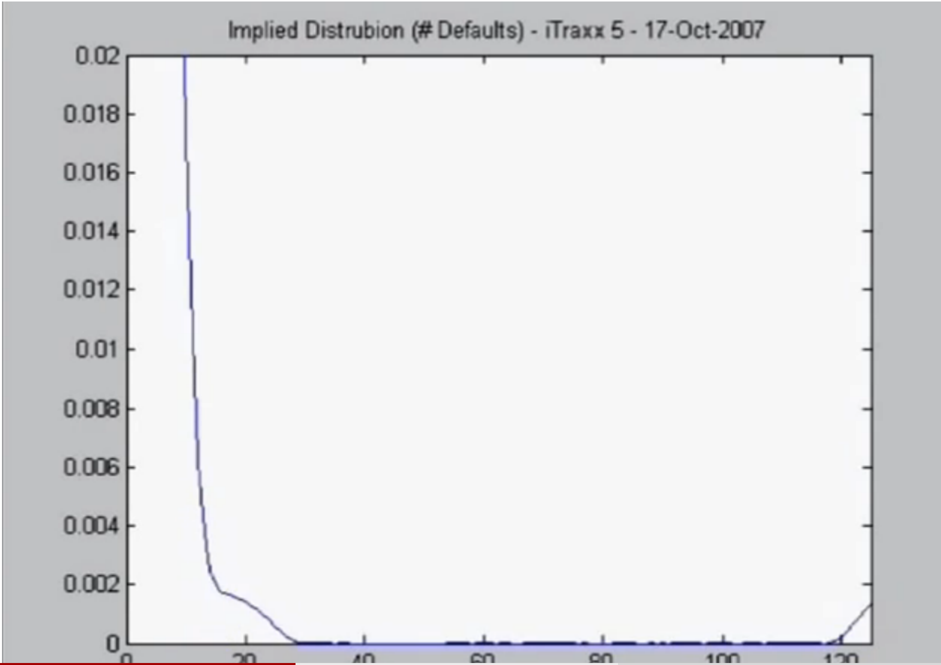

Implied Distrubion (# Defaults) - iTraxx 5 - 11-Aug-2006
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0

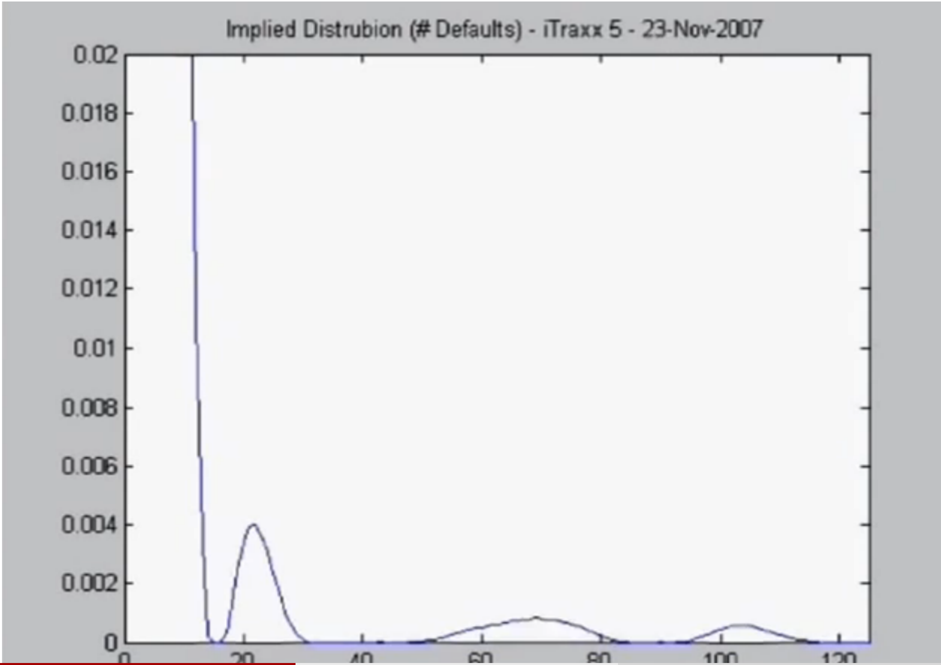

Implied Distrubion (# Defaults) - iTraxx 5 - 01-Nov-2006
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0

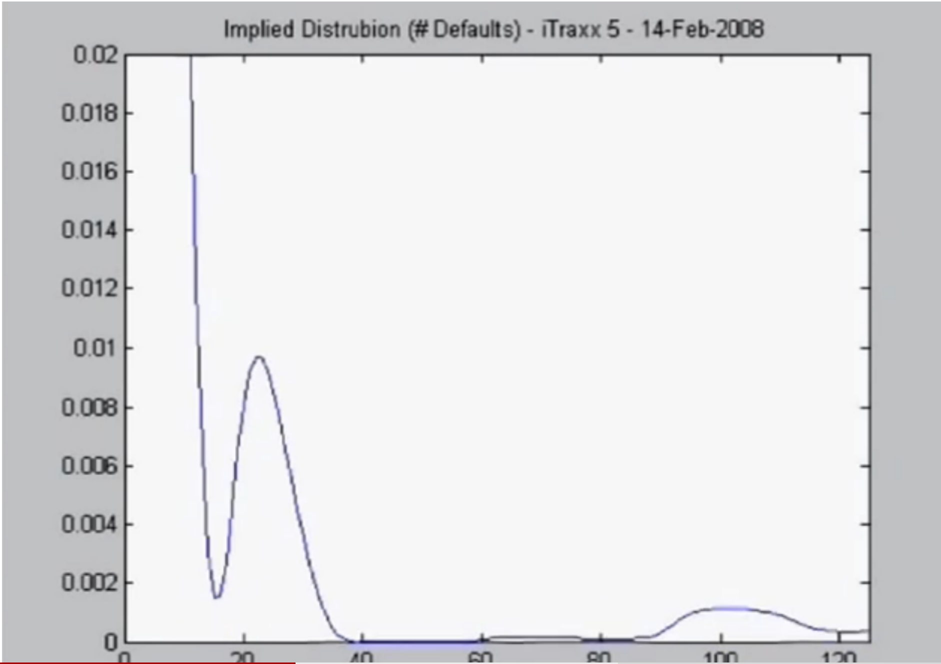

Implied Distrubion (# Defaults) - iTraxx 5 - 29-Mar-2007
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0

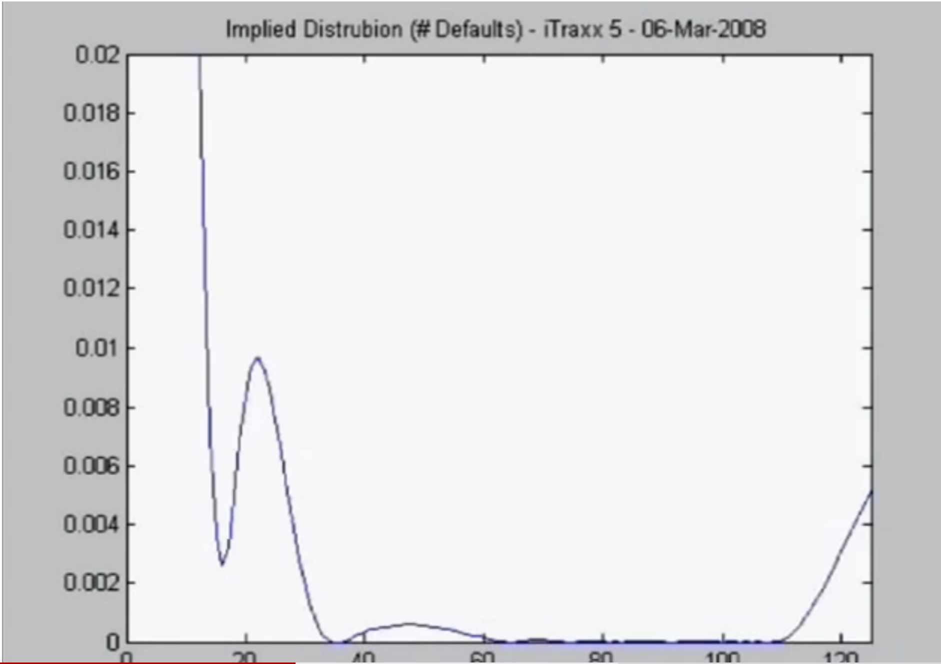

Implied Distrubion (# Defaults) - iTraxx 5 - 22-May-2007
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0

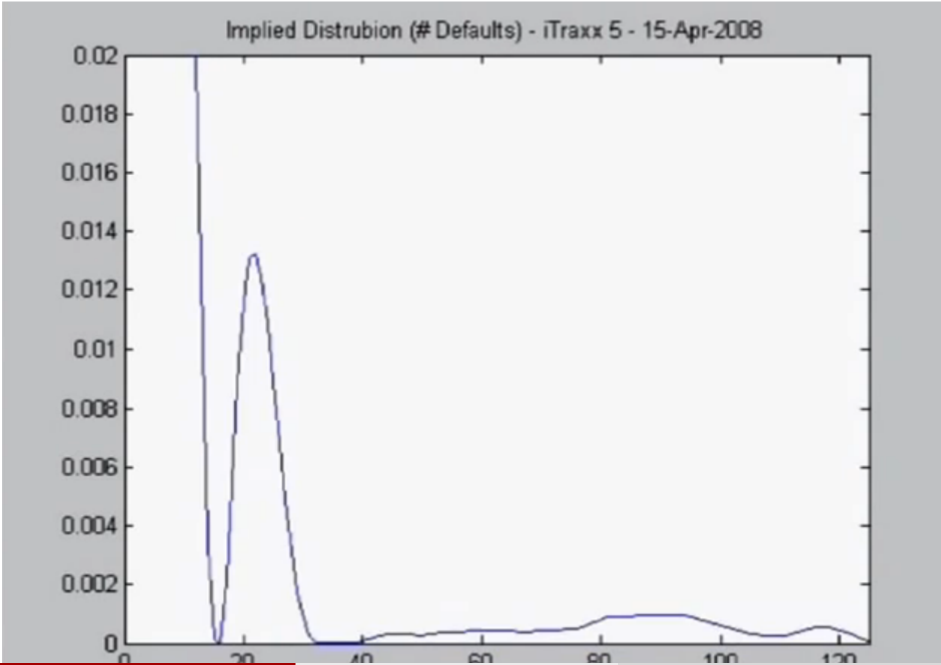

Implied Distrubion (# Defaults) - iTraxx 5 - 02-Aug-2007
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0

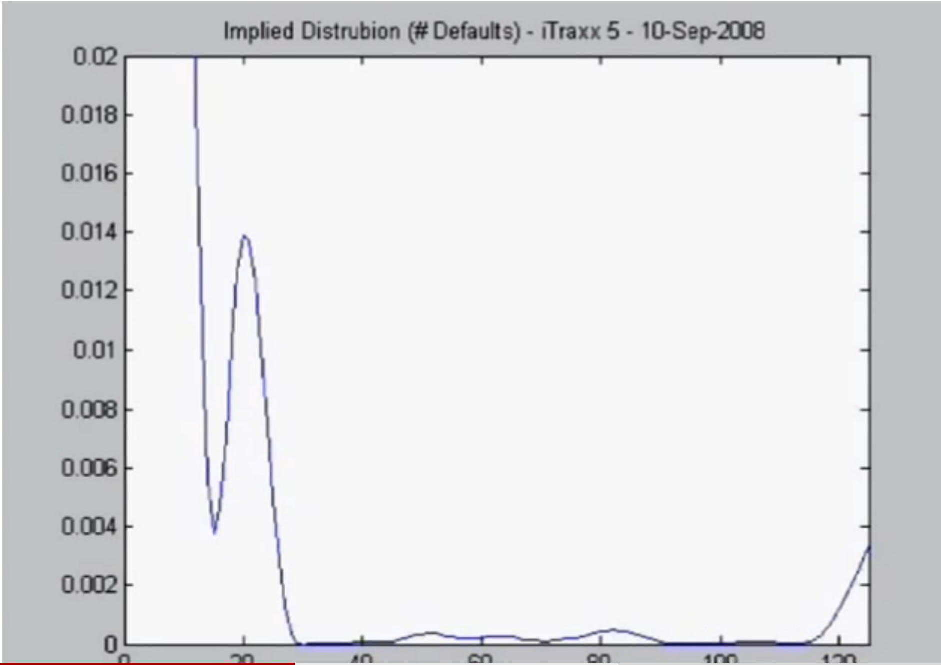

Implied Distrubion (# Defaults) - iTraxx 5 - 17-Oct-2007
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0

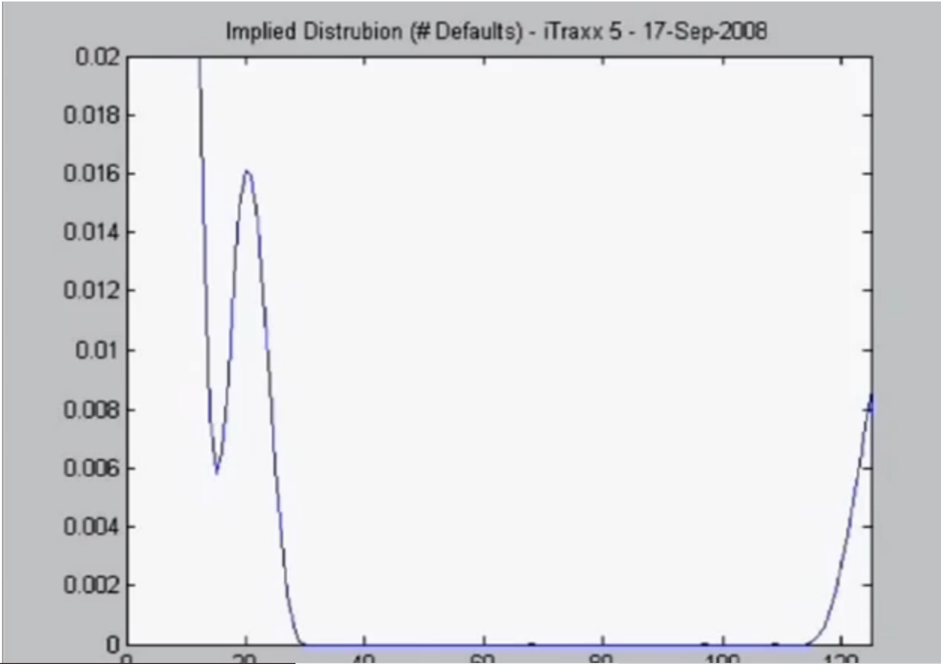

Implied Distrubion (# Defaults) - iTraxx 5 - 23-Nov-2007
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
20
40
60
80
100
120

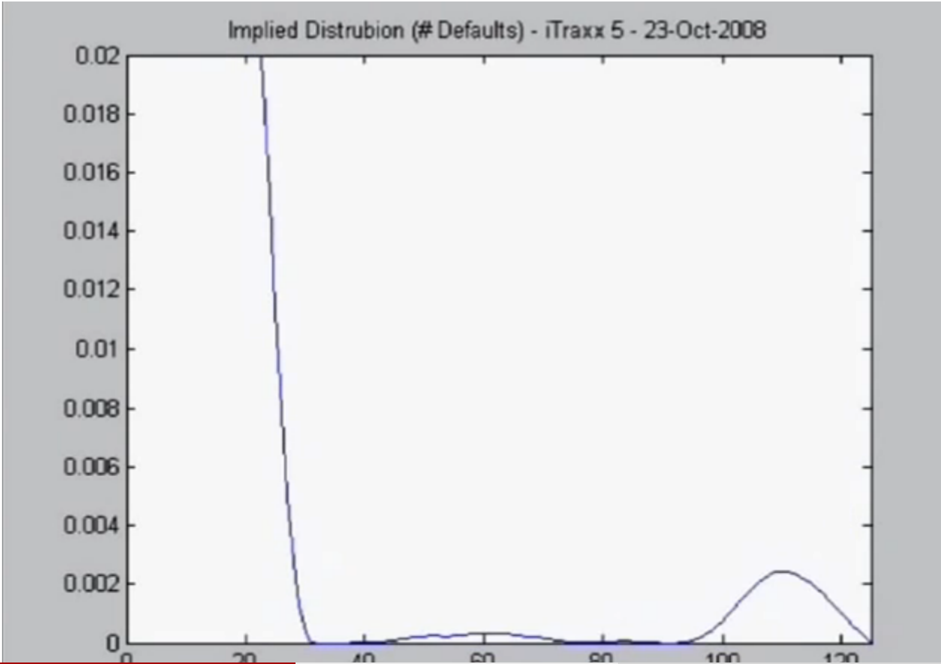

Implied Distrubion (# Defaults) - iTraxx 5 - 14-Feb-2008
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
20
40
60
80
100
120

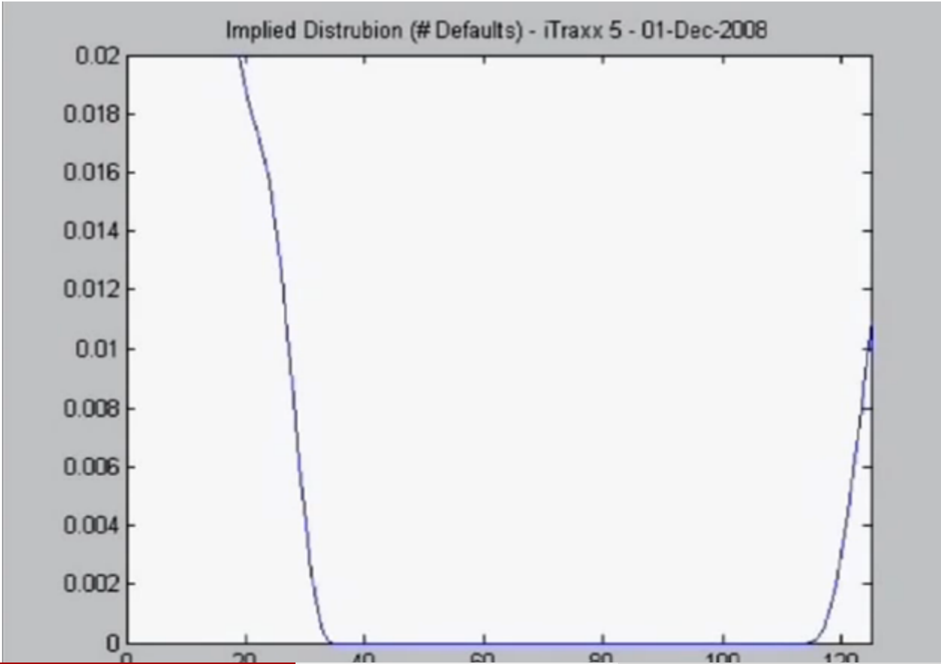

Implied Distrubion (# Defaults) - iTraxx 5 - 06-Mar-2008
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
20
40
60
80
100
120

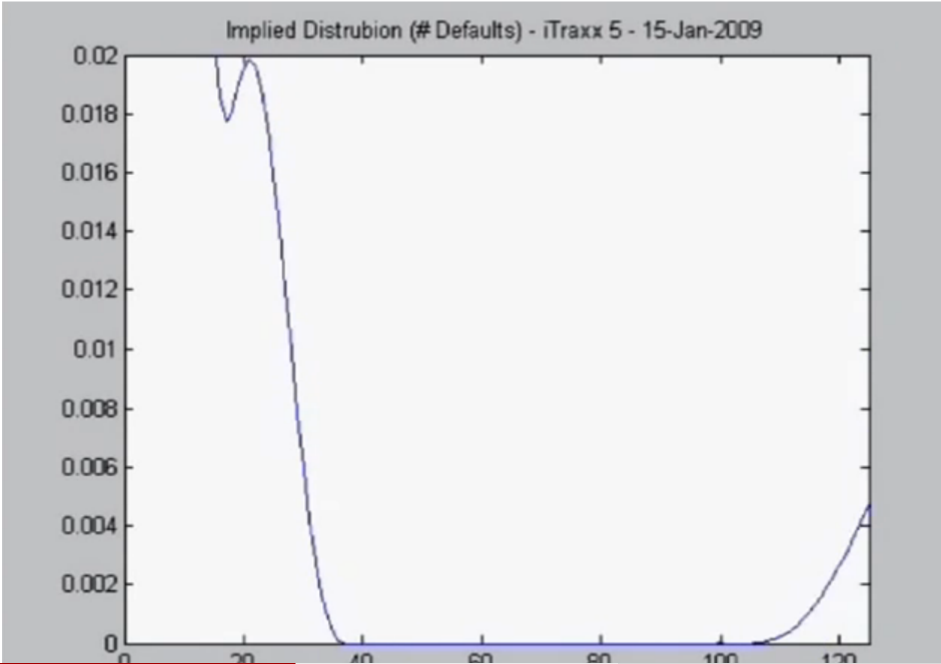

Implied Distrubion (# Defaults) - iTraxx 5 - 15-Apr-2008
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0


Implied Distrubion (# Defaults) - iTraxx 5 - 10-Sep-2008
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0


Implied Distrubion (# Defaults) - iTraxx 5 - 17-Sep-2008
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0


Implied Distrubion (# Defaults) - iTraxx 5 - 23-Oct-2008
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0


Implied Distrubion (# Defaults) - iTraxx 5 - 01-Dec-2008
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0


Implied Distrubion (# Defaults) - iTraxx 5 - 15-Jan-2009
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0

Calibration in-crisis

A full treatment of the calibration in crisis and a model extension is given in the book ”Credit Models and the Crisis” by Brigo, Pallavicini and Torresetti (2010), Wiley.

The synthetic CDO case?

- We have illustrated how a complex situation in CDO markets has been trivialized by media and even regulators
- Models (such as base correlation) were indeed inadequate, but the industry and researchers had been looking for much more powerful and consistent alternatives
- We have seen the example of the GPL model, a fully consistent arbitrage free dynamic model for CDOs
- So why didn't the media pick this up? Why didn't the media realize the glitches they were signalling were the same the Wall Street Journal had reported years earlier in 2005?
- We hope the CDO case study illustrates the lack of rigour in a broad part of investigative journalism, especially in connection with complex and technical subjects.
- We cannot blame (even poor) modeling for policy, regulation, incentives, banking model, governance, lack of culture...
- We have a duty to make our research visible and heard to society

Is Maths Guilty and Wrong?

- Mathematics is not wrong. We have to be careful in understanding what is meant when saying that one uses *mathematical models*.
- Mathematical models are a simplification of reality, and as such, are always ”wrong”, even if they try to capture the salient features of the problem at hand.
- **”All models are wrong, but some models are useful”** (Prof. George E.P. Box)
- The core mathematical theory behind derivatives valuation is correct, but the assumptions on which the theory is based may not reflect the real world when the market evolves over the years.

Is Mathematics guilty?

- Although the models used in Credit Derivatives and counterparty risk have limits that have been highlighted before the crisis by several researchers, the ongoing crisis is due to factors that go well beyond any methodological inadequacy: the killer formula

$$\boxed{\int_{-\infty}^{+\infty} \prod_{i=1}^{125} \Phi\left(\frac{\Phi^{-1}(1-\exp(-\Lambda_i(T))) - \sqrt{\rho_i}m}{\sqrt{1-\rho_i}}\right)\varphi(m)dm.}$$

Versus

The Crisis:
US real estate policy, Originate to Distribute (to Hold?) system fragility, volatile monetary policies,
myopic compensation and incentives system, lack of homogeneity in regulation, underestimation of liquidity risk, lack of data, fraud corrupted data...(Szegö 2009, The crash sonata in D major, JRMFI).

And what about the data?

Data and Inputs quality

For many financial products, and especially RMBS (Residential Mortgage Backed Securities), quite related to the asset class that triggered the crisis, the problem is in the data rather than in the models.

Risk of fraud

At times data for valuation in mortgages CDOs (RMBS and CDO of RMBS) can be distorted by fraud (see for example the FBI Mortgage fraud report, 2007, www.fbi.gov/publications/fraud/mortgage_fraud07.htm.

Pricing a CDO on this underlying:

Figure: The above photos are from condos that were involved in a mortgage fraud. The appraisal described ”recently renovated condominiums” to include Brazilian hardwood, granite countertops, and a value of 275,000 USD

And what about the data?

At times it is not even clear what is in the portfolio: From the offering circular of a huge RMBS (more than 300.000 mortgages)

Type of property	% of Total
Detached Bungalow	2.65%
Detached House	16.16%
Flat	13.25%
Maisonette	1.53%
Not Known	2.49%
New Property	0.02%
Other	0.21%
Semi Detached Bungalow	1.45%
Semi Detached House	27.46%
Terraced House	34.78%
Total	100.00%

Mathematics or Magic?

All this is before modeling. Models obey a simple rule that is popularly summarized by the acronym GIGO (Garbage In → Garbage Out). As Charles Babbage (1791–1871) famously put it:

On two occasions I have been asked,
"Pray, Mr. Babbage, if you put into the machine wrong figures, will the right answers come out?"
I am not able rightly to apprehend
the kind of confusion of ideas
that could provoke such a question.

So, in the end, how can the crisis be mostly due to models inadequacy, and to quantitative analysts and academics pride and unawareness of models limitations?

Valuation of financial products post 2008

Pre-2007 the emphasis was PRICING/HEDGING COMPLEX DERIVATIVES on simple risks (pure equity risk, pure interest rate risk, etc)

Now we need to price SIMPLE DERIVATIVES such as Interest Rate Swaps under COMPLEX RISKS (credit, liquidity, funding, collateral, gap risk, multiple curves...)

This new task is much harder, not least because many of the new risks are INTERCONNECTED. More on this in a minute...

New risks are often added one by one by valuation adjustments: CVA, DVA, LVA, FVA, KVA... In this course we will see how the inclusion of credit, liquidity, collateral and funding should work in principle, embracing full nonlinearity and interconnectedness, and how the industry forces an often misleading separation in more and more adjustments.

CVA and DVA

We are a investment bank ”I” trading with a counterparty ”C”.

Credit Valuation Adjustment (CVA)

is the reduction in price we ask to ”C” for the fact that ”C” may default. See B. and Tarenghi (2004) and B. and Masetti (2005).

Debit Valuation Adjustment (DVA)

is the increase in price we face towards ”C” for the fact that *we* may default. See B. and Capponi (2008). In very simple contexts, DVA can also be interpreted as a funding benefit.

CVA/DVA are complex options on netting sets...

containing hundreds of risk factors and with a random maturity given by the first to default between ”I” and ”C”

CVA, DVA

CVA and DVA can be sizeable

Citigroup in its press release on the first quarter revenues of 2009 reported a *positive* mark to market due to its *worsened* credit quality: “Revenues also included [...] a net 2.5$ billion positive CVA on derivative positions, excluding monolines, mainly due to the widening of Citi’s CDS spreads” (DVA)

CVA mark to market losses: BIS

”During the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults.”

The case of symmetric counterparty risk

"I" : the investor; "C": the counterparty;
τ_I, τ_C: default times of "I" and "C". T: final maturity.
$\Pi(t, T)$: Sum of the trade discounted cash flows from t to T, discounted back at t at the risk free rate, without credit risk of "I" or "C".
We consider the following events partition (no simult. defaults)

Four events ordering the default times

$$\begin{aligned} \mathcal{A} &= \{\tau_B \le \tau_C \le T\} & E = \{T \le \tau_B \le \tau_C\} \\ \mathcal{B} &= \{\tau_B \le T \le \tau_C\} & F = \{T \le \tau_C \le \tau_B\} \\ \mathcal{C} &= \{\tau_C \le \tau_B \le T\} & \\ \mathcal{D} &= \{\tau_C \le T \le \tau_B\} & \end{aligned}$$

Define $\mathrm{NPV}_{\{B,C\}}(t) := \mathbb{E}_t[\Pi_{\{B,C\}}(t, T)]$, and recall $\Pi_B = -\Pi_C$.

CVA and DVA: Derivation from simple cash flows

$$\Pi^D_B(t,T) = \mathbf{1}_{E\cup F}\Pi_B(t,T)$$
$$+\mathbf{1}_{\mathcal{C}\cup\mathcal{D}}\left[\Pi_B(t,\tau_C) + D(t,\tau_C)\left(REC_C\,(\mathrm{NPV}_B(\tau_C))^+ - (-\mathrm{NPV}_B(\tau_C))^+\right)\right]$$
$$+\mathbf{1}_{\mathcal{A}\cup\mathcal{B}}\left[\Pi_B(t,\tau_B) + D(t,\tau_B)\left((\mathrm{NPV}_B(\tau_B))^+ - REC_B\,(-\mathrm{NPV}_B(\tau_B))^+\right)\right]$$

1. If no early default $\Rightarrow$ payoff of a default-free claim (1st term).
2. In case of early default of the counterparty, the payments due before default occurs are received (second term),
3. and then if the residual net present value is positive only the recovery value of the counterparty REC_C is received (third term),
4. whereas if negative, it is paid in full by the investor/ Bank (4th term).
5. In case of early default of the investor, the payments due before default occurs are received (fifth term),
6. and then if the residual net present value is positive it is paid in full by the counterparty to the investor/ Bank (sixth term),
7. whereas if it is negative only the recovery value of the investor/ Bank REC_B is paid to the counterparty (seventh term).

CVA, DVA: A useful derivation in view of funding

$$\mathbb{E}_t\left\{\Pi_I^D(t,T)\right\} = \mathbb{E}_t\left\{\Pi_I(t,T)\right\} + \mathrm{DVA}_I(t) - \mathrm{CVA}_I(t)$$

$$\mathrm{DVA}_I(t) = \mathbb{E}_t\left\{\mathrm{LGD_I}\cdot \mathbf{1}(t < \tau^{1st} = \tau_I < T)\cdot D(t,\tau_I)\cdot[-\mathrm{NPV_I}(\tau_I)]^+\right\}$$

$$\mathrm{CVA}_I(t) = \mathbb{E}_t\left\{\mathrm{LGD_C}\cdot \mathbf{1}(t < \tau^{1st} = \tau_C < T)\cdot D(t,\tau_C)\cdot[\mathrm{NPV_I}(\tau_C)]^+\right\}$$

- Obtained simplifying closeout cash flows and taking expectation.
- 2nd term : adj due to scenarios $\tau_I < \tau_C$. This is positive to the investor/ Bank B and is called ”Debit Valuation Adjustment” (DVA)
- 3d term : Counterparty risk adj due to scenarios $\tau_C < \tau_I$
- Bilateral Valuation Adjustment as seen from B:
$\mathrm{BVA}_I = \mathrm{DVA}_I - \mathrm{CVA}_I$.
- If computed from the opposite point of view of “C” having counterparty “B”, $\mathrm{BVA}_C = -\mathrm{BVA}_I$. Symmetry.

DVA problems

Strange consequences of the formula new mid term, i.e. DVA

- credit quality of investor WORSENS $\Rightarrow$ books POSITIVE MARK TO MKT
- credit quality of investor IMPROVES $\Rightarrow$ books NEGATIVE MARK TO MKT
- Citigroup in its press release on the first quarter revenues of 2009 reported a *positive* mark to market due to its *worsened* credit quality: “Revenues also included [...] a net 2.5$ billion positive CVA on derivative positions, excluding monolines, mainly due to the widening of Citi’s CDS spreads”

DVA Hedging?

`October 18, 2011, 3:59 PM ET, WSJ. Goldman Sachs Hedges Its Way to Less Volatile Earnings.`

Goldman's DVA gains in the third quarter totaled $450 million [...] That amount is comparatively smaller than the $1.9 billion in DVA gains that J.P. Morgan Chase and Citigroup each recorded for the third quarter. Bank of America reported $1.7 billion of DVA gains in its investment bank. Analysts estimated that Morgan Stanley will record $1.5 billion of net DVA gains when it reports earnings on Wednesday [...]

Is DVA real? **DVA Hedging**. Buying back bonds? Proxying?

DVA hedge? One should sell protection on oneself, impossible, unless one buys back bonds that he had issued earlier. Very Difficult. Most times: proxying. Instead of selling protection on oneself, one sells protection on a number of names that one thinks are highly correlated to oneself.

DVA?

Again from the WSJ article above:

[...] Goldman Sachs CFO David Viniar said Tuesday that the company attempts to hedge [DVA] using a basket of different financials.
A Goldman spokesman confirmed that the company did this by selling CDS on a range of financial firms. [...] Goldman wouldn't say what specific financials were in the basket, but Viniar confirmed [...] that the basket contained 'a peer group.'

This can approximately hedge the spread risk of DVA, but not the jump to default risk. Merrill hedging DVA risk by selling protection on Lehman would not have been a good idea. Worsens systemic risk.

DVA or no DVA? Accounting VS Capital Requirements

NO DVA: Basel III, page 37, July 2011 release

This CVA loss is calculated without taking into account any offsetting debit valuation adjustments which have been deducted from capital under paragraph 75.

YES DVA: FAS 157

Because nonperformance risk (the risk that the obligation will not be fulfilled) includes the reporting entitys credit risk, the reporting entity should consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value under other accounting pronouncements FAS 157 (see also IAS 39)

DVA or no DVA? Accounting VS Capital Requirements

Stefan Walter says:

”The potential for perverse incentives resulting from profit being linked to decreasing creditworthiness means capital requirements cannot recognise it, says Stefan Walter, *secretary-general of the Basel Committee*: The main reason for not recognising DVA as an offset is that it would be inconsistent with the overarching supervisory prudence principle under which we do not give credit for increases in regulatory capital arising from a deterioration in the firms own credit quality.”

CVA, DVA: Closeout

$$\bar{V}_t = \mathbb{E}_t\left\{\Pi_I^D(t,T)\right\} = \overbrace{\mathbb{E}_t\{\Pi_I(t,T)\}}^{V_t^0} + \mathrm{DVA}_I(t) - \mathrm{CVA}_I(t)$$

$$\mathrm{DVA}_I(t) = \mathbb{E}_t\left\{\mathrm{LGD_I}\cdot \mathbf{1}(t < \tau^{1st} = \tau_I < T)\cdot D(t,\tau_I)\cdot\left[-\underbrace{\mathrm{NPV}_I(\tau_I)}_{V^0_{\tau_I}\text{ or }\bar{V}_{\tau_I}?}\right]^+\right\}$$

$$\mathrm{CVA}_I(t) = \mathbb{E}_t\left\{\mathrm{LGD_C}\cdot \mathbf{1}(t < \tau^{1st} = \tau_C < T)\cdot D(t,\tau_C)\cdot\left[\underbrace{\mathrm{NPV}_I(\tau_C)}_{V^0_{\tau_C}\text{ or }\bar{V}_{\tau_C}?}\right]^+\right\}$$

V^0 risk free closeout (much easier but discontinuity, this is our derviation above), $\bar{V}$ replacement closeout - recursive problem but more continuous.

CVA, DVA: Closeout

B. and Morini [28] point out several problems with both forms of closeout. Even in a simple trade as a loan (or zero coupon bond), there are problems with both closeouts. The default of the lender should not impact the value of the loan. And yet:

Impact of an early default of the Lender when the loan is valued with bilateral CVA and DVA

Dependence τ_I, τ_C $\rightarrow$ Closeout$\downarrow$	independence	co-monotonicity
Risk Free	Negatively affects Borrower	No contagion
Replacement	No contagion	Further Negatively affects Lender

For numerical examples see the paper. ISDA suggests a replacement closeout but subsimulations...

PAYOUT RISK

The exact payout corresponding with the Credit and Debit valuation adjustment is not always clear.

- DVA or not?
- Which Closeout?
- First to default risk or not? (some banks don't check who defaults first in the formula)
- How are collateral and funding accounted for exactly?

Worse than model risk: Payout risk. WHICH PAYOUT?

At a 2012 industry panel (WBS) on CVA it was stated that 5 banks might compute CVA in 15 different ways.

Intensity Models for default times τ

In intensity models the random default time τ is assumed to be exponentially distributed.

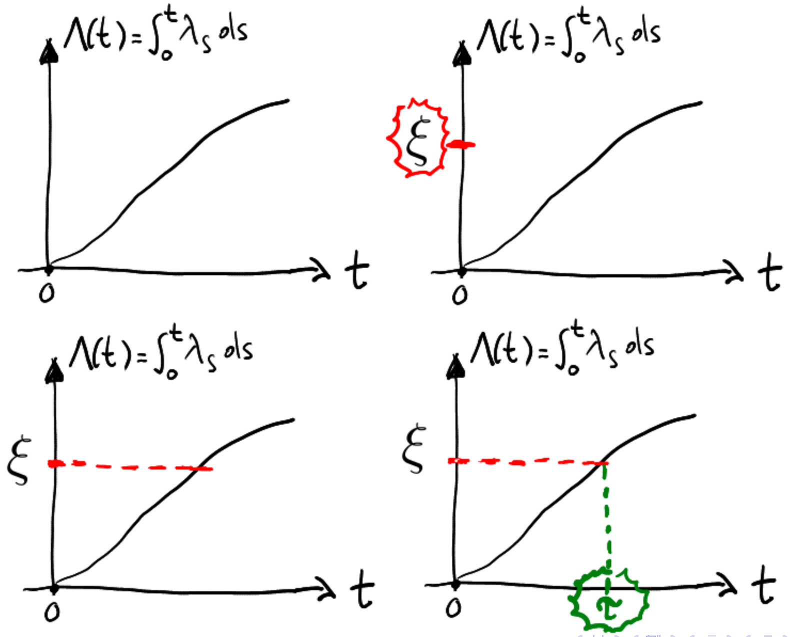
A strictly positive stochastic process $t \mapsto \lambda_t$ called *default intensity* (or hazard rate) is given for the bond issuer or the CDS reference name.

The *cumulated intensity* (or hazard function) is the process $t \mapsto \int_0^t \lambda_s \, ds =: \Lambda_t$. Since λ is positive, Λ is increasing in time.

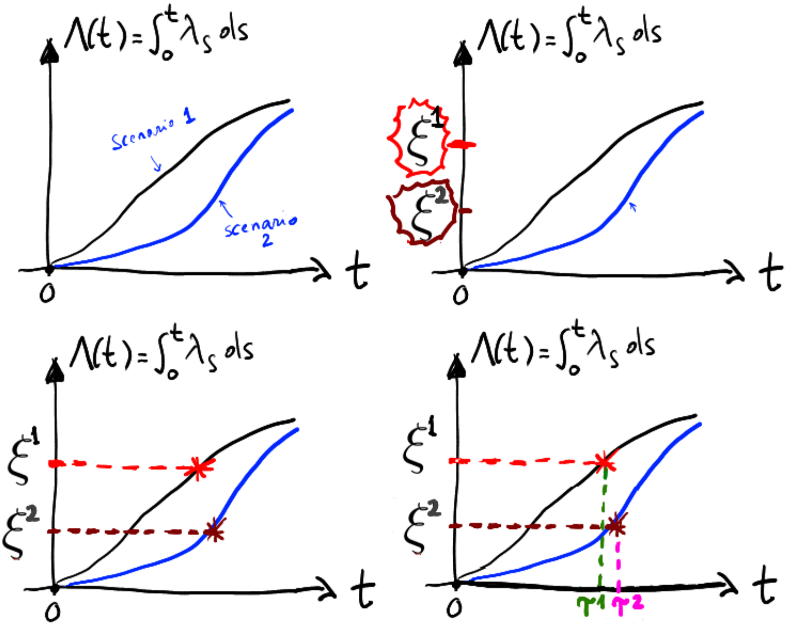
The default time is defined as the inverse of the cumulative intensity on an exponential random variable ξ with mean 1 and independent of λ

$$\tau = \Lambda^{-1}(\xi) \quad (\mathbb{Q}(\xi > u) = e^{-u}, \quad \mathbb{E}(\xi) = 1).$$

Why is this definition interesting and suitable? More in a minute.



$\Lambda(t) = \int_0^t \lambda_s ds$
ξ
τ
0
t


$\Lambda(t) = \int_0^t \lambda_s \, ds$
Scenario 1
scenario 2
t
0
$\Lambda(t) = \int_0^t \lambda_s \, ds$
ξ^1
ξ^2
t
0
$\Lambda(t) = \int_0^t \lambda_s \, ds$
ξ^1
ξ^2
t
0
$\Lambda(t) = \int_0^t \lambda_s \, ds$
ξ^1
ξ^2
τ^1
τ^2
t
0

Intensity Models for default times τ

A few calculations: Probability of surviving time t:

$$\mathbb{Q}(\tau > t) = \mathbb{Q}(\Lambda^{-1}(\xi) > t) = \mathbb{Q}(\xi > \Lambda(t)) = \rightarrow$$

Let's use the tower property of conditional expectation and the fact that Λ is independent of ξ:

$$\rightarrow = \mathbb{E}[\mathbb{Q}(\xi > \Lambda(t) | \Lambda(t))] = \mathbb{E}[e^{-\Lambda(t)}] = \mathbb{E}[e^{-\int_0^t \lambda_s\, ds}]$$

This looks exactly like a bond price if we replace r by λ!

Intensity Models for default times τ

Let's price a defaultable zero coupon bond with zero recovery. Assume that ξ is also independent of r.

$$\bar{P}(0,T) = \mathbb{E}[D(0,T)1_{\{\tau>T\}}] = \mathbb{E}[e^{-\int_0^T r_s\,ds}1_{\{\Lambda^{-1}(\xi)>T\}}] =$$

$$= \mathbb{E}[e^{-\int_0^T r_s\,ds}1_{\{\xi>\Lambda(T)\}}] = \mathbb{E}[\mathbb{E}\{e^{-\int_0^T r_s\,ds}1_{\{\xi>\Lambda(T)\}}|\Lambda,r\}]$$

$$= \mathbb{E}[e^{-\int_0^T r_s\,ds}\mathbb{E}\{1_{\{\xi>\Lambda(T)\}}|\Lambda,r\}]$$

$$= \mathbb{E}[e^{-\int_0^T r_s\,ds}\mathbb{Q}\{\xi>\Lambda(T)|\Lambda\}] = \mathbb{E}[e^{-\int_0^T r_s\,ds}e^{-\Lambda(T)}] =$$

$$= \mathbb{E}[e^{-\int_0^T r_s\,ds-\int_0^T \lambda_s\,ds} = \mathbb{E}[e^{-\int_0^T \boxed{(r_s+\lambda_s)}\,ds}]$$

So the price of a defaultable bond is like the price of a default-free bond *where the risk free discount short rate r has been replaced by r plus a spread* λ.

Intensity Models for default times τ

This is why the intensity is interpreted as a credit spread.

Because of properties of the exponential random variable, one can also prove that

$$\mathbb{Q}(\tau \in [t, t+dt) | \tau > t, \text{''}\lambda[0,t]\text{''}) = \lambda_t \, dt \qquad (*)$$

and the intensity $\lambda_t \, dt$ is also a local probability of defaulting around t. For example, if λ is deterministic,

$$\mathbb{Q}(\tau \in [t, t+dt) | \tau > t) = \frac{\mathbb{Q}(\tau \in [t, t+dt) \cap \tau > t)}{\mathbb{Q}(\tau > t)} = \frac{\mathbb{Q}(\tau \in [t, t+dt))}{\mathbb{Q}(\tau > t)}$$

$$= \frac{-d\mathbb{Q}(\tau > t)}{\mathbb{Q}(\tau > t)} = -\frac{d \; e^{-\int_0^t \lambda_s ds}}{e^{-\int_0^t \lambda_s ds}} = \lambda_t \; dt$$

Notice also if we take (*) as a starting point (instead of $\tau = \Lambda^{-1}(\xi)$) and proceed as in the last calculation except the last two steps, we get to

Intensity Models for default times τ

$$\lambda_t dt = \mathbb{Q}(\tau \in [t, t+dt) | \tau > t) = \frac{-d\mathbb{Q}(\tau > t)}{\mathbb{Q}(\tau > t)} = -\frac{dS(t)}{S(t)}$$

where we called S the survival probability. Then we get the equation $dS(t) = -\lambda_t S(t) dt$, leading to the exponential formula $S(t) = e^{-\int_0^t \lambda_s ds}$.

Summing up, given $\tau = \Lambda^{-1}(\xi)$:

λ is an instantaneous credit spread or local default probability

ξ is pure jump to default risk

Intensity models and Interest Rate Models

As is now clear, the exponential structure of τ in intensity models makes the modeling of credit risk very similar to interest rate models.

The spread/intensity λ behaves exactly like an interest rate in discounting

Then it is possible to use a lot of techniques from interest rate modeling for credit as well.

CVA, DVA: A useful derivation in view of funding

- Immersion hypothesis for credit risk: work under default-free filtration $\mathcal{F}_t$ as much as possible.
- Assume conditional independence of defaults: spreads λ's may be correlated, but jump to defaults ξ's will be independent.

Conditional independence of defaults I

Recall that we are assuming

$$\mathcal{G}_t = \mathcal{F}_t \vee \sigma(\{\tau_i \leq u\}, u \leq t)$$

with i indexing all the default times in the system. Working under $\mathcal{F}$-immersion usually means that the risks in the basic cash flows Π are assumed not to be credit sensitive but to depend only on the filtration $\mathcal{F}$ of pre-default or default-free market information, eg default free interest rate swaps portfolio.

Conditional independence of defaults II

We are also assuming default times to be $\mathcal{F}$-conditionally independent:

$$\text{if } \tau_I = \Lambda_I^{-1}(\xi_I), \quad \tau_C = \Lambda_C^{-1}(\xi_C),$$

then this means assuming that ξ_I and ξ_C are independent. Intensities $\lambda_I(t)$ and $\lambda_C(t)$ are taken $\mathcal{F}_t$ adapted (& can be correlated) and

$$\mathbb{Q}(\tau > t) = \mathbb{Q}(\min(\tau_I, \tau_C) > t) = \mathbb{Q}(\tau_I > t \cap \tau_C > t) =$$

We use the tower property + independence of ξ's on each other and $\mathcal{F}$:

$$= \mathbb{E}[\mathbb{Q}(\tau_I > t \ \cap \ \tau_C > t|\mathcal{F}_t)] = \mathbb{E}[\mathbb{Q}(\tau_I > t|\mathcal{F}_t)\mathbb{Q}(\tau_C > t|\mathcal{F}_t)] =$$

$$= \mathbb{E}[e^{-\Lambda_I(t)}e^{-\Lambda_C(t)}] = \mathbb{E}[e^{-\Lambda_I(t)-\Lambda_C(t)}] = \mathbb{E}[e^{-\int_0^t(\lambda_I(s)+\lambda_C(s))ds}]$$

Similarly, one can show the first to default time τ intensity λ

$$\text{is} \quad \mathbb{Q}(\tau \in [t, t+dt)|\tau > t, \mathcal{F}_t) = \lambda_t \, dt = (\lambda_I(t) + \lambda_C(t))dt.$$

Conditional independence of defaults III

Summing up:
Whenever we use the immersion hypothesis, meaning that we switch filtration from $\mathcal{G}$ to $\mathcal{F}$, we assume the ξ to be conditionally independent and the basic cash flows $\Pi(s,t)$ to be $\mathcal{F}_t$ adapted for all $s \le t$.

Switching to the filtration $\mathcal{F}$ typically transforms indicators such as $1_{\{\tau>t\}}$ into their $\mathcal{F}$ expectations $e^{-\int_0^t (\lambda_I(s)+\lambda_C(s))ds}$. This is often collected in the discount term $D(0,t;r)$ that becomes $D(0,t;r+\lambda)$.

$$D(0,t;r)1_{\{\tau>t\}} = e^{-\int_0^t r_s ds} 1_{\{\tau>t\}} \text{ goes } e^{-\int_0^t r_s ds} e^{-\int_0^t \lambda_s ds} = D(0,t;r+\lambda)$$

The switching also transforms $1_{\{\tau \in dt\}}$ into $\lambda_t e^{-\int_0^t \lambda_s ds} dt$.

CVA, DVA: A useful derivation in view of funding

$$\text{CVA}_I(t) = \mathbb{E}_t \left\{ \text{LGD}_C \cdot \mathbf{1}(t < \tau^{1st} = \tau_C < T) \cdot D(t, \tau_C) \cdot [V(\tau_C)]^+ \right\}$$

$$= \mathbb{E}_t \left\{ LGD_C \int_t^T 1_{\{\tau^{1st} \in du\}} 1_{\{\tau_I > u\}} D(t, u) \, [V(u)]^+ \right\}$$

$$= LGD_C \int_t^T \mathbb{E}_t \left\{ 1_{\{\tau_C \in du\}} 1_{\{\tau_I > u\}} D(t, u) \, (V(u))^+ \right\}$$

$$= LGD_C \int_t^T \mathbb{E}_t \left\{ \mathbb{E}_u \left[1_{\{\tau_C \in du\}} 1_{\{\tau_I > u\}} D(t, u) \, (V(u))^+ \, | \mathcal{F}_{u+du} \right] \right\}$$

$$= LGD_C \int_t^T \mathbb{E}_t \left\{ D(t, u) \, (V(u))^+ \, \mathbb{E}_u \left[1_{\{\tau_C \in du\}} 1_{\{\tau_I > u+du\}} | \mathcal{F}_{u+du} \right] \right\} = \ldots$$

$$\Big(\mathbb{E}_u \left[1_{\{\tau_C \in du\}} 1_{\{\tau_I > u+du\}} | \mathcal{F}_{u+du} \right] = \mathbb{E}_u \left[1_{\{\tau_C \in du\}} | \mathcal{F} \right] \mathbb{E}_u \left[1_{\{\tau_I > u+du\}} | \mathcal{F} \right] =$$

$$= \lambda_C(u) du \, e^{\left(-\int_t^u \lambda_C(s) ds\right)} e^{\left(-\int_t^u \lambda_I(s) ds\right)} = \lambda_C(u) du \, e^{\left(-\int_t^u (\lambda_C(s) + \lambda_I(s)) ds\right)}$$

$$= \lambda_C(u) e^{-\int_t^u \lambda(s) ds} \, du \Big) = \mathbb{E}_t \left\{ \text{LGD}_C \int_t^T D(t, u; r + \lambda) \lambda_C(u) \, (V(u))^+ \, du \right\}$$

CVA, DVA: A useful derivation in view of funding

$$\mathrm{CVA}_I(t) = \mathbb{E}_t\left\{\int_t^T D(t,u;r+\lambda)\mathrm{L}_{\mathrm{GD}_C}\,\lambda_C(u)\,(V(u))^+\,du\right\}$$

$$\mathrm{DVA}_I(t) = \mathbb{E}_t\left\{\int_t^T D(t,u;r+\lambda)\mathrm{L}_{\mathrm{GD}_I}\,\lambda_I(u)\,(-V(u))^+\,du\right\}$$

and we will see later that (without collateral and under the Reduced Borrowing Benefit case) Funding Cost and Benefit Adjustments (FCA, FBA) are (notice the formal analogies, used in industry)

$$\mathrm{FCA}_I(t) = \mathbb{E}_t\left\{\int_t^T D(t,u;r+\lambda)\mathrm{L}_{\mathrm{GD}_I}\,\lambda_I(u)\,(V(u))^+\,du\right\}$$

$$\mathrm{FBA}_I(t) = \mathbb{E}_t\left\{\int_t^T D(t,u;r+\lambda)\mathrm{L}_{\mathrm{GD}_I}\,\lambda_I(u)\,(-V(u))^+\,du\right\} = \mathrm{DVA}_I(t)$$

CVA and DVA: Collateral and Gap Risk

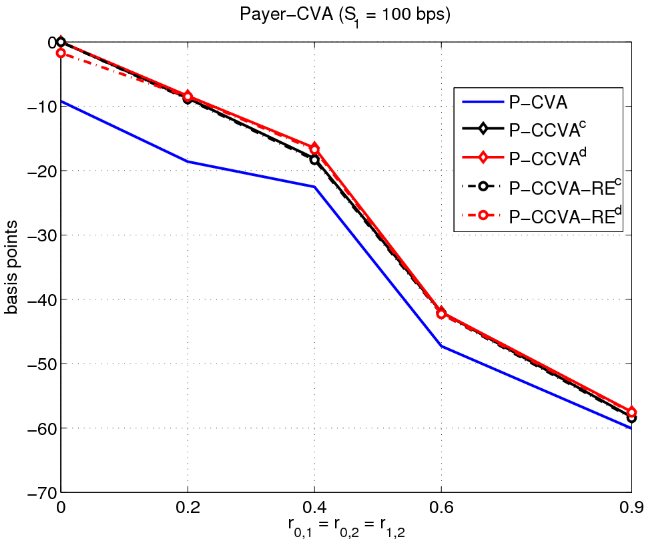
Collateral is a guarantee following mark to market...

and posted from the party that is facing a negative variation of mark to market in favour of the other party. If one party defaults, the other party may use collateral to cover their losses.

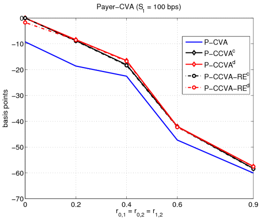
However, even under daily collateralization...

there can be large mark to market swings due to contation that make collateral rather ineffective. This is called GAP RISK and is one of the reasons why Central Clearing Counterparties (CCPs) and the new standard CSA have an initial margin as well.

Example of Gap Risk (from B. Capponi Pallavicini (2011)):


Payer–CVA (S_1 = 100 bps)
basis points
0
–10
–20
–30
–40
–50
–60
–70
0
0.2
0.4
0.6
0.9
$r_{0,1} = r_{0,2} = r_{1,2}$
P–CVA
P–CCVA^c
P–CCVA^d
P–CCVA–RE^c
P–CCVA–RE^d

Collateral Management and Gap Risk



The figure refers to a payer CDS contract as underlying See full paper B., Capponi and Pallavicini (2011) for more cases. Figure: relevant CVA component starting at 10 and ending up at 60 under high correlation. Here we plot it negative to remind it is to be subtracted from the standard price.

Collateral very effective in removing CVA when correlation $= 0$

CVA goes from 10 to 0 basis points.

Collateral not effective as default dependence grows

Collateralized and uncollateralized CVA become closer and closer, and for high correlations we get 60 bps CVA even with collateral.

Collateral Management and Gap Risk

Instantaneous contagion makes collateralization ineffective

We are “A”, buying from “B” protection (CDS) against default of “C”.

Under positive “correlation” “B”-“C”, default contagion pushes up the intensity of the CDS protection at default of the counterparty.

- Indeed, the term structure of the default probabilities for “C” after “B”’s default lies significantly above the levels pre “B”’s default, especially for large default correlation B-C.
- $\Rightarrow$ the default leg of the CDS will increase in value due to contagion at default of “B”, and instantaneously the Payer CDS will be worth more than what is in the collateral account.
- This increases the loss to us (“A”) when we buy protection from a new bank B2, and most of the CVA value will come from this jump.
- This explains the limited effectiveness of collateral.

Inclusion of Funding Cost

We now move to the inclusion of funding costs.

This is an important part of valuation, as shown by the financial news concerning JPMorgan as from January 2014, showing that Funding costs impacted the firm for 1.5 Billion $.

Where does the problem of funding costs originate from?

Inclusion of Funding Cost

When managing a trading position, one needs to obtain cash in order to do a number of operations:

- hedging the position,
- posting collateral,
- paying coupons or notionals, or interest on received collateral
- set reserves in place

and so on. Where are such founds obtained from?

- Obtain cash/assets from Treasury department or market.
- receive cash as a consequence of being in the position:
  - a coupon or notional reimbursement,
  - a positive mark to market move,
  - getting some collateral or interest on posted collateral,
  - a closeout payment.

All such flows need to be remunerated:

- if one is ”borrowing”, this will have a cost,
- and if one is ”lending”, this will provide revenues.

Inclusion of Funding Cost

Funding is not just different discounting

- CVA and DVA are not obtained just by adding a spread to the discount factor of assets cash flows
- Similarly, a hypothetical FVA is not simply applying spreads to borrowing and lending cash flows.

One has to carefully and properly analyze and price the real cash flows rather than add an artificial spread. The simple spread may emerge for very simple deals and under simplifying assumptions (no correlations, uni-directional cash flows, etc)

Inclusion of Funding Cost I

We restart from scratch from the product cash flows and add collateralization, cost of collateral, CVA and DVA after collateral, and funding costs for collateral and for the replication of the product.

In the following τ_I denotes the default time of our bank, or “us”.

We still denote by τ_C the default time of the counterparty,

while τ_F will be the default of the funder our bank treasury is using to borrow externally.

Our approach is based on modelling all cash flows that happen because of collateralization, default and funding.

Inclusion of Funding Cost II

We will use a risk neutral valuation framework and will discount at the risk free short rate r_t, assuming initially existence of a risk neutral measure (no arbitrage) with cash numeraire the theoretical bank account

$$dB_t = \boxed{r_t} B_t \, dt.$$

The related stochastic discount factor between dates s and t is $D(s,t) = \exp\left(-\int_s^t r_u \, du\right)$.

Our approach is based on a theoretical rate r that will *vanish* from our final valuation equation. Our valuation will be based only on observable market rates (invariance theorem).

However, in some useful industry approximation this invariance theorem for r cannot be invoked, as we will see, and we will need to make a choice for a proxy for r.

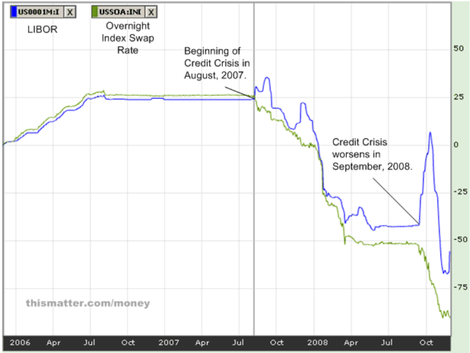
Inclusion of Funding Cost III

So in such cases the question is: What should we use as a proxy of the short rate?

Following the default of Lehman (and 7 other financial entities in 1 month of 2008) LIBOR and Overnight rates diverged:

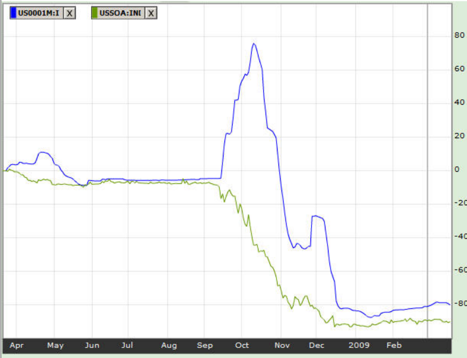
(From:
`http://thismatter.com/money/banking/libor-ois-spread.htm`
Accessed on Nov 26, 2014)

Inclusion of Funding Cost IV



Bloomberg.com: Investment Tools
http://www.bloomberg.com/apps/cbuilder?ticker1=USSOA%3AIND
Screen clipping taken: 11/30/2008

Here's an updated graph ending on March 19, 2009.



Bloomberg.com: Investment Tools
http://www.bloomberg.com/apps/cbuilder?ticker1=USSOA%3AIND
Screen clipping taken: 3/20/2009, 12:02 AM

Inclusion of Funding Cost V

Prior to Lehman's default LIBOR was used for discounting

Following the above divergence, it was decided that overnight rates are a better proxy of risk free rates.

The following table is taken by a presentation of Marco Bianchetti

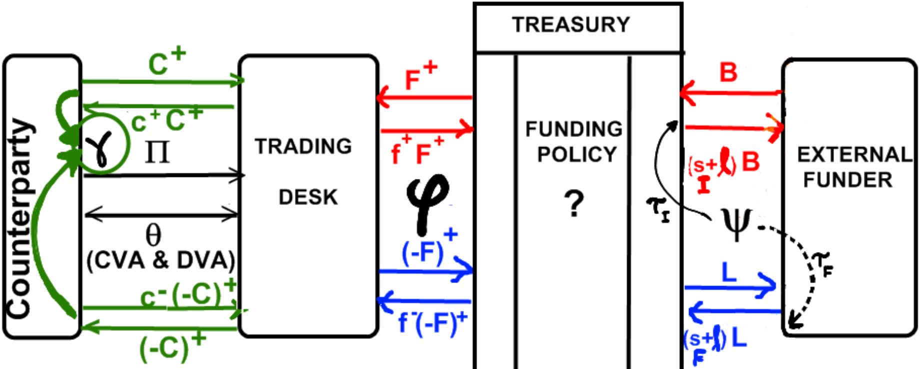
Inclusion of Funding Cost VI

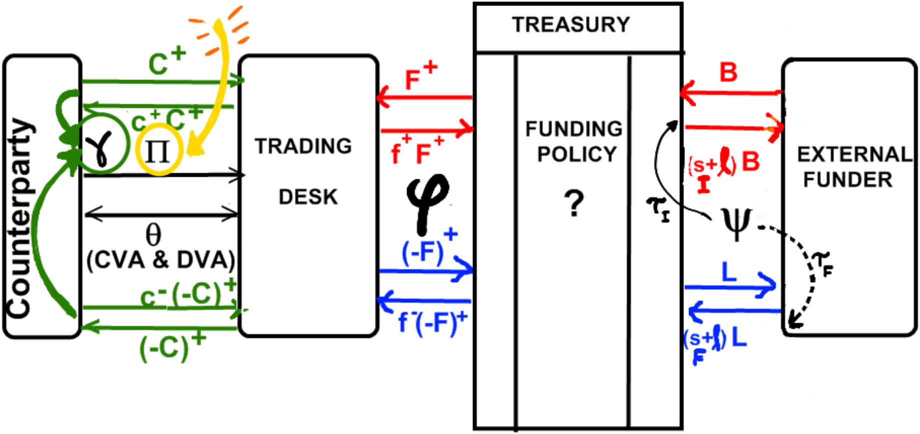
	Libor	Euribor	Eonia	Eurepo
Definition	London InterBank Offered Rate	Euro InterBank Offered Rate	Euro OverNight Index Average	Euro Repurchase Ageement rate
Market	London Interbank	Euro Interbank	Euro Interbank	Euro Interbank
Side	Offer	Offer	Offer	Offer
Rate quotation specs	EURLibor = Euribor, Other currencies: minor differences (e.g. act/365, T+0, London calendar for GBPLibor).	TARGET calendar, settlement $T+2$, act/360, three decimal places, modified following, end of month, tenor variable.	TARGET calendar, settlement $T+1$, act/360, three decimal places, tenor 1d.	As Euribor
Maturities	1d-12m	1w, 2w, 3w,1m,...,12m	1d	T/N-12m
Publication time	12.30 CET	11:00 am CET	6:45-7:00 pm CET	As Euribor
Panel banks	8-16 banks (London based) per currency	42 banks from 15 EU countries + 4 international banks	Same as Euribor	34 EU banks plus some large international bank from non-EU countries
Calculation agent	Reuters	Reuters	European Central Bank	Reuters
Transactions based	No	No	Yes	No
Collateral	No (unsecured)	No (unsecured)	No (unsecured)	Yes (secured)
Counterparty risk	Yes	Yes	Low	Negligible
Liquidity risk	Yes	Yes	Low	Negligible
Tenor basis	Yes	Yes	No	No

Inclusion of Funding Cost VII

As we said earlier, our approach is based on modelling all cash flows that happen because of collateralization, default and funding.

We start now.


Counterparty
TRADING
DESK
TREASURY
FUNDING
POLICY
?
EXTERNAL
FUNDER
c+
c+C+
Π
θ
(CVA & DVA)
c-(-C)+
(-C)+
F+
f+F+
φ
(-F)+
f-(-F)+
B
(s+λ)B
L
(s+λ)L
Ψ


TREASURY
Counterparty
TRADING
DESK
FUNDING
POLICY
?
EXTERNAL
FUNDER
c+
c+C+
(CVA & DVA)
c-(-C)+
(-C)+
F+
f+F+
(-F)+
f-(-F)+
B
L

Basic Payout Cash Flows Π

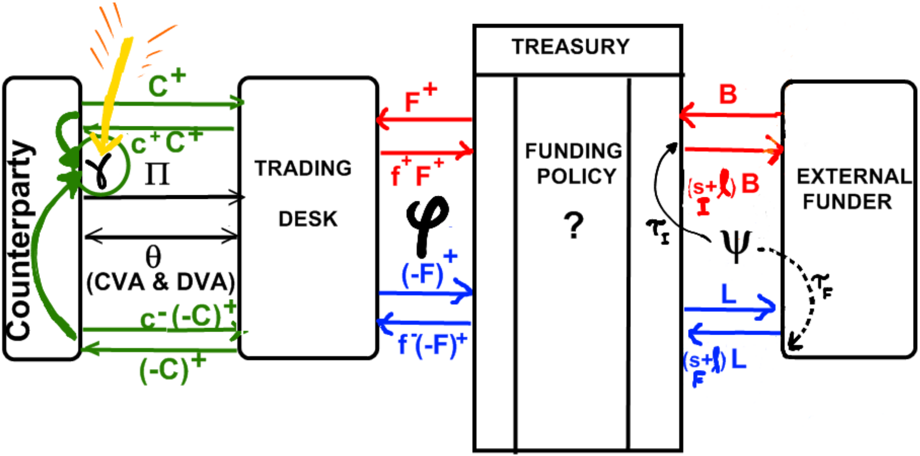
- We calculate prices by discounting cash-flows under the pricing measure. Collateral and funding are modeled as additional cashflows (as for CVA and DVA)
- We start from derivative's basic cash flows without credit, collateral of funding risks

$$\bar{V}_t := \mathbb{E}_t[\,\Pi(t, T \wedge \tau) + \ldots]$$

$\longrightarrow$ $\tau := \tau_C \wedge \tau_I$ is the first default time, and
$\longrightarrow$ $\Pi(t, u)$ is the sum of all discounted payoff terms up from t to u,

Cash flows are stopped either at the first default or at portfolio's expiry if defaults happen later.

- Define $V_t^0 := \mathbb{E}_t[\,\Pi(t, T)\,]$ (credit and funding-) "risk-free" price.


Counterparty
TRADING
DESK
TREASURY
FUNDING
POLICY
?
EXTERNAL
FUNDER
c+
c+C+
Π
θ
(CVA & DVA)
c-(-C)+
(-C)+
F+
f+F+
φ
(-F)+
f-(-F)+
B
(s+l) B
I
L
(s+l) L
F
ψ

Collateral costs cash flows γ I

- As second contribution we consider the collateralization procedure and we add its cash flows.

$$\bar{V}_t := \mathbb{E}_t[\,\Pi(t, T \wedge \tau)\,] + \mathbb{E}_t[\,\gamma(t, T \wedge \tau; C) + \dots\,]$$

where

$\longrightarrow$ C_t is the collateral account defined by the CSA,

$\longrightarrow$ $\gamma(t, u; C)$ are the collateral margining costs up to time u.

- The second expected value originates what is occasionally called Liquidity Valuation Adjustment (LVA) in simplified versions of this analysis. We will show this in detail later.
- If $C > 0$ collateral has been overall posted by the counterparty to protect us, and we have to pay interest c^+.
- If $C < 0$ we posted collateral for the counterparty (and we are remunerated at interest c^-).

Collateral costs cash flows γ II

- The cash flows due to the margining procedure on the time grid $\{t_k\}$ are equal to

$$\gamma(t,u;C) := -\sum_{k=1}^{n-1} 1_{\{t\le t_k<u\}} D(t,t_k) C_{t_k}\left(P_{t_k}(t_{k+1})(1+\alpha_k \tilde{c}_{t_k}(t_{k+1}))-1\right)$$

where $\alpha_k = t_{k+1} - t_k$ and the collateral accrual rates are given by

$$\tilde{c}_t := c_t^+ 1_{\{C_t>0\}} + c_t^- 1_{\{C_t<0\}}$$

- A few first order approximations/linearizations allow us to simplify.

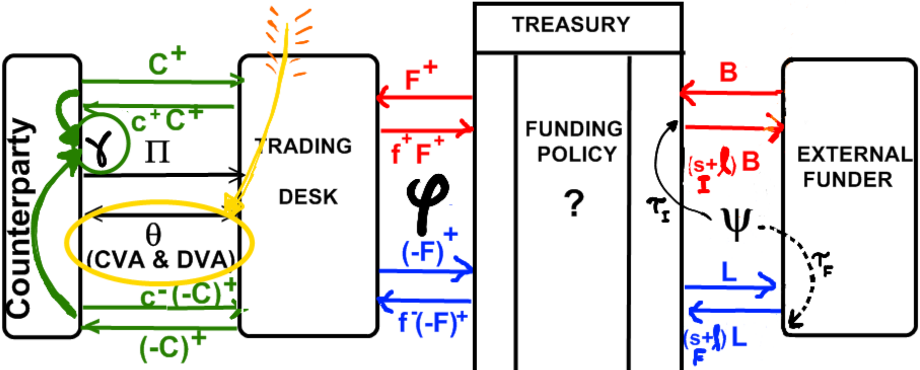
$$P_{t_k}(t_{k+1})(1+\alpha c) = e^{-r_{t_k}(t_{k+1})\alpha}(1+\alpha c) \approx e^{-r\alpha} e^{c\alpha} = e^{(c-r)\alpha} \approx 1+(c-r)\alpha$$

Collateral costs cash flows γ III

- Note that this becomes exact if we take collateralization as happening in continuous time (later).

$$\gamma(t,u;C) \approx -\sum_{k=1}^{n-1} 1_{\{t \le t_k < u\}} D(t,t_k) C_{t_k} \alpha_k (\tilde{c}_{t_k}(t_{k+1}) - r_{t_k}(t_{k+1}))$$

Note that if the collateral rates in $\tilde{c}$ are both equal to the risk free rate, then this term is zero.


Counterparty
c+
c+C+
γ
Π
θ
(CVA & DVA)
c-(-C)+
(-C)+
TRADING
DESK
F+
f+F+
φ
(-F)+
f-(-F)+
TREASURY
FUNDING
POLICY
?
B
(s+λI) B
τI
Ψ
τF
L
(s+λF) L
EXTERNAL
FUNDER

Close-Out θ: Trading-CVA/DVA under Collateral – I

- As third contribution we consider the cash flow happening at 1st default, and we have

$$\begin{aligned}\bar{V}_t &:= \mathbb{E}_t[\,\Pi(t, T\wedge\tau)\,] \\ &+ \mathbb{E}_t[\,\gamma(t, T\wedge\tau; C)\,] \\ &+ \mathbb{E}_t\left[\,1_{\{\tau<T\}} D(t,\tau)\theta_\tau(C,\varepsilon) + \dots\right]\end{aligned}$$

where

$\longrightarrow$ ε_τ is the close-out amount, or residual value of the deal at default, which we called NPV earlier, and

$\longrightarrow$ $\theta_\tau(C,\varepsilon)$ is the on-default cash flow.

- θ_τ will contain collateral adjusted CVA and DVA payouts for the instument cash flows
- We define θ_τ including the pre-default value of the collateral account since it is used by the close-out netting rule to reduce exposure

Close-Out θ: Trading-CVA/DVA under Collateral – II

- The close-out amount is not a symmetric quantity w.r.t. the exchange of the role of two parties, since it is valued by one party after the default of the other one.

$$\varepsilon_\tau := 1_{\{\tau=\tau_C\}}\varepsilon_{I,\tau} + 1_{\{\tau=\tau_I\}}\varepsilon_{C,\tau}$$

- Without entering into the detail of close-out valuation we can assume a close-out amount equal to the risk-free price of remaining cash flows inclusive of collateralization and funding costs. More details in the examples.
  - $\longrightarrow$ See ISDA document “Market Review of OTC Derivative Bilateral Collateralization Practices” (2010).
  - $\longrightarrow$ See, for detailed examples, Parker and McGarry (2009) or Weeber and Robson (2009)
  - $\longrightarrow$ See, for a review, Brigo, Morini, Pallavicini (2013).

Close-Out θ: Trading-CVA/DVA under Collateral – III

- At transaction maturity, or after applying close-out netting, the originating party expects to get back the remaining collateral.
- Yet, prevailing legislation's may give to the Collateral Taker some rights on the collateral itself.
  - ⟶ In presence of re-hypothecation the collateral account may be used for funding, so that cash requirements are reduced, but counterparty risk may increase.
  - ⟶ See Brigo, Capponi, Pallavicini and Papatheodorou (2011).
- In case of collateral re-hypothecation the surviving party must consider the possibility to recover only a fraction of his collateral.
  - ⟶ We name such recovery rate $\text{R}\textsc{ec}'_I$, if the investor is the Collateral Taker, or $\text{R}\textsc{ec}'_C$ in the other case.
  - ⟶ In the worst case the surviving party has no precedence on other creditors to get back his collateral, so that

$$\text{R}\textsc{ec}_I \leq \text{R}\textsc{ec}'_I \leq 1\ , \quad \text{R}\textsc{ec}_C \leq \text{R}\textsc{ec}'_C \leq 1$$

Close-Out θ: Trading-CVA/DVA under Collateral – IV

- The on-default cash flow $\theta_\tau(C,\varepsilon)$ can be calculated by following ISDA documentation. We obtain

$$\begin{aligned}\theta_\tau(C,\varepsilon) &:= 1_{\{\tau=\tau_C<\tau_I\}}\left(\varepsilon_{I,\tau} - \mathrm{L{\small GD}}_C(\varepsilon^+_{I,\tau} - C^+_{\tau-})^+ - \mathrm{L{\small GD}}'_C(\varepsilon^-_{I,\tau} - C^-_{\tau-})^+\right)\\ &+ 1_{\{\tau=\tau_I<\tau_C\}}\left(\varepsilon_{C,\tau} - \mathrm{L{\small GD}}_I(\varepsilon^-_{C,\tau} - C^-_{\tau-})^- - \mathrm{L{\small GD}}'_I(\varepsilon^+_{C,\tau} - C^+_{\tau-})^-\right)\end{aligned}$$

  where loss-given-defaults are defined as $\mathrm{L{\small GD}}_C := 1 - \mathrm{R{\small EC}}_C$, and so on.
- If both parties agree on exposure, namely $\varepsilon_{I,\tau} = \varepsilon_{C,\tau} = \varepsilon_\tau$ then

$$\begin{aligned}\theta_\tau(C,\varepsilon) &:= \varepsilon_\tau - 1_{\{\tau=\tau_C<\tau_I\}}\Pi_{\mathrm{CVAcoll}} + 1_{\{\tau=\tau_I<\tau_C\}}\Pi_{\mathrm{DVAcoll}}\\ \Pi_{\mathrm{CVAcoll}} &= \mathrm{L{\small GD}}_C(\varepsilon^+_\tau - C^+_{\tau-})^+ + \mathrm{L{\small GD}}'_C(\varepsilon^-_\tau - C^-_{\tau-})^+\\ \Pi_{\mathrm{DVAcoll}} &= \mathrm{L{\small GD}}_I((-\varepsilon_\tau)^+ - (-C_{\tau-})^+)^+ + \mathrm{L{\small GD}}'_I(C^+_{\tau-} - \varepsilon^+_\tau)^+\end{aligned}$$

Close-Out θ: Trading-CVA/DVA under Collateral – V

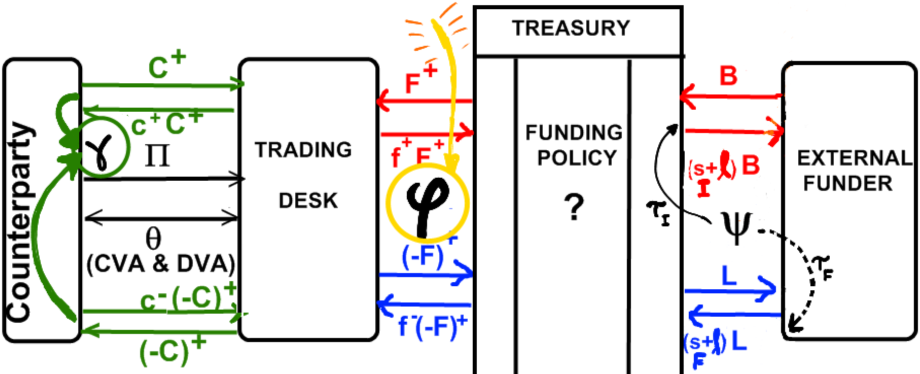
- In case of re-hypothecation, when $\text{LGD}_C = \text{LGD}'_C$ and $\text{LGD}_I = \text{LGD}'_I$, we obtain a simpler relationship

$$\begin{aligned} \theta_\tau(C,\varepsilon) &:= \varepsilon_\tau \\ &- 1_{\{\tau=\tau_C<\tau_I\}}\text{LGD}_C(\varepsilon_{I,\tau} - C_{\tau^-})^+ \\ &- 1_{\{\tau=\tau_I<\tau_C\}}\text{LGD}_I(\varepsilon_{C,\tau} - C_{\tau^-})^- \end{aligned}$$

- With re-hypothecation, we can set

$$\Pi_{\text{DVAcoll}} = (-(\varepsilon_\tau - C_{\tau^-}))^+, \quad \Pi_{\text{CVAcoll}} = (\varepsilon_\tau - C_{\tau^-})^+.$$

Under replacement closeout, $\epsilon_\tau = \bar{V}_\tau$ (nonlinearity/recursion!)
Under risk-free closeout, $\epsilon_\tau = V^0_\tau$ (easier)


Counterparty
TRADING
DESK
TREASURY
FUNDING
POLICY
?
EXTERNAL
FUNDER
c+
c+C+
Π
θ
(CVA & DVA)
c-(-C)+
(-C)+
F+
f+F+
φ
(-F)+
f-(-F)+
B
(s+λI) B
L
(s+λF) L
Ψ
τI
τF
γ

Funding Costs of the Replication Strategy φ – I

- As fourth contribution we consider the cost of funding for the hedging procedures and we add the relevant cash flows.

$$\begin{aligned}\bar{V}_t &:= \mathbb{E}_t[\,\Pi(t, T\wedge\tau)\,] + \mathbb{E}_t\left[\,\gamma(t, T\wedge\tau; C) + 1_{\{\tau<T\}}D(t,\tau)\theta_\tau(C,\varepsilon)\,\right] \\ &+ \mathbb{E}_t[\,\varphi(t, T\wedge\tau; F, H)\,] + \dots\end{aligned}$$

  The last term, especially in simplified versions, is related to what is called FVA in the industry. We will point this out once we get rid of the rate r.

  $\longrightarrow$ F_t is the cash account for the replication of the trade,
  $\longrightarrow$ H_t is the risky-asset account in the replication,
  $\longrightarrow$ $\varphi(t, u; F, H)$ are the cash F and hedging H funding costs up to u.

- In classical Black Scholes on Equity, for a call option (no credit risk, no collateral, no funding costs),

$$\bar{V}_t^{\text{Call}} = \Delta_t S_t + \eta_t B_t =: H_t + F_t, \quad \tau = +\infty, \ \ C = \gamma = \varphi = 0.$$

Funding Costs of the Replication Strategy φ – II

- Cash flows due to funding of the replication strategy are

$$\begin{aligned}\varphi(t,u) &:= \sum_{j=1}^{m-1} 1_{\{t\le t_j<u\}} D(t,t_j)(F_{t_j}+H_{t_j})\left(1-P_{t_j}(t_{j+1})(1+\alpha_k \tilde{f}_{t_j}(t_{j+1}))\right)\\ &- \sum_{j=1}^{m-1} 1_{\{t\le t_j<u\}} D(t,t_j)H_{t_j}\left(1-P_{t_j}(t_{j+1})(1+\alpha_k \tilde{h}_{t_j}(t_{j+1}))\right)\end{aligned}$$

where the funding and lending rates for F and H are given by

$$\tilde{f}_t := f_t^+ 1_{\{F_t>0\}} + f_t^- 1_{\{F_t<0\}} \quad \tilde{h}_t := h_t^+ 1_{\{H_t>0\}} + h_t^- 1_{\{H_t<0\}}$$

Funding Costs of the Replication Strategy φ – III

- Continuously compounding and linearizing exponentials:

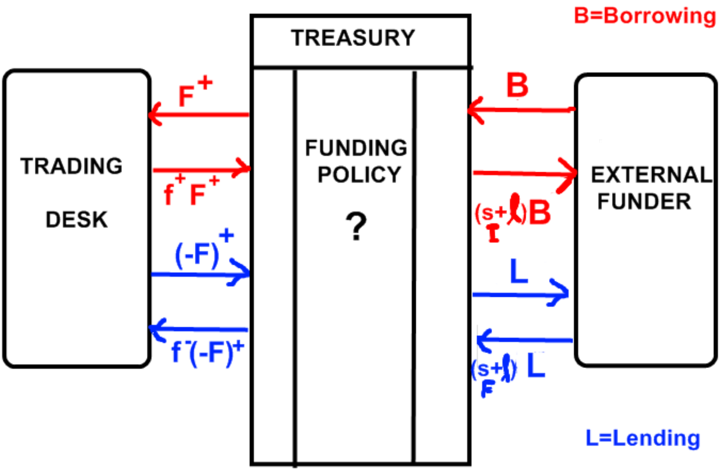
$$\varphi(t,u) := \sum_{j=1}^{m-1} 1_{\{t\le t_j<u\}} D(t,t_j)(F_{t_j}+H_{t_j})\alpha_k\left(r_{t_j}(t_{j+1})-\tilde f_{t_j}(t_{j+1})\right)$$
$$- \sum_{j=1}^{m-1} 1_{\{t\le t_j<u\}} D(t,t_j)H_{t_j}\alpha_k\left(r_{t_j}(t_{j+1})-\tilde h_{t_j}(t_{j+1})\right)$$

- Note: the expected value of φ is related to the so called FVA. If the treasury funding rates $\tilde f$ are the same as the asset lending/borrowing rates $\tilde h$ then the funding cash flows simplify to

$$\varphi(t,u) := \sum_{j=1}^{m-1} 1_{\{t\le t_j<u\}} D(t,t_j)F_{t_j}\alpha_k\left(r_{t_j}(t_{j+1})-\tilde f_{t_j}(t_{j+1})\right)$$

- If further the treasury borrows and lends at the risk free rate, $\tilde f = r$, then $\varphi = 0$ and FVA$= 0$.

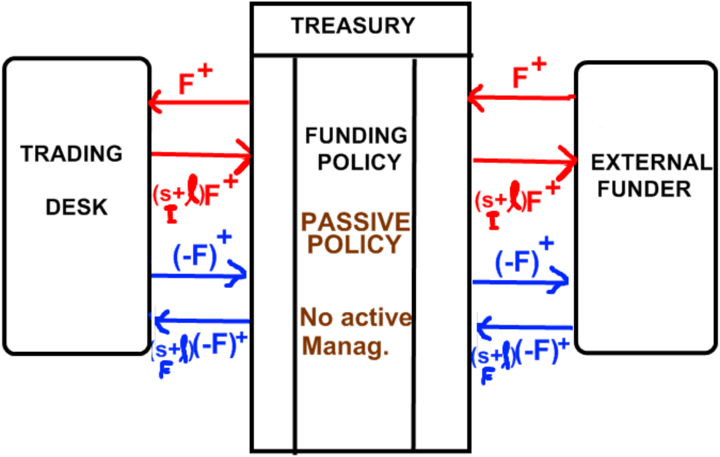
Funding Costs of the Replication Strategy φ – IV



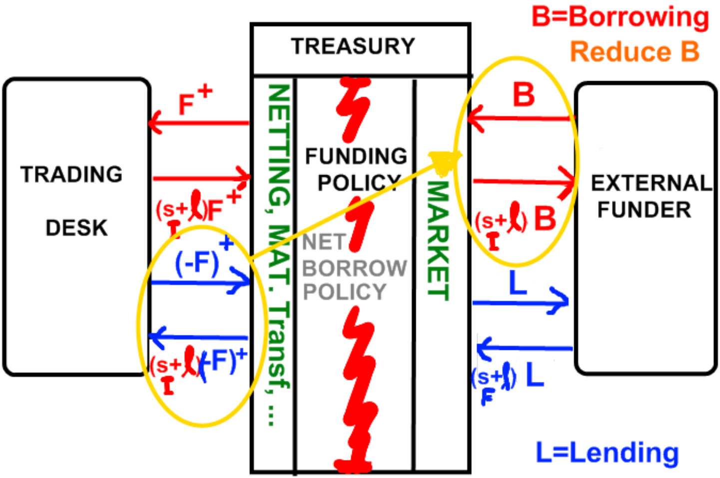
Funding rates depend on Treasury policies

- In real applications the funding rate $\tilde{f}_t$ is determined by the party managing the funding account for the investor, eg the bank's treasury:
  - $\longrightarrow$ trading positions may be netted before funding on the mkt
  - $\longrightarrow$ a Funds Transfer Pricing (FTP) process may be implemented to gauge the performances of different business units;
  - $\longrightarrow$ a maturity transformation rule can be used to link portfolios to effective maturity dates;
  - $\longrightarrow$ sources of funding can be mixed into the internal funding curve ...

Funding rates depend on Treasury policies

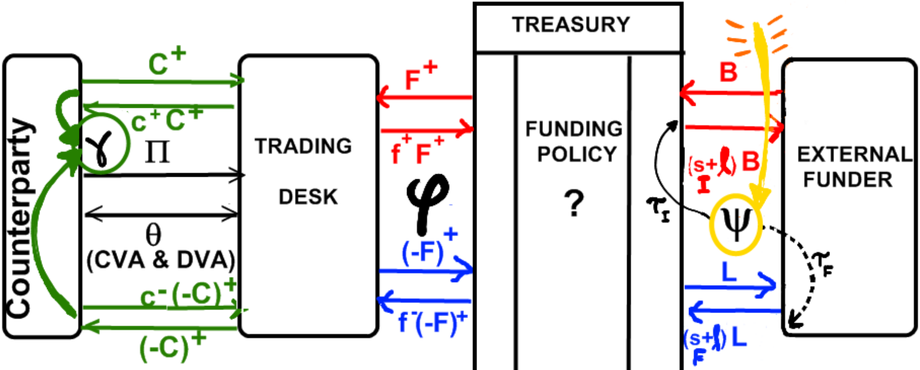


Funding rates depend on Treasury policies



Funding rates depend on Treasury policies

- In part of the literature the role of the treasury is usually neglected, leading to controversial results particularly when the funding positions are not distinguished from the trading positions.
- See partial claims “funding costs $=$ DVA”, or “there are no funding costs”, cited in the literature (Hull White, ”FVA =0”). We’ll clarify these points in a minute.


Counterparty
TRADING
DESK
TREASURY
FUNDING
POLICY
?
EXTERNAL
FUNDER
c+
c+C+
Π
θ
(CVA & DVA)
c-(-C)+
(-C)+
F+
f+F+
φ
(-F)+
f-(-F)+
B
(s+l) B
L
(s+l) L
Ψ

Default flows ψ for the Funding part I

$$\begin{aligned}\bar{V}_t &:= \mathbb{E}_t[\Pi(t, T\wedge\tau)] + \mathbb{E}_t\left[\gamma(t, T\wedge\tau; C) + 1_{\{\tau<T\}} D(t,\tau)\theta_\tau(C,\varepsilon)\right] \\ &+ \mathbb{E}_t[\varphi(t, T\wedge\tau; F, H)] + \mathbb{E}_t[\psi(t, \tau_F, \tau, T)]\end{aligned}$$

When our bank treasury is borrowing in the market from bank F, F charges our bank a CVA due to our credit risk. Seen from our bank, this charge is a DVA_F that makes the loan more expensive.

This means that if we fix the final notional, we will be able to borrow less than if we were default free. If we fix the amount borrowed now, we will have to repay more at the end. Overall the loan will be more expensive because of our bank credit risk. This is a cost.

Similarly, when our bank treasury lends externally, it measures a CVA_F on the loan due to the possibility that the borrower defaults. Loan is more remunerative due to upfront CVA_F charged to external Borrower (External Funder Benefit).

Default flows ψ for the Funding part II

Often one assumes that H generates no funding costs because it is fully and perfectly collateralized with re-hypothecation of collateral.

IMPORTANT

We are adding the ψ treasury DVA-CVA term to our Equation but the Eq terms would ideally sit in different parts of the bank.

- The value of the ψ part is with the treasury,
- while the other parts are with the trading desk.
- We will shortly see the different ways the treasury may pass the cost/benefits in ψ to the desk
- This is controlled with the rates f^+ and f^- in the funding cost-benefit term φ through suitable credit spreads

Default flows ψ for the Funding part III

Total value of claim then includes cash flows from debit and credit risk in the funding strategy that are seen by the treasury:

$$\psi_{EFB}(t,\tau_F,\tau,T) = D(t,\tau)1_{\{\tau=\tau_I<T\}}\text{L}_{\text{GD}I}(F_\tau)^+$$

$$-D(t,\tau_F)1_{\{\tau\wedge\tau_F=\tau_F<T\}}\text{L}_{\text{GD}F}(-F_{\tau_F})^+$$

The first term on the right hand side is the funding DVA cash flow (leading to what is called occasionally DVA_2 or FDA, “Funding Debit Adjustment”). We will call the value of this cash flow DVA_F. This is triggered when our treasury is borrowing and defaults first, causing a loss to the external lender.

The second term on the right hand side is the funding credit valuation adjustment cash flow, that is triggered when our treasury is lending externally and the borrower defaults first. The value of this cash flow is called -CVA_F.

Default flows ψ for the Funding part IV

There is a possibly different definition for ψ.
If the treasury considers the desk as net borrowing, the lending of $(-F)^+$ will be considered not as a loan but as a reduction in borrowing.

In this sense there will be no CVA_F term now, since no lending is considered by the treasury.

In this case the cash flows of the credit adjustment for the funding part consist only of the debit adjustment part and are called Reduced Borrowing Benefit:

$$\psi_{RBB}(t, \tau_F, \tau, T) = D(t, \tau) 1_{\{\tau=\tau_I<T\}} \mathrm{L}_{\mathrm{GD}_I}(F_\tau)^+$$

The two cases of External Funder Benefit (EFB) and Reduced Borrowing Benefit (RBB) will be discussed shortly also in connection with interest rates $\tilde{f}$.

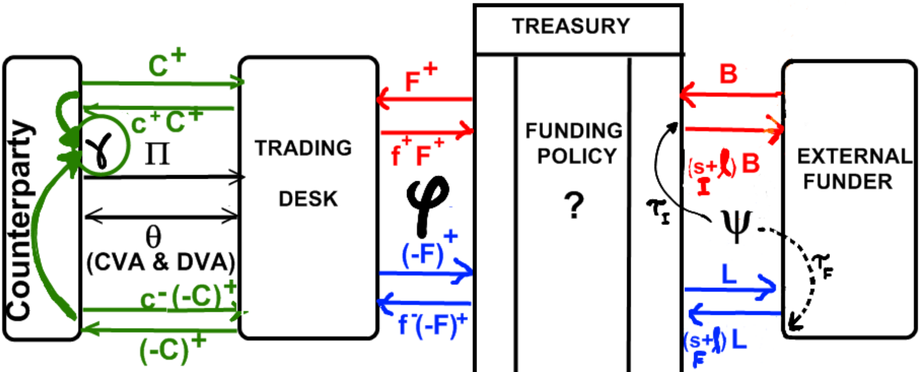
Recursive non-decomposable Nature of Pricing – I

We have now included all the cash flows.

Notice that we are going to add up all cash flows but, as we mentioned earlier, different parts of this valuation may sit in different part of the bank and may be exchanged, in particular, between the Treasury and the Trading Desk.

We now move to discussing the final valuation equation we obtained more in detail.

Recursive non-decomposable Nature of Pricing – II



Recursive non-decomposable Nature of Pricing – III

$$(*) \quad \bar{V}_t = \mathbb{E}_t\big[\,\Pi(t, T\wedge\tau) + \gamma(t) + 1_{\{\tau<T\}}D(t,\tau)\theta_\tau + \varphi(t) + \psi(t,\tau_F,\tau)\,\big]$$

Can we interpret:

$\mathbb{E}_t\big[\,\Pi(t, T\wedge\tau) + 1_{\{\tau<T\}}D(t,\tau)\theta_\tau(C,\varepsilon)\,\big]$: RiskFree Price + DVA - CVA?

$\mathbb{E}_t[\,\gamma(t, T\wedge\tau) + \varphi(t, T\wedge\tau; F, H)\,]$: Funding adjustment LVA+FVA?

$\mathbb{E}_t[\,\psi(t,\tau_F,\tau,T)\,]$: Treasury CVA and DVA

Not really. This is not a decomposition. It is an equation. In fact since

$$\bar{V}_t = F_t + H_t + C_t \quad \text{(re–hypo)}$$

we see that the φ present value term depends on future $F_t = \bar{V}_t - H_t + C_t$ and generally the closeouts θ ψ, via ϵ, F and C, depend on future $\bar{V}$ too. All terms feed each other and there is no neat separation of risks. *Recursive pricing: Nonlinear PDE's / BSDEs* for $\bar{V}$

Recursive non-decomposable Nature of Pricing – IV

$$(*) \quad \bar{V}_t = \mathbb{E}_t\big[\,\Pi(t, T \wedge \tau) + \gamma(t) + 1_{\{\tau < T\}} D(t,\tau)\theta_\tau + \varphi(t) + \psi(t, \tau_F, \tau)\,\big]$$

"FinalPrice = RiskFreePrice (+ DVA?) - CVA + FVA" not possible.

See Pallavicini Perini B. (2011, 2012) for $\bar{V}$ equations and algorithms.

Each term inside the expectation on the right hand side depends on future $\bar{V}$ and hence on all risks at the same time.

If we remove asymmetry of borrowing and lending rates, assuming $f^+ = f^-, c^+ = c^-, h^+ = h^-$ and adopt a risk free closeout at default, replacing $\bar{V}_\tau$ with V^0_τ, then the problem becomes linear again and the split works.

Recursive non-decomposable Nature of Pricing – V

We'll discuss "linearization" and the related NONLINEARITY VALUATION ADJUSTMENT in a minute.

We can obtain a more specific valuation equation (equivalent to a PDE or BSDE) by further steps:

1. Write the equation for $\bar{V}_{t_j}$ starting from $\bar{V}_{t_{j+1}}$, backwards.
2. Take the continuous time limit, where funding happens instantaneously and collateral is posted continuosly (still gap risk, unless you assume NPV to be left continuous)
3. credit risk: work under default-free filtration $\mathcal{F}_t$ and assume basic cash flows are $\mathcal{F}_t$ adapted.
4. Assume conditional independence of defaults: spreads λ's may be correlated, but jump to defaults ξ's will be independent.

Immersion hypothesis and conditional independence of defaults I

Recall that we are assuming

$$\mathcal{G}_t = \mathcal{F}_t \vee \sigma(\{\tau_i \leq u\}, u \leq t)$$

with i indexing all the default times in the system. Working under $\mathcal{F}$-immersion usually means that the risks in the basic cash flows Π are assumed not to be credit sensitive but to depend only on the filtration $\mathcal{F}$ of pre-default or default-free market information, eg default free interest rate swaps portfolio.

Credit in funding rates $\tilde{f}$: external funder benefit

We now discuss the funding cost-benefit rates $\tilde{f}$ in the funding cash flows φ.

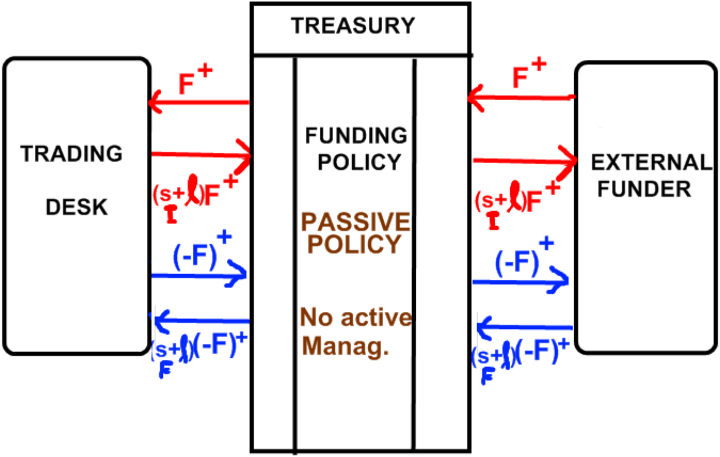
These rates play a fundamental role also with respect to double counting and need to be defined very carefully.

We will give two possible definitions, leading to two different set of valuation equations and to two different sets of rules to avoid double counting.

The two cases were briefly intrduced earlier with the term ψ (treasury $DVA_F - CVA_F$ term) and we expand the discussion now.

- External Funder Benefit (EFB)
- Reduced Borrower Benefit (RBB)

Credit in funding rates $\tilde{f}$: External Funder Benefit



Credit in funding rates $\tilde{f}$: External Funder Benefit

We now investigate whether the rates $\tilde{f}$ should include credit effects.

When we borrow or lend X from/to a funder F to maintain the funding strategy, we will need to pay/receive interest on the amount borrowed/lent to the funder (the treasury will be in-between us and the external funder).

If we **borrow** X in t and pay back in $t + dt$ with interest f^+ we write

$$1_{\{\tau > t\}} X[1 - D(t, t+dt) e^{f^+ dt} (1_{\{\tau_I > t+dt\}} + \text{R}\text{EC}_I 1_{\{\tau_I \in dt\}})]$$

Present valuing this conditional on $\mathcal{G}_t$ and $\tau > t$ leads to

$$\approx X[1 - e^{-(r_t + \lambda_I(t) - f^+)\ dt} - \text{R}\text{EC}_I \lambda_I(t) dt] = X(r_t + \lambda_I(t)\text{L}\text{GD}_I - f^+)dt = 0$$

This means that for the loan to be valued at par we need to assume

$$f^+ = r_t + \lambda_I(t)\text{L}\text{GD}_I + \ell_+ =: r_t + s_I(t) + \ell_+$$

where we added a funding liquidity basis ℓ_+ (see CDS-Bond basis).

Credit in funding rates $\tilde{f}$: External Funder Benefit

If we **lend** X in t and receive it back in $t + dt$ from a funder F, we see similarly that the interest to be paid is coming from setting to zero the total loan value inclusive of credit risk:

$$1_{\{\tau>t\}}X[-1 + D(t, t + dt)e^{f^- dt}(1_{\{\tau_F>t+dt\}} + \mathrm{REC}_F 1_{\{\tau_F\in dt\}})]$$

and amounts to

$$-(r_t + \lambda_F(t)\mathrm{LGD}_F - f^-)dt = 0$$

This means that the total interest f_- that we should receive is

$$f^- = r_t + \lambda_F(t)\mathrm{LGD}_F + \ell^-(t) =: r_t + s_F(t) + \ell^-(t)$$

where we added a funding liquidity basis ℓ^-. Summing up, with credit included we have

$$f_-(t) = r_t + \lambda_F(t)\mathrm{LGD}_F + \ell_-(t), \quad f_+(t) = r_t + \lambda_I(t)\mathrm{LGD}_I + \ell_+(t).$$

Without credit effects in the treasury borrowing/lending, we'd have

$$f_-(t) = r_t + \ \ell_+(t), \quad f_+(t) = r_t + \ell_+(t).$$

Credit in funding rates $\tilde{f}$: External Funder Benefit

We now discuss briefly how the rates f^+ and f^- control the funding costs transfer between treasury and trading desks.

The treasury when borrowing and lending externally to service the desk faces credit costs-benefits, the $DVA_F - CVA_F$ terms in ψ.

The treasury, when borrowing, marks a cost with the initial DVA_F, that measures the credit cost of the external loan with the funder. This cost will be charged to the trading desk via the rate f^+. In fact both quantities are indexed to s_I, the spread of the bank.

The treasury, when lending, marks a benefit with the initial CVA_F, that measures the increased remuneration of the loan due to the funder credit risk. This benefit is passed to the trading desk with the rate f^-.

We will see how this ”passing” can be made precise in a minute with calculations, and how we can avoid double counting.

Credit in funding rates $\tilde{f}$: EFB vs Reduced Borrowing

We call the above setting for f^- the External Funder Benefit (EFB).

There is an alternative approach.

If the trading desk is always considered as net borrowing (other trades), by "lending" X to treasury it is reducing the net borrowing.

Say desk has borrowed total outstanding L from treasury. Treasury is borrowing L externally at cost $(s_l + \ell)L$ that passes to desk.

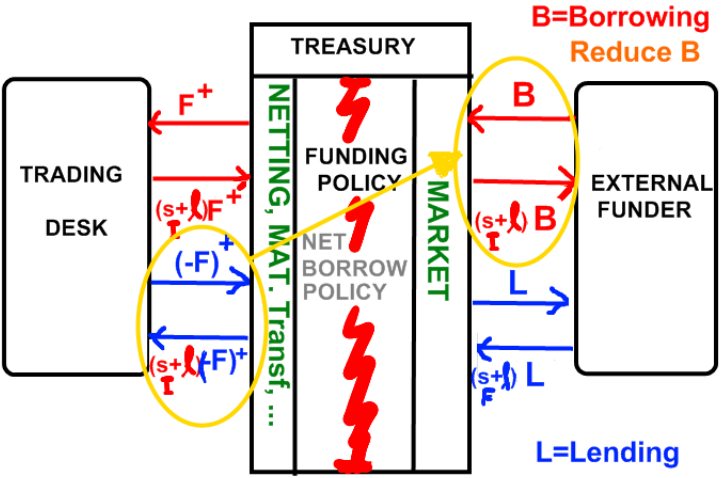
Now desk lends X back to treasury with new trade ($X < L$). Net borrowing lowers to $L - X$, & desk pays now interest $(s_l + \ell)(L - X)$

The benefit in this case is the reduction in cost, namely

$$(s_l + \ell)L - (s_l + \ell)(L - X) = (s_l + \ell)X$$

as opposed to the direct benefit of lending X externally for $s_F X$. We call this setting the Reduced Borrowing Benefit (RBB).

Credit in funding rates $\tilde{f}$: reduced borrowing benefit



Credit in funding rates $\tilde{f}$: reduced borrowing benefit

At the same time, recall our earlier discussion on the CVA_F and DVA_F measured by treasury. If we adopt RBB, there will be no CVA_F, as discussed earlier, and $\psi = \psi_{RBB}$.

Indeed, as the Treasury is considering the desk lending as simply reducing the borrowing to the external funder, it has no right to charge the upfront CVA_F to the external borrower and then give this back to the desk, since it is not treating the amount $(-F)^+$ as if actually lending to the external funder.

Credit in funding rates $\tilde{f}$: reduced borrowing benefit

To fix ideas, suppose the desk is borrowing overall 100 million USD for all its trades, and that the specific netting set we are considering now has a negative cash component $F = 10$ Million on the replica. We can lend this amount F to the treasury.

Previously, we assumed that the treasury would lend this to an external funder and get a interest compensation of $f^- = r + s_F + \ell_-$, that would then be passed to the trading desk.

However, in reality, the trading desk simply reduces its borrowing from 100 to $100 - 10 = 90$ millions. The benefit then is that it will no longer pay interest $s_I + \ell_+$ on 10 of the 100 millions. Hence the benefit in the rate f^- is indexed at s_I rather than s_F.

Still, the treasury, due to funding policies, may maintain a basis ℓ_- different from ℓ_+

$$f^- = r + s_F + \ell_- \quad \rightarrow \quad f^- = r + s_I + \ell_-$$

ℓ? CDS-Bond Basis as funding liquidity component

We now discuss one possible component for the liquidity bases ℓ.

Consider a reference credit entity where we have both Bonds issued by that entity and CDS traded on the default of that entity.

We can strip default intensities both from bonds (Z-spread) $\lambda^b(t)$ and CDS (fair spread) $\lambda^c(t)$. Why can the two λ's be different?

Funded vs Unfunded instruments

When I buy the bond I am using cash to pay its price at time 0. The bond is a funded instrument. I need to find/mobilize the cash.
When I sell protection via a CDS it costs me zero to enter the CDS at fair premium at time zero.

Hence, all things being equal, due to the need of putting liquidity on the table, I will require a further premium (liquidity premium) ℓ from the Bond, compared to the CDS.

ℓ? CDS-Bond Basis as funding liquidity component

Bonds λ^b or CDS λ^c?

It follows that we should use the bond-implied default intensities λ rather than the CDS implied ones when computing funding costs.

However, CDS are more standardized and can be more liquid too, so that it is easier and often more reliable to deduce λ's from CDS and then correct them a posteriori for a CDS-Bond Basis that can be proxied from other names as well.

The CDS-bond basis is considered to be an indicator of funding liquidity

$$\ell(t) = \lambda^c(t) - \lambda^b(t).$$

Given the liquidity premium in bonds, one would expect the basis to be negative in general. However, the situation is more complicated.

ℓ? CDS-Bond Basis as funding liquidity component

$$\ell(t) = \lambda^c(t) - \lambda^b(t).$$

The basis has been both "$+$" and "$-$" through history. Traders may set up basis trades if convinced arbitrage opportunities are showing up.

- Bond funding cost: $\ell \downarrow$
- CDS counterparty risk: $\ell \downarrow$
- Shorting credit: Easier buying CDS protection than shorting bonds. CDS more attractive and default leg more expensive $\ell \uparrow$.
- CDS protect from more general defaults than bonds and have cheapest do deliver advantages when buying protection, as one delivers a less valuable bond in exchange for face value: $\ell \uparrow$.

Funding incusive valuation equations I

- With the above steps, we obtain (here $\pi_t\, dt = \Pi(t, t + dt)$)

$$\bar{V}_t = \int_t^T \mathbb{E}\{D(t, u; r + \lambda)[\pi_u + (r_u - \tilde{c}_u)C_u + \lambda_u \theta_u + \qquad \text{EQFund1}$$

$$+(r_u - \tilde{f}_u)(F_u + H_u) + (\tilde{h}_u - r_u)H_u + \lambda_u^I \mathrm{L}_{\mathrm{GD}_I}(F_u)^+ - \lambda_u^F \mathrm{L}_{\mathrm{GD}_F}(-F_u)^+]|\mathcal{F}_t\}du$$

- Set $Z_u = \lambda_u^I \mathrm{L}_{\mathrm{GD}_I}(F_u)^+ - \lambda_u^F \mathrm{L}_{\mathrm{GD}_F}(-F_u)^+$, the Treasury DVA-CVA term, and subtract $\epsilon = \bar{V}$, assuming replacement closeout, from θ, so as to isolate the Trading CVA and DVA terms. Use V=F+H+C

$$\bar{V}_t = \int_t^T \mathbb{E}\{D(t, u; r + \lambda)[\pi_u + \lambda_u(\theta_u - \bar{V}_u) + (\tilde{f}_u - \tilde{c}_u)C_u + \qquad \text{EQFund2}$$

$$+(r_u - \tilde{f}_u + \lambda_u)\bar{V}_u + (\tilde{h}_u - r_u)H_u + Z_u]|\mathcal{F}_t\}du$$

Funding incusive valuation equations II

- Use Feynman Kac: we know that

$$\bar{V}_t = \mathbb{E}_t\left[\int_t^T D(t,u;\mu)[\alpha_u + \beta_u \bar{V}_u]du\right] = \mathbb{E}_t\left[\int_t^T D(t,u;\mu-\beta)\alpha_u du\right]$$

- Then from EQFund2 we have, absorbing λV in the discount:

$$\bar{V}_t = \int_t^T \mathbb{E}\{D(t,u;r)[\pi_u + \lambda_u(\theta_u - \bar{V}_u) + (\tilde{f}_u - \tilde{c}_u)C_u + \qquad \text{EQFund3}$$

$$+(r_u - \tilde{f}_u)\bar{V}_u + (\tilde{h}_u - r_u)H_u + Z_u]|\mathcal{F}_t\}du$$

Funding incusive valuation equations III

- or alternatively, absorbing the whole $(r - f + \lambda)V$

$$\bar{V}_t = \int_t^T \mathbb{E}\{D(t,u;\tilde{f})[\pi_u + \lambda_u(\theta_u - \bar{V}_u) + (\tilde{f}_u - \tilde{c}_u)C_u + \quad \text{EQFund4}$$

$$+(\tilde{h}_u - r_u)H_u + Z_u]|\mathcal{F}_t\}du$$

- Assuming $H = 0$ (rolled par swaps or, better, perfectly collateralized hedge with collateral incuded)

$$\bar{V}_t = \int_t^T \mathbb{E}\{D(t,u;\tilde{f})[\pi_u + \lambda_u(\theta_u - \bar{V}_u) + (\tilde{f}_u - \tilde{c}_u)C_u + Z_u]|\mathcal{F}_t\}du \quad \text{EQFund4'}$$

Funding incusive valuation equations IV

- If $H \neq 0$, assume now generalized delta hedging (in vector sense)

$$H_u = S_u \frac{\partial \bar{V}(u,S)}{\partial S}$$

and use Feynam Kac again:

$$\bar{V}_t = \mathbb{E}^r \int_t^T D(t,u;\mu)[\alpha_u + m(u,S_u)\frac{\partial \bar{V}}{\partial S}]du = \mathbb{E}_t^{r+m}\left[\int_t^T D(t,u;\mu)\alpha_u du\right]$$

where in general E^m is a probability measure where S grows at rate m, ie with drift mS.

Funding incusive valuation equations V

- EqFund4 with delta hedging becomes ($(h-r)H = (h-r)\partial_S \bar{V}$)

$$\bar{V}_t = \int_t^T \mathbb{E}^h\{D(t,u;\tilde{f})[\pi_u + \lambda_u(\theta_u - \bar{V}_u) + (\tilde{f}_u - \tilde{c}_u)C_u + Z_u]|\mathcal{F}_t\}du \quad \text{EQFund5}$$

- **This last equation depends only on market rates. There is no theoretical risk free rate or risk neutral measure in this Eq. Invariance Theorem: The pricing equation is invariant wrt the specification of the short rate r_t.**
- Recall: h are repo/stock lending rates for underlying risky assets,
- $(\theta_u - \bar{V}_u)$ are trading CVA and DVA after collateralization
- $(\tilde{f}_u - \tilde{c}_u)C_u$ is the cost of funding collateral with the treasury
- Z_u is the treasury CVA_F and DVA_F on the funding process
- NO Explicit funding term for the replica **as this has been absorbed in the discount curve and in the collateral cost**

Funding incusive valuation equations VI

- The last equation can be written as a semi-linear PDE or a BSDE
- As we explained, $\mathbb{E}^h$ is the expected value under a probability measure where the underlying assets evolve with a drift rate (return) of $\tilde{h}$. Remember that $\tilde{h}$ depends on H, and hence on V.
- Therefore the PRICING MEASURE DEPENDS ON THE FUTURE VALUES OF THE VERY PRICE V WE ARE COMPUTING. NONLINEAR EXPECTATION. THE PRICING MEASURE BECOMES DEAL DEPENDENT.
- Under the assumption $H = 0$ (H perfectly collateralized with re-hypothecation) we can avoid the last Feynman Kac step and the deal dependent measure: we still price under r_t ($\approx$ OIS) but the terms in EQFund4' bear the same description as EQFund5 we just commented.

Funding incusive valuation equations VII

- Notice that in EQFund5 or the simpler EQFund4' we DISCOUNT AT FUNDING directly. Some industry parties use this version and a funding discount curve.

Funding incusive valuation equations VIII

Let's take a step back. Write EqFund1-2 more in detail.

$$\bar{V}_t = \int_t^T \mathbb{E}\{D(t,u;r+\lambda)[\pi_u + \lambda_u\theta_u + (r_u - \tilde{c}_u)C_u + \qquad \text{EQFund1'}$$

$$+(r_u - \tilde{f}_u)(\bar{V}_u - C_u) + (\tilde{h}_u - r_u)H_u + Z_u]|\mathcal{F}_t\}du$$

We can see easily that

$$\int_t^T \mathbb{E}\{D(t,u;r+\lambda)[\pi_u + \lambda_u\bar{V}_u]\}du = V_t^0$$

and, given $\theta_u = \varepsilon_u - 1_{\{u=\tau_C<\tau_I\}}\Pi_{\text{CVAcoll}}(u) + 1_{\{u=\tau_I<\tau_C\}}\Pi_{\text{DVAcoll}}(u)$, under replacement closeout ($\varepsilon = \bar{V}$), rehypotecation and under $\mathcal{F}$ it is tempting to write EQFund1' as

Funding incusive valuation equations IX

$\bar{V}$ = RiskFreePrice - CVA + DVA + LVA + FVA -CVA$_F$+ DVA$_F$

$$RiskFreePrice = V_t^0, \quad LVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)(r_u - \tilde{c}_u)C_u|\mathcal{F}_t\right\}du$$

$$-CVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[-\mathrm{L}_{\mathrm{GD}_C}\lambda_C(u)(\bar{V}_u - C_{u-})^+\right]|\mathcal{F}_t\right\}du$$

$$DVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[\mathrm{L}_{\mathrm{GD}_I}\lambda_I(u)(-(\bar{V}_u - C_{u-}))^+\right]|\mathcal{F}_t\right\}du$$

$$FVA = -\int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[(\tilde{f}_u - r_u)(\bar{V}_u - C_u) - (\tilde{h}_u - r_u)H_u\right]|\mathcal{F}_t\right\}du$$

$$-CVA_F = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[\mathrm{L}_{\mathrm{GD}_F}\lambda_F(u)(-(\bar{V}_u - C_u - H_u))^+\right]|\mathcal{F}_t\right\}du$$

$$DVA_F = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[\mathrm{L}_{\mathrm{GD}_I}\lambda_I(u)(\bar{V}_u - C_u - H_u)^+\right]|\mathcal{F}_t\right\}du$$

Funding incusive valuation equations X

If we insist in applying these equations, rather than the r-independent EQFund5, then we need to find a proxy for r. This can be taken as the overnight rate (OIS discounting).

Further, if we assume that H_u is zero as it is perfectly collateralized and includes its collateral, then

$$FVA = -\int_t^T \mathbb{E}\Big\{D(t,u;r+\lambda)\Big[(\tilde{f}_u - r_u)(\bar{V}_u - C_u)\Big]|\mathcal{F}_t\Big\}du$$

Notice that when we are borrowing cash $F = V - C$, since usually $f > r$, FVA is negative and is a cost. Also LVA can be negative. Occasionally LVA and FVA are added together in a sort of total $\text{FVA}_{tot} = LVA + FVA$.

$$FVA_{tot} = \int_t^T \mathbb{E}\Big\{D(t,u;r+\lambda)\Big[-(\tilde{f}_u - r_u)\bar{V}_u + (\tilde{f}_u - \tilde{c}_u)C_u\Big]|\mathcal{F}_t\Big\}du$$

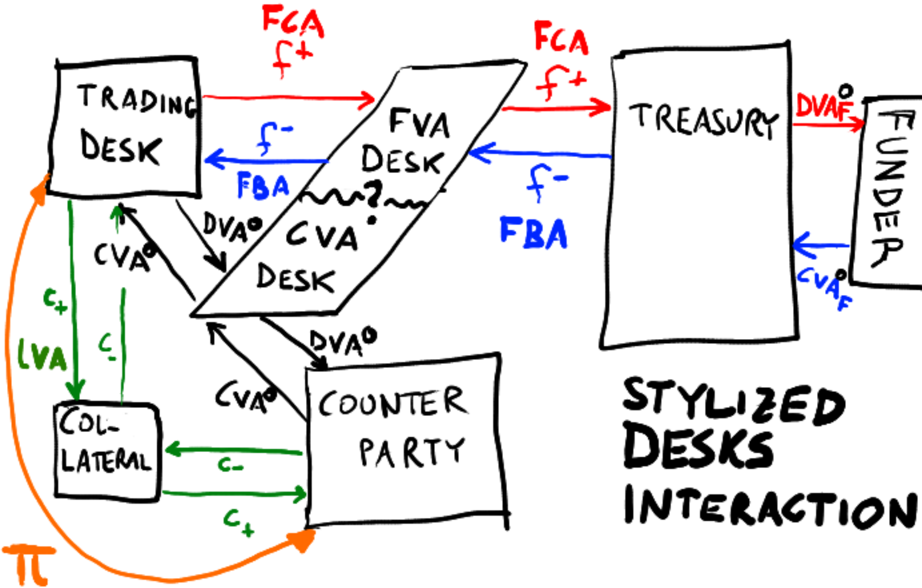
Funding incusive valuation equations XI

Define $FVA = -FCA + FBA$ where $-FCA$ will be a Cost, and hence negative, while FBA will be a Benefit, hence positive.

$$FCA = \int_t^T \mathbb{E}\left\{ D(t,u;r+\lambda)\left[(f_u^+ - r_u)(\bar{V}_u - C_u)^+\right] |\mathcal{F}_t \right\} du$$

$$FBA = \int_t^T \mathbb{E}\left\{ D(t,u;r+\lambda)\left[(f_u^- - r_u)(-(\bar{V}_u - C_u))^+\right] |\mathcal{F}_t \right\} du$$

Notice the structural analogies with the expressions for CVA and DVA respectively.

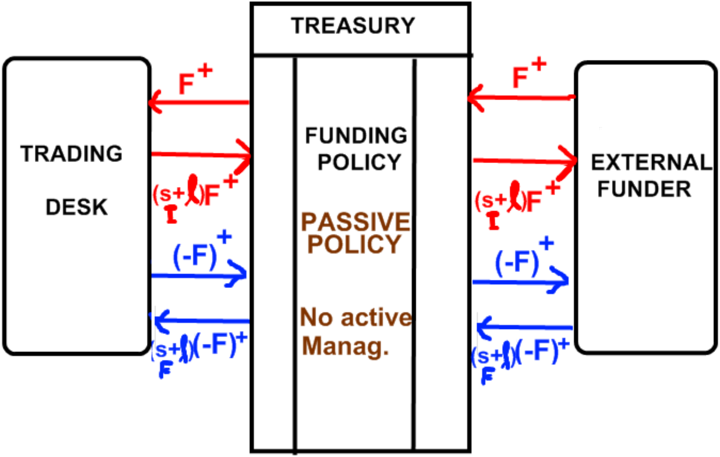

FCA
f+
TRADING
DESK
FVA
DESK
f-
FBA
DVA°
CVA°
CVA
DESK
FCA
f+
TREASURY
DVA°F
FUNDER
f-
FBA
CVA°F
c+
LVA
c-
DVA°
CVA°
COL-
LATERAL
COUNTER
PARTY
c-
c+
π
STYLIZED
DESKS
INTERACTION

Funding incusive valuation equations: EFB vs RBB

To further specify the equations we need to distinguish the assumptions on external lending by the treasury, and we will deal now separately with the two cases:

- External Funder Benefit (EFB)
- Reduced Borrower Benefit (RBB)

Funding incusive valuation equations: EFB case



Funding incusive valuation equations: EFB case

Assume that we use the EFB funding rates $\tilde{f}$ inclusive of credit risk as we have seen before, so that (set $s_{I,C,F} = \lambda_{I,C,F}\text{L}_{\text{GD}\,I,C,F}$) where
$f^+ - r = s_I + \ell^+, \quad f^- - r = s_F + \ell^-$

$$-FCA = -\int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[(s_I+\ell^+)(\bar{V}_u - C_u)^+\right]|\mathcal{F}_t\right\}du$$

$$FBA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[(s_F+\ell^-)(-(\bar{V}_u - C_u))^+\right]|\mathcal{F}_t\right\}du$$

We see $-FCA =: -DVA_F - FCA_\ell, \quad FBA =: CVA_F + FBA_\ell$
where FCA_ℓ is the part in ℓ^+, and FBA_ℓ is the part in ℓ^-.
The presence of Credit Spreads in $\tilde{f}$ leads to components in FBA and FCA that offset the Treasury DVA$_F$ and CVA$_F$.
Summing up:
$V = V_0 - CVA + LVA + DVA - FCA + FBA + DVA_F - CVA_F$ where

$$V_0 = \int_t^T \mathbb{E}\left\{D(t,u;r)\pi_u|\mathcal{F}_t\right\}du, \quad LVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)(r_u-\tilde{c}_u)C_u|\mathcal{F}_t\right\}du$$

$$-FCA(= -DVA_F - FCA_\ell) = -\int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)(s_I(u)+\ell^+(t))(\bar{V}_u - C_u)^+\right]|\mathcal{F}_t\right\}du$$

$$FBA(= CVA_F + FBA_\ell) = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[(s_F+\ell^-)(-(\bar{V}_u - C_u))^+\right]|\mathcal{F}_t\right\}du$$

$$-CVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)[-s_C(\bar{V}_u - C_{u-})^+]|\mathcal{F}_t\right\}du$$

$$DVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)[s_I(-(\bar{V}_u - C_{u-}))^+]|\mathcal{F}_t\right\}du$$

$$DVA_F = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)s_I(u)(\bar{V}_u - C_u)^+\right]|\mathcal{F}_t\right\}du$$

$$-CVA_F = -\int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[s_F(-(\bar{V}_u - C_u))^+\right]|\mathcal{F}_t\right\}du$$

Double Counting: EFB Case

Summing up: $V = V_0(risk\ free) +$

$$\underbrace{-CVA + DVA}_{Trading\ CVA\ DVA} \quad \overbrace{+LVA}^{Coll\ cost\ \&\ benefit} \quad \underbrace{-FCA + FBA}_{Replica\ funding\ cost\ \&\ benefit} \quad + \overbrace{DVA_F - CVA_F}^{Funding\ CVA\ DVA}$$

Remember also what we just found for FCA and FBA:

$$V = V_0 - CVA + DVA + LVA \quad \underbrace{-FCA}_{-DVA_F - FCA_\ell} \quad + \quad \underbrace{FBA}_{CVA_F + FBA_\ell} \quad + DVA_F - CVA_F$$

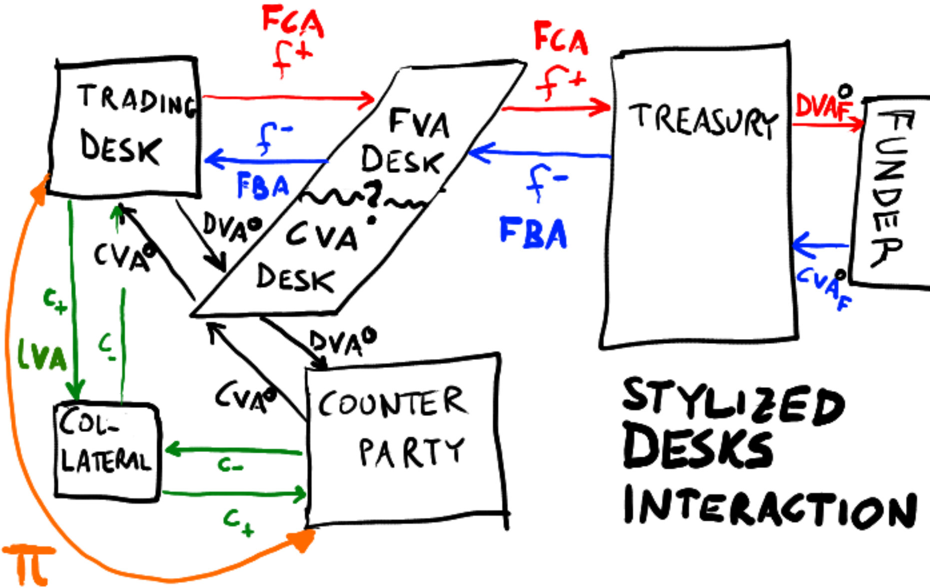
The blue and red terms are passed by the treasury to the desk so the total net value for the whole bank cancels

Total value to bank: EFB Case

Keeping the full formula without simplifying

$$V = V_0 - CVA + DVA + LVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{CVA_F + FBA_\ell} + DVA_F - CVA_F$$

- If bases $\ell = 0$ then Funding costs are offset by the treasury CVA_F and DVA_F and "there are no funding costs" overall.
- However, for the trading desk (TDesk) there is still a cost $FCA = DVA_F + FCA_\ell$ to be paid to Treasury. This happens via the FVA desk if that exists, or via the CVA desk otherwise.
- TDesk also sees a benefit $FBA = CVA_F + FBA_\ell$ received from treasury via the FVA desk if existing, or CVA desk otherwise.
- Treasury pays DVA_F at time 0 to Funder, charging that as a cost FCA to Tdesk, and receives CVA_F at time 0 from funder, and passes that to the TDesk as benefit. All this via FVA desk if existing, if not CVA desk
- CVA desk still deals with trading CVA and DVA


FCA
f+
TRADING
DESK
FVA
DESK
f-
FBA
FCA
f+
TREASURY
DVA°F
FUNDER
f-
FBA
CVA°F
DVA°
CVA
DESK
CVA°
c+
LVA
c-
DVA°
CVA°
COL-
LATERAL
COUNTER
PARTY
c-
c+
π
STYLIZED
DESKS
INTERACTION

Double Counting: EFB Case

Priority to credit adjustments wrt funding ones

$$V = V_0 - CVA + DVA + LVA \underbrace{-FCA}_{\cancel{-DVA_F} - FCA_\ell} + \underbrace{FBA}_{\cancel{CVA_F} + FBA_\ell} \cancel{+DVA_F} - \cancel{CVA_F}$$

$$V = V_0 - CVA + DVA + LVA - FCA_\ell + FBA_\ell$$

Keep CVA & DVA unchanged (CVA DESK) and reduce FCA and FBA to the basis terms (FVA Desk if ∃, else CVA Desk).

- If bases $\ell = 0$ then all funding terms vanish ("FVA=0").
- If $\ell \neq 0$ for TDesk there is still cost FCA_ℓ to be paid to Treasury via FVADesk if ∃, else CVA Desk.
- TDesk also receives benefit FBA_ℓ from Treasury (via FVADesk if ∃ else CVADesk)
- CVA Desk still manages trading CVA and DVA

Double Counting: EFB Case

Priority to funding adjustments over credit ones

$$V = V_0 + LVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{CVA_F + FBA_\ell} - \underbrace{CVA_{tot}}_{CVA - DVA_F} + \underbrace{DVA_{tot}}_{DVA - CVA_F}$$

Here we keep FCA and FBA unchanged and reduce CVA and DVA by the treasury DVA_F and CVA_F terms to avoid double counting.

- TDesk faces a cost $FCA = DVA_F + FCA_\ell$ to be paid to Treasury. This happens via the FVA desk if ∃, or via CVADesk otherwise.
- TDesk also sees a benefit $FBA = CVA_F + FBA_\ell$ received from treasury via the FVA desk if ∃, or CVADesk otherwise.
- CVA Desk takes charge of both trading and funding CVA and DVA, registering on positive exposures a CVA reduced by DVA_F, and on negative exposures a DVA reduced by CVA_F

Double Counting: EFB Case I

$$V = V_0 - CVA + DVA + LVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{CVA_F + FBA_\ell} + DVA_F - CVA_F$$

If one fails to offset correctly double contributions (reds and blues) one is double-counting.

Double counting 1: failing to reduce funding adjustments

$$OK: \quad V = V_0 - CVA + DVA - FCA_\ell + FBA_\ell$$

$$NO: \quad V = V_0 - CVA + DVA - FCA + FBA$$

The "NO" equation has excess CVA_F and DVA_F in the funding terms since these are not offset any longer by the CVA_F and DVA_F terms we removed (for example kept in treasury).

Double Counting: EFB Case II

$$V = V_0 - CVA + DVA + LVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{CVA_F + FBA_\ell} + DVA_F - CVA_F$$

Double counting 2: not reducing credit adjustments

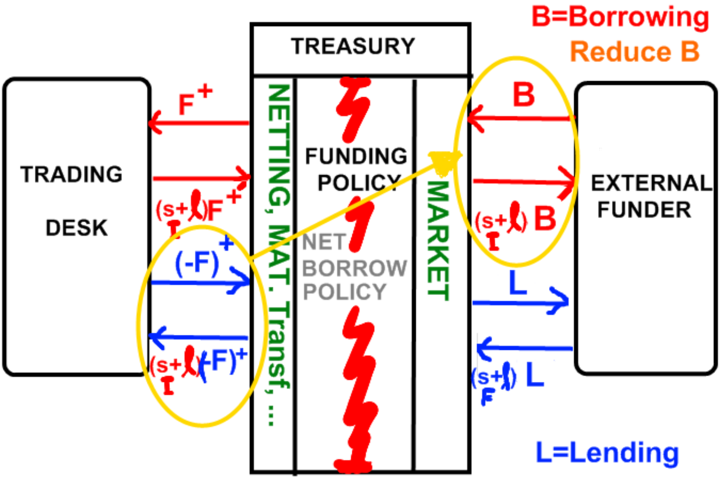
If one does not offset credit adjustments:

$$OK: \quad V = V_0 + LVA - FCA + FBA - (CVA - DVA_F) + (DVA - CVA_F)$$

$$NO: \quad V = V_0 + LVA - FCA + FBA - CVA + DVA$$

The "NO" equations neglect the reduction in CVA and DVA due to the funding credit risk and thus forget to offset the corresponding terms in FCA and FBA

Funding incusive valuation equations: RBB case



Funding incusive valuation equations: RBB case

We are now going to specialize the funding equations

$$FCA = \int_t^T \mathbb{E}\Big\{D(t,u;r+\lambda)\Big[(f_u^+ - r_u)(\bar{V}_u - C_u)^+\Big]|\mathcal{F}_t\Big\}du$$

$$FBA = \int_t^T \mathbb{E}\Big\{D(t,u;r+\lambda)\Big[(f_u^- - r_u)(-(\bar{V}_u - C_u))^+\Big]|\mathcal{F}_t\Big\}du$$

to the RBB case where

$$f^+ - r = s_I + \ell^+, \quad f^- - r = s_I + \ell^-.$$

We also take $\psi = \psi_{RBB}$ (no CVA_F part).

Funding incusive valuation equations: RBB case

The FCA term remains as in the EFB case.

However, notice what happens to FBA now, in the RBB case.

$$FBA = \int_t^T \mathbb{E}\left\{ D(t,u;r+\lambda)\left[(s_l+\ell^-)(-(\bar{V}_u-C_u))^+ \right] | \mathcal{F}_t \right\} du = DVA + FBA_\ell$$

We have that FBA includes a copy of the *trading* DVA

$$V_0 = \int_t^T \mathbb{E}\left\{D(t,u;r)\pi_u|\mathcal{F}_t\right\}du, \quad LVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)(r_u-\tilde{c}_u)C_u|\mathcal{F}_t\right\}du$$

$$-FCA(=-DVA_F-FCA_\ell) = -\int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)(s_l(u)+\ell^+(t))(\bar{V}_u-C_u)^+\right]|\mathcal{F}_t\right\}du$$

$$FBA(=\color{red}{DVA}+FBA_\ell) = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[(\color{red}{s_l}+\ell^-)(-(\bar{V}_u-C_u))^+\right]|\mathcal{F}_t\right\}du$$

$$-CVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[-s_C(\bar{V}_u-C_{u-})^+\right]|\mathcal{F}_t\right\}du$$

$$DVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[s_l(-(\bar{V}_u-C_{u-}))^+\right]|\mathcal{F}_t\right\}du$$

$$DVA_F = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)s_l(u)(\bar{V}_u-C_u)^+\right]|\mathcal{F}_t\right\}du$$

$-CVA_F = 0;$ One of the two DVA must go.

Funding incusive valuation equations: RBB case I

$$V = V_0 - CVA + DVA + LVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{DVA + FBA_\ell} + DVA_F$$

Now we no longer have exact offsetting terms. The DVA inside FBA will not be offset by a CVA_F. The problem is that the formula contains two identical DVA's.

Compare with the EFB case:

$$V = V_0 - CVA + DVA + LVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{CVA_F + FBA_\ell} + DVA_F - CVA_F$$

Funding incusive valuation equations: RBB case II

When we did the analysis to compute the funding rate f^- in a mini-loan in $t, t+dt$ we used our own $s_I = \lambda_I \mathrm{L{\scriptstyle GD}}_I$ as a gain spread, based on the "reduced borrowing" argument.

But receiving back interest s_I as a benefit of reduced borrowing means we are in fact computing a rolling-DVA for F as $[t, t+dt)$ spans the whole trading interval. Since $F = V - C$, we are basically computing again the trading DVA by means of the funding rate f^-.

We are thus counting our own default risk twice on the *same* exposure scenario $(-(V-C))^+$. This is why, save for the basis term ℓ_-, we should take one of the two DVA's out to avoid double counting.

Funding incusive valuation equations: RBB case III

We thus have two possible choices:

1: Privilege Credit Adjustments over Funding ones

$$V = V_0 - CVA + DVA + LVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{\not{DVA} + FBA_\ell} + DVA_F$$

Treasury is charged initially DVA_F, and charges this back to TDesk as part of FCA via FVADesk if $\exists$, else CVADesk.
For the reduced borrowing TDesk sees a benefit FBA_ℓ, obtained from treasury via FVADesk as a payment reduction, and TDesk is still charged DVA at time 0 and receives CVA at time 0 from counterparty via CVADesk. Overall (notice that if $\ell = 0$ there's no funding adjustment)

$$V = V_0 - CVA + DVA + LVA - FCA_\ell + FBA_\ell.$$

Funding incusive valuation equations: RBB case IV

2: Privilege Funding adjustments over the Credit ones

$$V = V_0 - CVA + \not{DVA} + LVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{DVA + FBA_\ell} + DVA_F$$

resulting in

$$V = V_0 - CVA + LVA - FCA_\ell + \underbrace{FBA}_{DVA + FBA_\ell}$$

Now DVA is managed by the FVA Desk. Notice that if liquidity basis $\ell = 0$ then $V = V_0 - CVA + LVA + \underbrace{FBA}_{DVA}$ and the only funding term is the benefit term given by trading DVA

Advanced Modeling Problems

We have seen above a decomposition in several terms:

Additive Valuation Adjustments contributing to total $\bar{V}$

V^0, CVA, DVA, LVA, FCA, FBA, CVA_F, DVA_F

However, as pointed out earlier, with asymmetric borrowing and lending rates and with replacement closeout at default, all terms depend on the value $\bar{V}$ itself and hence contain all risks.

This implies nonlinearity of the valuation process.

It also implies the different adjustments do not really separate risks.

We illustrate nonlinear valuation in a simple case in the benchmark Black Scholes model

Nonlinear valuation: Black Scholes I

Go back to the r-indepdendent formula EQFund5.

$$\bar{V}_t = \int_t^T \mathbb{E}^h\{D(t,u;\tilde{f})[\pi_u + \lambda_u(\theta_u - \bar{V}_u) + (\tilde{f}_u - \tilde{c}_u)C_u + Z_u]|\mathcal{F}_t\}du \quad \text{EQFund5}$$

- Write this last eq as a BSDEs by completing the martingale term. Add and subtract $\int_0^t$, then notice that one term becomes $\int_0^T$ and its E_t is a martingale M_t. Use the martingale representation theorem (see B. and Pallavicini [35], JFE 1, pp 1-60 for details).

$$d\bar{V}_t - [\tilde{f}_t\bar{V}_t + (\tilde{f}_t - \tilde{c}_t)C_t + \pi_t + \lambda_t(\theta(C_t, \bar{V}_t) - \bar{V}_t) - (r - \tilde{h})H_t + Z_t]dt = dM_t,$$

$$\bar{V}_t = H_t + F_t + C_t, \quad \varepsilon_t = \bar{V}_t \ \text{(replacement closeout)}, \quad \bar{V}_T = 0.$$

Recall that $\tilde{f}$ depends on $\bar{V}$ nonlinearly, and so does $\tilde{c}$ on C and $\tilde{h}$ on H. M is a martingale under the pre-default filtration.

Nonlinear valuation: Black Scholes II

- Assume a Markovian vector of underlying assets S (pre- credit and funding) with diffusive generator $\mathcal{L}^{r,\sigma}$ under $\mathbb{Q}$. Let this be associated with brownian W under $\mathbb{Q}$.

$$dS = rSdt + \sigma(t,S)SdW_t, \quad \mathcal{L}^{r,\sigma}u(t,S) = rS\partial_S u + \frac{1}{2}\sigma(t,S)^2 S^2 \partial_S^2 u$$

- Use Ito's formula on $\bar{V}(t,S)$ and match dt (and dW) terms from BSDE: obtain PDE (& explicit representation for BSDE term ZdW). Details are given in the Pallavicini Perini and B. (2011, 2012) reports.

Nonlinear valuation: Black Scholes III

This leads to the following PDE with terminal condition $\bar{V}_T = 0$.

$$(\partial_t - \tilde{f}_t - \lambda_t + \mathcal{L}^{r,\sigma})\bar{V}_t + (\tilde{f}_t - \tilde{c}_t)C_t + \pi_t + \lambda_t\theta(C_t, \bar{V}_t) - (r - \tilde{h})H_t + Z_t = 0 \ \text{[NPDE1]}$$

$$\bar{V}_t = H_t + F_t + C_t, \ \ \varepsilon_t = \bar{V}_t \ \text{(replacement closeout)}$$

Alternatively, the funding/credit risk free price can be used for closeout (risk free closeout), simplifying calculations.

Nonlinear valuation: Black Scholes IV

The above PDE can be simplified further by assuming Delta Hedging:

$$H_t = S_t \frac{\partial \bar{V}_t}{\partial S} \text{ (delta hedging), leading to}$$

$$(\partial_t - \tilde{f}_t - \lambda_t + \mathcal{L}^{\tilde{h},\sigma})\bar{V}_t + (\tilde{f}_t - \tilde{c}_t)C_t + \pi_t + \lambda_t \theta(C_t, \bar{V}_t) + Z_t(F_t) = 0, \quad \text{[NPDE2]}$$

This PDE is NON-LINEAR not only because of θ, but also because $\tilde{f}$ depends on F, and $\tilde{h}$ on H, and hence both on $\bar{V}$ itself.

IMPORTANT: Again invariance theorem.
PDE DOES NOT DEPEND ON r.
This is good, since r is a theoretical rate that does not correspond to any market observable.

Nonlinear valuation: Black Scholes V

We now try to bring this PDE closer to the classical Black Scholes PDE. Assume collateral is a variable fraction $\alpha_t > 0$ of mark to market, with α_t being $\mathcal{F}_t$ adapted, typically non-negative and smaller than one. Recall that we assume

$$\tilde{f}_t = f_+ 1_{F \geq 0} + f_- 1_{F \leq 0}, \quad \tilde{c}_t = c_+ 1_{\bar{V}_t \geq 0} + c_- 1_{\bar{V}_t \leq 0}, \quad f_{+,-} \text{ and } c_{+,-} \text{ constants.}$$

We further assume $\tilde{h} = \tilde{f}$. One obtains

$$\partial_t V - (f_+ - s^I)(V - S_t \partial_S V_t - \alpha V)^+ + (f_- - s^F)(-V + S_t \partial_S V_t + \alpha V)^+ - \lambda_t V +$$

$$+ \frac{1}{2}\sigma^2 S^2 \partial_S^2 V - c_+ \alpha_t (V_t)^+ + c_- \alpha_t (-V_t)^+ + \pi_t + \lambda_t \theta_t(V_t) = 0$$

NONLINEAR PDE (SEMILINEAR).

Nonlinear valuation: Black Scholes VI

$$\partial_t V - (f_+ - s^I)(V - S_t \partial_S V_t - \alpha V)^+ + (f_- - s^F)(-V + S_t \partial_S V_t + \alpha V)^+ - \lambda_t V +$$

$$+ \frac{1}{2}\sigma^2 S^2 \partial_S^2 V - c_+ \alpha_t (V_t)^+ + c_- \alpha_t (-V_t)^+ + \pi_t + \lambda_t \theta_t (V_t) = 0$$

λ is the first to default intensity, π is the ongoing dividend cash flow process of the payout, θ are the complex optional contractual cash flows at default including CVA and DVA payouts after collateral. c_+ and c_- are the borrowing and lending rates for collateral, $s^{I,F} = \lambda^{I,F} \, \mathrm{L}_{\mathrm{GD}\,I,F}$, spread of investment bank & funder from Z (treasury CVA and DVA).

We can use Lipschitz coefficients results to investigate $\exists!$ of viscosity solutions. Classical soultions may also be found but require much stronger assumptions and regularizations.

None of this is much applicable in practical situations.

The Black Scholes Benchmark Case I

$$\partial_t V - (f_+ - s^I)(V - S_t \partial_S V_t - \alpha V)^+ + (f_- - s^F)(-V + S_t \partial_S V_t + \alpha V)^+ - \lambda_t V +$$

$$+ \frac{1}{2}\sigma^2 S^2 \partial_S^2 V - c_+ \alpha_t (V_t)^+ + c_- \alpha_t (-V_t)^+ + \pi_t + \lambda_t \theta_t(V_t) = 0$$

Notice that

- if $f_+ = f_- = r$ (symmetric risk free borrowing and lending),
- $\alpha = 0$ (no collateral),
- $\lambda = 0$ (no credit risk),

then we get back the Black Scholes LINEAR (parabolic) PDE.

$$\partial_t V + r S_t \partial_S V_t + \frac{1}{2}\sigma^2 S^2 \partial_S^2 V - rV + \pi = 0.$$

In Theory: Nonlinearities due to funding I

So what is the THEORY telling us?
We know that NONLINEAR PDEs cannot be solved as Feynman Kac expectations.

Backward Stochastic Differential Equations (BSDEs)

For NPDEs, the correct translation in stochastic terms are BSDEs. The equations have a recursive nature and simulation is quite complicated. Or we keep the PDE.
BSDEs due to asymmetric rates had been briefly introduced in El Karoui, Peng and Quenez (1997). We added credit gap risk & collateral processes, adding more nonlinearity into the picture.

In Theory: Nonlinearities due to funding II

Aggregation–dependent and asymmetric valuation

Worse, the valuation of a portfolio is aggregation dependent and is different for the two parties in a deal. In the classical pricing theory a la Black Scholes, if we have 2 or more derivatives in a portfolio we can price each separately and then add up. Not so with funding and replacement closeout at default. Moreover, without funding the price to one entity is minus the price to the other one. This is no longer true.

Aggregation levels decided a priori and somewhat arbitrarily.

In Theory: Nonlinearities due to funding III

Consistent global modeling across asset classes and risks

Once the level of aggregation is set, the funding valuation problem is non–separable. An holistic approach is needed and consistent modeling across trading desks and asset classes is needed. Internal competition in banks does not favour this.

Furthermore, the classical transaction-independent arbitrage free price is lost, now the price depends on the specific entities trading the product and on their policies (λ, f, ℓ). Recall $\mathbb{E}^h$ and PDE coefficients depending on $\bar{V}$ nonlinearly.

In Theory: Nonlinearities due to funding IV

The end of Platonic pricing?

There is no Platonic measure $\mathbb{Q}$ in the sky to price all derivatives with an expectation where all assets have the risk free return r.
Now the pricing measure is product dependent, and every trade will have a specific measure. This is an implication of the PDE non-linearity.

When basic financial sense leads to complex mathematics

Notice that, in theory, adherence to real banking policies does not make the problem ”boring, purely accounting–like and trivial”. Rather, valuation becomes aggregation dependent and holistic. We need BSDEs rather than expected values, or nonlinear PDEs rather than linear ones.

In Theory: Nonlinearities due to funding V

This would open many problems of operational efficiency and efficiency of implementation.

However, in practice things are implemented quite differently, as we'll see in a minute...

Before looking at that, now that we have seen how to compute funding costs, a fundamental question.

Price of Value?

Why should the client pay for our funding policy choices?

Again recall entity specific (λ, f, ℓ), $\mathbb{E}^h$ and PDE coefficients depending on $\bar{V}$.

Each entity computes a different funding adjusted price for the same product

and “prices” change with aggregation.

The funding adjusted ”price” is not a price in the conventional sense. We may use it for cost/profitability analysis or to pay our treasury, but can we charge it to a client?

Can the client charge us too as she has funding costs?

Price of Value?

Accessibility of valuation parameters

How can the client check our price is fair if she has no access to our funding policy (less transparent than credit standing) and vice versa?

It is more a ”value” than a ”price”.

Provocative question. Why do not we charge an Electricity Bill Valuation Adjustment (EBVA)?

Should funding costs be zero?

In a number of papers, Hull and White argued that there should be no funding costs.

They invoked the Modigliani Miller theorem. A folk version of the theorem is this:

“If market price processes follow random walks, and there are no

- taxes,
- bankruptcy costs,
- agency costs,
- asymmetric information

and if the market is efficient *then* the value of a firm does not depend on how the firm is financed.

Should funding costs be zero?

However the above assumptions do not hold in practice.

The very presence of liquidity bases ℓ violates the assumptions.

However we saw in the above calculations that if $\ell = 0$ then there are indeed no funding costs. For example, in the EFB framework

$$V = V_0 - CVA + DVA + LVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{CVA_F + FBA_\ell} + DVA_F - CVA_F$$

we see that if $\ell = 0$ and $\tilde{c} = r$ we end up with

$$V = V_0 - CVA + DVA$$

and there are no funding costs indeed.

Should funding costs be zero?

So it is a matter of qualifying the assumptions in the Modigliani Miller theorem.

Market imperfections such as the bases ℓ, among others, may make the theorem not valid and hence funding costs become relevant.

We now go back to the implications of nonlinearities of aggregation dependent values and nonlinear valuation. We analyzed the theoretical implications. But are banks taking those into account?

Nonlinearities in theory. What about practice?

... in practical implementation, in many cases one forces symmetries and linearization so as to go back to a linear setting and have either funding included as simple discounting or a linear pricing problem. This is not accurate in general but allows the quick calculation of a funding valuation adjustment (FVA).

In our earlier formulas for the Reduced Borrowing Benefit (RBB) case:

$$V_0 = \int_t^T \mathbb{E}\left\{ D(t,u;r)\pi_u | \mathcal{F}_t \right\} du, \quad LVA = \int_t^T \mathbb{E}\left\{ D(t,u;r+\lambda)(r_u - \tilde{c}_u)C_u | \mathcal{F}_t \right\} du$$

$$-FCA(= -DVA_F - FCA_\ell) = -\int_t^T \mathbb{E}\left\{ D(t,u;r+\lambda)(s_l(u) + \ell^+(t))(\bar{V}_u - C_u)^+ \Big] | \mathcal{F}_t \right\} du$$

$$FBA(= DVA + FBA_\ell) = \int_t^T \mathbb{E}\left\{ D(t,u;r+\lambda)\Big[(s_l + \ell^-)(-(\bar{V}_u - C_u))^+ \Big] | \mathcal{F}_t \right\} du$$

$$-CVA = \int_t^T \mathbb{E}\left\{ D(t,u;r+\lambda)\big[-s_C(\bar{V}_u - C_{u-})^+\big] | \mathcal{F}_t \right\} du$$

$$DVA = \int_t^T \mathbb{E}\left\{ D(t,u;r+\lambda)\big[s_l(-(\bar{V}_u - C_{u-}))^+\big] | \mathcal{F}_t \right\} du$$

$$DVA_F = \int_t^T \mathbb{E}\left\{ D(t,u;r+\lambda)s_l(u)(\bar{V}_u - C_u)^+ \Big] | \mathcal{F}_t \right\} du$$

$-CVA_F = 0$; One of the two DVA must go.

In Theory: Nonlinearities due to funding

Here if we assume $\ell+ \approx \ell^-$, and closeout term is the risk free price $V^0(\tau)$ rather than the replacement value $\bar{V}(\tau)$, then the problem becomes linear and is much more manageable. In practice everyone assumes this and applies a posteriori corrections if needed.

NVA

In the recent paper `http://ssrn.com/abstract=2430696` we introduce a Nonlinearity Valuation Adjustment (NVA), which analyzes the double counting involved in forcing linearization. Our numerical examples for simple call options show that NVA can easily reach 2 or 3% of the deal value even in relatively standard settings.

Equity call option (long or short), $r = 0.01$, $\sigma = 0.25$, $S_0 = 100$, $K = 80$, $T = 3y$, $V_0 = 28.9$ (no credit risk or funding/collateral costs). Precise credit curves are given in the paper. No ψ (value for Desk)

$$NVA = \bar{V}_0(nonlinear) - \bar{V}_0(\text{linearized})$$

NVA

Table: NVA with default risk and collateralization

Funding Rates bps			Default risk, low[a] Long	Default risk, low[a] Short	Default risk, high[b] Long	Default risk, high[b] Short
f^+	f^-	$\hat{f}$				
300	100	200	-3.27 (11.9%)	-3.60 (10.5%)	-3.16 (11.4%)	-3.50 (10.1%)
100	300	200	3.63 (10.6%)	3.25 (11.8%)	3.52 (10.2%)	3.13 (11.3%)

The percentage of the total call price corresponding to NVA is reported in parentheses.
[a] Based on the joint default distribution D_{low} with low dependence.
[b] Based on the joint default distribution D_{high} with high dependence.

NVA

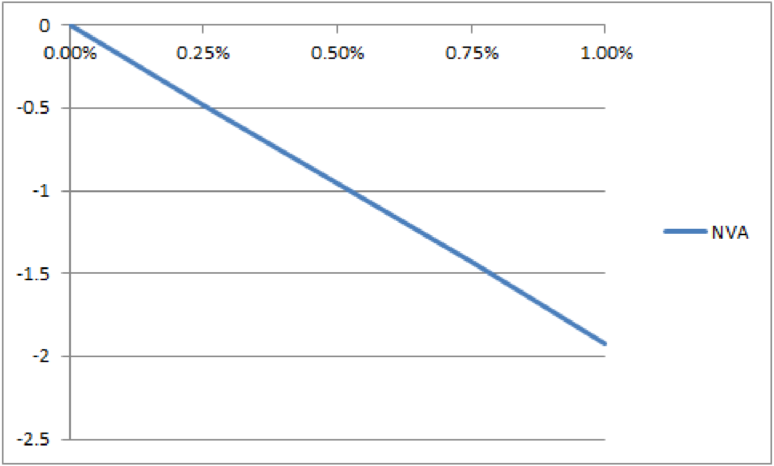
Table: NVA with default risk, collateralization and rehypothecation

Funding Rates bps			Default risk, low[a] Long	Default risk, low[a] Short	Default risk, high[b] Long	Default risk, high[b] Short
f^+	f^-	$\hat{f}$				
300	100	200	-4.02 (14.7%)	-4.45 (12.4%)	-3.91 (14.0%)	-4.35 (12.0%)
100	300	200	4.50 (12.5%)	4.03 (14.7%)	4.40 (12.2%)	3.92 (14.0%)

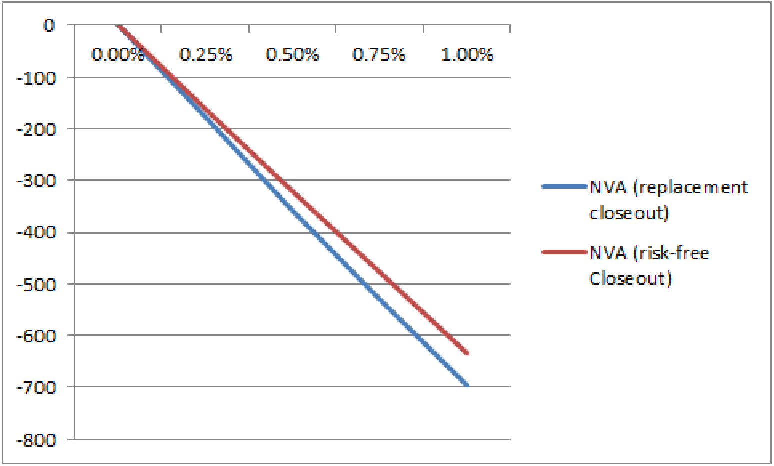
The percentage of the total call price corresponding to NVA is reported in parentheses.

[a] Based on the joint default distribution D_{low} with low dependence.

[b] Based on the joint default distribution D_{high} with high dependence.



NVA for long call as a function of $f^+ - f^-$, with $f^- = 1\%$, $f+$ increasing over 1% and $\hat{f}$ increasing accordingly. NVA expressed as an additive price component on a notional of 100, risk free option price 29. Risk free closeout. For example, $f + -f- = 25bps$ results in NVA=-0.5 circa, 50 bps $\Rightarrow$ NVA = -1



NVA for long call as a function of $f^+ - f^-$, with $f^- = 1\%$, $f+$ increasing over 1% and $\hat{f}$ increasing accordingly. NVA expressed as a percentage (in bps) of the linearized $\hat{f}$ price. For example, $f^+ - f^- = 25$bps results in NVA=-100bps = -1% circa, replacement closeout relevant (red/blue) for large $f^+ - f^-$

Multiple Interest Rate Curves

Derive interest rate dynamics consistently with credit, collateral and funding costs as per the above master valuation equations.

- We use our maket based (no r_t) master equation to price OIS & find OIS equilibrium rates. Collateral fees will be relevant here, driving forward OIS rates.
- Use master equation to price also one period swaps based on LIBOR market rates. LIBORs are market given and not modeled from first principles from bonds etc. Forward LIBOR rates obtained by zeroing one period swap and driven both from primitive market LIBOR rates and by collateral fees.
- We'll model OIS rates and forward LIBOR/SWAP jointly, using a mixed HJM/LMM setup
- In the paper we look at non-perfectly collateralized deals too, where we need to model treasury funding rates.
- See `http://ssrn.com/abstract=2244580`

Pricing under Initial Margins: SCSA and CCPs I

CCPs: Default of Clearing Members, Delays, Initial Margins...

Our general theory can be adapted to price under Initial Margins, both under CCPs and SCSA.

The type of equations is slightly different but quantitative problems are quite similar.

See B. and Pallavicini (2014) for details. See also "Brigo, D. and A. Pallavicini (2014). Nonlinear consistent valuation of CCP cleared or CSA bilateral trades with initial margins under credit, funding and wrong-way risks. Journal of Financial Engineering 1 (1), 1-60." Here we give a summary.

Pricing under Initial Margins: SCSA and CCPs II

So far all the accounts that need funding have been included within the funding netting set defining F_t.

If additional accounts needed, for example segregated initial margins, as with CCP or SCSA, their funding costs must be added.

Initial margins kept into a segregated account, one posted by the investor ($N_t^I \leq 0$) and one by the counterparty ($N_t^C \geq 0$):

$$\begin{aligned}\varphi(t,u) &:= \int_t^u dv\,(r_v - f_v)F_v D(t,v) - \int_t^u dv\,(f_v - h_v)H_v D(t,v) \quad (1)\\ &+ \int_t^u dv(f_v^{N^C} - r_v)N_v^C + \int_t^u dv(f_v^{N^I} - r_v)N_v^I\,,\end{aligned}$$

with $f_t^{N^C}$ & $f_t^{N^I}$ assigned by the Treasury to the initial margin accounts. $f^N \neq f$ as initial margins not in funding netting set of the derivative.

Pricing under Initial Margins: SCSA and CCPs III

$$\ldots + \int_t^u dv (f_v^{N^C} - r_v) N_v^C + \int_t^u dv (f_v^{N^I} - r_v) N_v^I$$

Assume for example $f > r$. The party that is posting the initial margin has a penalty given by the cost of funding this extra collateral, while the party which is receiving it reports a funding benefit, but only if the contractual rules allow to invest the collatera in low-risk activity, otherwise $f = r$ and there are no price adjustments.

We can describe the default procedure with initial margins and delay by assuming that at 1st default τ the surviving party enters a deal with a cash flow ϑ, at maturity $\tau + \delta$ (DELAY!).

δ 5d (CCP) or 10d (SCSA).

Pricing under Initial Margins: SCSA and CCPs IV

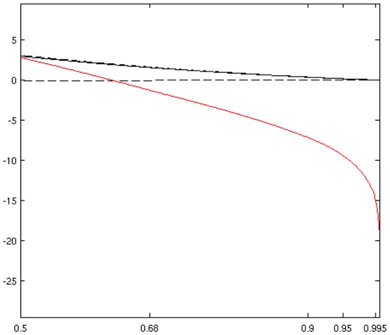
For a CCP cleared contract priced by the clearing member we have $N^I_{\tau^-} = 0$, whatever the default time, since the clearing member does not post the initial margin.

We assume that each margining account accrues continuously at collateral rate c_t.
We may further

- include funding default cloeseout and also
- define the Initial Margin as a percentile of the mark to market at time $\tau + \delta$.

This is done explicitly in the paper.

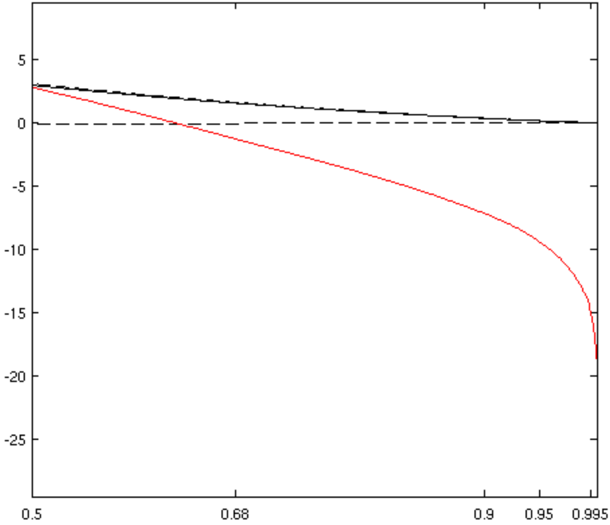
Now a few numerical examples:



Ten-year receiver IRS traded with a CCP.
Prices are calculated from the point of view of the CCP client.
Mid-credit-risk for CCP clearing member, high for CCP client.
Initial margin posted at various confidence levels q.

Prices in basis points with a notional of one Euro

Black continuous line: price inclusive of residual CVA and DVA after margining but not funding costs
Dashed black lines represent CVA and the DVA contributions.
Red line is the price inclusive both of credit & funding costs.
Symmetric funding policy. No wrong way correlation overnight/credit.


5
0
-5
-10
-15
-20
-25
0.5
0.68
0.9
0.95
0.995

CCP Pricing: Tables (see paper for WWR etc)

Table: Prices of a ten-year receiver IRS traded with a CCP (or bilaterally) with a mid-risk parameter set for the clearing member (investor) and a high-risk parameter set for the client (counterparty) for initial margin posted at various confidence levels q. Prices are calculated from the point of view of the client (counterparty). Symmetric funding policy. WWR correlation $\bar{\rho}$ is zero. Prices in basis points with a notional of one Euro.

	Receiver, CCP, $\beta^- = \beta^+ = 1$				Receiver, Bilateral, $\beta^- = \beta^+ = 1$			
q	**CVA**	**DVA**	**MVA**	**FVA**	**CVA**	**DVA**	**MVA**	**FVA**
50.0	-0.126	3.080	0.000	-0.1574	-2.1317	4.3477	0.0000	-0.0842
68.0	-0.066	1.605	-2.933	0.1251	-1.1176	2.2613	-4.1389	0.2491
90.0	-0.015	0.357	-8.037	0.5492	-0.2578	0.4997	-11.3410	0.7924
95.0	-0.007	0.154	-10.316	0.7205	-0.1149	0.2151	-14.5561	1.0250
99.0	-0.001	0.025	-14.590	1.0290	-0.0204	0.0346	-20.5869	1.4544
99.5	-0.001	0.013	-16.154	1.1402	-0.0107	0.0176	-22.7947	1.6107
99.7	-0.000	0.008	-17.233	1.2165	-0.0070	0.0114	-24.3164	1.7184
99.9	-0.000	0.004	-19.381	1.3684	-0.0035	0.0056	-27.3469	1.9326

XVA Desk?

We now move to a general discussion on the CVA/FVA (XVA?) desk and of its role in the bank.

FVA Desk or CVA Desk, or both? XVA Desk?

First recall the role of the CVA Desk.

How do banks price and trade/hedge CVA?

The idea is to move Counterparty Risk management away from classic asset classes trading desks by creating a specific counterparty risk trading desk, or ”CVA desk”.

Under simplifying assumptions, this would allow ”classical” traders to work in a counterparty risk-free world in the same way as before the counterparty risk crisis exploded.

CVA Desks and "Best practices"

What lead to CVA desks?

Roughly, CVA followed this historical path:

- Up to 1999/2000 no CVA. Banks manage counterparty risk through rough and static credit limits, based on exposure measurements (related to Credit VaR: Credit Metrics 1997).
- 2000-2007 CVA was introduced to assess the cost of counterparty credit risk. However, it would be charged upfront and would be managed mostly statically, with an insurance based approach.
- 2007 on, banks increasingly manage CVA dynamically. Banks become interested in CVA monitoring, in daily and even intraday CVA calculations, in real time CVA calculations and in more accurate CVA sensitivities, hedging and management.
- CVA explodes after 7[8] financials defaults occur in one month of 2008 (Fannie Mae, Freddie Mac, Washington Mutual, Lehman, [Merrill] and three Icelandic banks).

CVA Desks and "Best practices"

CVA desk location in a bank

- Trading floor: PROS works with other trading desk, direct use of hedge trades (especially CDS).
  CONS: competition and political problems.
- Treasury: PROS since it involves credit policy, collateral, good for coordination with funding. DVA as funding benefit.
  CONS: interface w/ other desks needs to be managed carefully.
- Often CVA desk does systemically important operations for the bank. Should it be part of RISK / CRO? See how Goldman CVA desk may have saved the firm in the AIG case.[a] Nonprofit desk, runs a service.
- Considerable operational implications too for the bank functioning. COO?

[a] "How Goldman's Counterparty Valuation Adjustment (CVA) Desk Saved The Firm From An AIG Blow Up" http://www.zerohedge.com/, accessed on Dec 1, 2014

CVA Desks and ”Best practices”

CVA desk and Classical Trading desks

The CVA desk charges classical trading desks a CVA fee in order to protect their trading activities from counterparty risk through hedging. This may happen also with collateral/CSA in place (Gap Risk, WWR, etc). The cost of implementing this hedge is the CVA fee the CVA desk charges to the classical trading desk. Often the hedge is performed via CDS trading.

CVA Desks and ”Best practices”

CVA desk in the trasury department

Charging a fee is not easy and can make a lot of P&L sensitive traders nervous. That is one reason why some banks set the CVA desk in the treasury for example. Being outside the trading floor can avoid some ”political” issues on P&L charges among traders.

—

Furthermore, given that the treasury often controls collateral flows and funding policies, this would allow to coordinate CVA and FVA calculations and charges after collateral.

CVA Desks and "Best practices"

How the CVA desk helps other trading desks

The CVA desk[a] would free the classical traders from the need to:

- develop advanced credit models to be coupled with classical asset classes models (FX, equity, rates, commodities...);
- know the whole netting sets trading portfolios; traders would have to worry only about their specific deals and asset classes, as the CVA desk takes care of "options on whole portfolios" embedded in counterparty risk pricing and hedging;
- Hedge counterparty credit risk, which is very complicated.

[a] See for example "CVA Desk in the Bank Implementation", *Global Market Solutions* white paper

CVA Desks and ”Best practices”

The CVA desk task looks quite difficult

The CVA desk has **little/no control** on inflowing trades, and has to:

- quote quickly to classical trading desks a ”incremental CVA” for specific deals, mostly for pre-deal analysis with the client;
- For every classical trade that is done, the CVA desk needs to integrate the position into the existing netting sets and in the global CVA analysis in real time;
- related to pre-deal analysis, after the trade execution CVA desk needs to allocate CVA results for each trade (”marginal CVA”)
- Manage the global CVA, and this is the core task: Hedge counterparty credit and classical risks, including credit-classical correlations (WWR), and check with the risk management department the repercussions on capital requirements.

CVA Desks and ”Best practices”

CVA Desks effectiveness if often questioned

Of course the idea of being able to relegate all CVA(/DVA/FVA) issues to a single specialized trading desk is a little delusional.

- WWR makes isolating CVA from other activities quite difficult.
- In particular WWR means that the idea of hedging CVA and the pure classical risks separately is not effective.
- CVA calculations may depend on the collateral policy, which does not depend on the CVA desk or even on the trading floor.
- We have seen FVA and CVA interact

In any case a CVA desk can have different levels of sophistication and effectiveness.

CVA Desks and "Best practices"

Classical traders opinions

Clearly, being P&L sensitive, the CVA desk role is rather delicate. There are mixed feelings.

- Because CVA is hard to hedge (especially the jump to default risk and WWR), occasionally classical traders feel that the CVA desk does not really hedge their counterparty risk effectively and question the validity of the CVA fees they pay to the CVA desk.
- Other traders are more optimistic and feel protected by the admittedly approximate hedges implemented by the CVA desk.
- There is also a psychological component of relief in delegating management of counterparty risk elsewhere.

Including FVA. XVA Desks?

As we have seen funding costs are now an important component of the valuation process, and FVA is calculated for the bank deals.

This may be charged internally to classical trading desks, who pay the FVA desk for the funding costs, and in turn charge the cost to clients externally.

XVA Desk

Both CVA and FVA reference collateral importantly, so they should be managed together, especially given analogies in these quantities, given DVA as funding benefit and given that one would like to avoid double counting.

Ideally, the XVA desk should immunize classical trading desk from credit risk and funding costs, using mirror trades that isolate those risks

XVA Desks?

XVA Desk and Mirror Trades

Isolating Credit Risk and Funding Costs away from traditional trading desks is made difficult by wrong way risk, where dependence makes all risks connected. One can manage this by assigning risk reserves to deal with wrong way risk losses.

One more difficulty is the little transparency on the bases ℓ. They depend on CDS-Bond basis & the bank funding policy: maturity transformation, netting of funding sets, fund transfer pricing policy, etc.

XVA Desks?

Cross Gammas

In this sense quantities that are helpful are cross gammas: sensitivities of computed values to joint shocks in credit and underlying risk factor, and possibly sensitivity to bases ℓ and underlying risk factors.

As own credit risk and the bases ℓ are difficult to hedge, a reserve is set in place for these risks.

Charging FVA to clients?

Charging FVA to Clients

From what we understand, *most of the banks we cited earlier charge FVA to clients*. The classical trading desk pays the funding costs to the FVA desk but then charges the FVA to the client. However, this is controversial. The client often has no transparency on our funding policy. Why should be pay for our choices? And what if the client decides to charge us her funding costs? Can this be done bilaterally given the lack of transparency?

We also debated the price vs value aspect of FVA earlier.

Possible objections to FVA charge are due to the Modigliani Miller theorem. We addressed these earlier via market imperfections and bases ℓ. *Banks are now satisfied with charging clients with FVA. Hence a bank that does not do that risks to be inconsistent with the market*.

FVA Desks?

FVA separate desk?

Some tier-2 banks are considering creating a FVA desk apart from the CVA Desk. However this is not a popular option with tier-1 banks and most banks are trying to incorporate the FVA function in the already existing CVA desk, that becomes a XVA desk. This is what may be happening with all the banks we mentioned earlier.

The reason is that the split between credit and fuding is not as clearcut as one may think. See our derivation of
CVA, DVA, LVA, FCA, FBA, CVA_F, DVA_F
and of all ways to recombine them.

All quantities are driven by s_I, s_C and ℓ^+, ℓ^-.

Recall also that in the full theory FVA and CVA are not really separable.

Thank you for your attention!

Questions?

References I

[1] Albanese, C., Brigo, D., and Oertel, F. (2011). Restructuring Counterparty Credit Risk. Deutsche Bundesbank Working Paper. Forthcoming.

[2] Assefa, S., Bielecki, T. R., Crèpey, S., and Jeanblanc, M. (2009). CVA computation for counterparty risk assessment in credit portfolios. In Recent advancements in theory and practice of credit derivatives, T. Bielecki, D. Brigo and F. Patras, eds, Bloomberg Press.

[3] Basel Committee on Banking Supervision "International Convergence of Capital Measurement and Capital Standards A Revised Framework Comprehensive Version" (2006), "Strengthening the resilience of the banking sector" (2009). Available at `http://www.bis.org`.

References II

[4] Bergman, Y. Z. (1995). Option Pricing with Differential Interest Rates. The Review of Financial Studies, Vol. 8, No. 2 (Summer, 1995), pp. 475-500.

[5] Bianchetti, M. (2010). Two Curves, One Price. Risk, August 2010.

[6] Bianchetti, M. (2012). The Zeeman Effect in Finance: Libor Spectroscopy and Basis Risk Management. Available at arXiv.com

[7] Bielecki, T., and Crepey, S. (2010). Dynamic Hedging of Counterparty Exposure. Preprint.

[8] Bielecki, T., Rutkowski, M. (2001). *Credit risk: Modeling, Valuation and Hedging*. Springer Verlag .

References III

[9] Biffis, E., Blake, D. P., Pitotti L., and Sun, A. (2011). The Cost of Counterparty Risk and Collateralization in Longevity Swaps. Available at SSRN.

[10] Blanchet-Scalliet, C., and Patras, F. (2008). Counterparty Risk Valuation for CDS. Available at defaultrisk.com.

[11] Blundell-Wignall, A., and P. Atkinson (2010). Thinking beyond Basel III. Necessary Solutions for Capital and Liquidity. OECD Journal: Financial Market Trends, No. 2, Volume 2010, Issue 1, Pages 9-33.
Available at
`http://www.oecd.org/dataoecd/42/58/45314422.pdf`

[12] Brigo, D. (2005). Market Models for CDS Options and Callable Floaters. Risk Magazine, January issue

References IV

[13] D. Brigo, Constant Maturity CDS valuation with market models (2006). Risk Magazine, June issue. Earlier extended version available at http://ssrn.com/abstract=639022

[14] D. Brigo (2012). Counterparty Risk Q&A: Credit VaR, CVA, DVA, Closeout, Netting, Collateral, Re-hypothecation, Wrong Way Risk, Basel, Funding, and Margin Lending. Circulated Internally, Banco Popolare (Claudio Nordio)

[15] Brigo, D., and Alfonsi, A. (2005) Credit Default Swaps Calibration and Derivatives Pricing with the SSRD Stochastic Intensity Model, Finance and Stochastic, Vol. 9, N. 1.

[16] Brigo, D., and Bakkar I. (2009). Accurate counterparty risk valuation for energy-commodities swaps. *Energy Risk*, March issue.

References V

[17] Brigo, D., and Buescu, C., and Morini, M. (2011). Impact of the first to default time on Bilateral CVA. Available at arXiv.org

[18] Brigo, D., Buescu, C., Pallavicini, A., and Liu, Q. (2012). Illustrating a problem in the self-financing condition in two 2010-2011 papers on funding, collateral and discounting. Available at ssrn.com and arXiv.org

[19] Brigo, D., and Capponi, A. (2008). Bilateral counterparty risk valuation with stochastic dynamical models and application to CDSs. Working paper available at `http://arxiv.org/abs/0812.3705`. Short updated version in *Risk*, March 2010 issue.

References VI

[20] Brigo, D., Capponi, A., and Pallavicini, A. (2011). Arbitrage-free bilateral counterparty risk valuation under collateralization and application to Credit Default Swaps. To appear in *Mathematical Finance*.

[21] Brigo, D., Capponi, A., Pallavicini, A., and Papatheodorou, V. (2011). Collateral Margining in Arbitrage-Free Counterparty Valuation Adjustment including Re-Hypotecation and Netting. Working paper available at http://arxiv.org/abs/1101.3926

[22] Brigo, D., and Chourdakis, K. (2008). Counterparty Risk for Credit Default Swaps: Impact of spread volatility and default correlation. *International Journal of Theoretical and Applied Finance*, **12**, 7.

References VII

[23] D. Brigo, L. Cousot (2006). A Comparison between the SSRD Model and the Market Model for CDS Options Pricing. International Journal of Theoretical and Applied Finance, Vol 9, n. 3

[24] Brigo, D., and El–Bachir, N. (2010). An exact formula for default swaptions pricing in the SSRJD stochastic intensity model. Mathematical Finance, Volume 20, Issue 3, Pages 365-382.

[25] Brigo, D., and Masetti, M. (2005). Risk Neutral Pricing of Counterparty Risk. In Counterparty Credit Risk Modelling: Risk Management, Pricing and Regulation, *Risk Books*, Pykhtin, M. editor, London.

[26] Brigo, D., Mercurio, F. (2001). Interest Rate Models: Theory and Practice with Smile, Inflation and Credit, Second Edition 2006, Springer Verlag, Heidelberg.

References VIII

[27] Brigo, D., Morini, M. (2006) Structural credit calibration, *Risk*, April issue.

[28] Brigo, D., and Morini, M. (2010). Dangers of Bilateral Counterparty Risk: the fundamental impact of closeout conventions. Preprint available at `ssrn.com` or at `arxiv.org`. Short version in Risk Magazine.

[29] Brigo, D., and Morini, M. (2010). Rethinking Counterparty Default, Credit flux, Vol 114, pages 18–19

[30] Brigo D., Morini M., and Tarenghi M. (2011). Equity Return Swap valuation under Counterparty Risk. In: Bielecki, Brigo and Patras (Editors), Credit Risk Frontiers: Sub- prime crisis, Pricing and Hedging, CVA, MBS, Ratings and Liquidity, Wiley, pp 457–484

References IX

[31] Brigo, D., and Nordio, C. (2010). Liquidity Adjusted Market Risk Measures with Random Holding Period. Available at SSRN.com, arXiv.org, and sent to BIS, Basel.

[32] Brigo, D., and Pallavicini, A. (2007). Counterparty Risk under Correlation between Default and Interest Rates. In: Miller, J., Edelman, D., and Appleby, J. (Editors), Numercial Methods for Finance, Chapman Hall.

[33] D. Brigo, A. Pallavicini (2008). Counterparty Risk and Contingent CDS under correlation, Risk Magazine, February issue.

[34] Brigo, D., and Pallavicini, A. (2014). CCP Cleared or Bilateral CSA Trades with Initial/Variation Margins under credit, funding and wrong-way risks: A Unified Valuation Approach. SSRN.com and arXiv.org

References X

[35] Brigo, D. and A. Pallavicini (2014). Nonlinear consistent valuation of CCP cleared or CSA bilateral trades with initial margins under credit, funding and wrong-way risks. Journal of Financial Engineering 1 (1), 1–60.

[36] Brigo, D., Pallavicini, A., and Papatheodorou, V. (2011). Arbitrage-free valuation of bilateral counterparty risk for interest-rate products: impact of volatilities and correlations, International Journal of Theoretical and Applied Finance, 14 (6), pp 773–802

[37] Brigo, D., Pallavicini, A., and Torresetti, R. (2010). Credit Models and the Crisis: A journey into CDOs, Copulas, Correlations and Dynamic Models. Wiley, Chichester.

References XI

[38] Brigo, D. and Tarenghi, M. (2004). Credit Default Swap Calibration and Equity Swap Valuation under Counterparty risk with a Tractable Structural Model. Working Paper, available at www.damianobrigo.it/cdsstructural.pdf. Reduced version in *Proceedings of the FEA 2004 Conference at MIT, Cambridge, Massachusetts, November 8-10* and in *Proceedings of the Counterparty Credit Risk 2005 C.R.E.D.I.T. conference*, Venice, Sept 22-23, Vol 1.

[39] Brigo, D. and Tarenghi, M. (2005). Credit Default Swap Calibration and Counterparty Risk Valuation with a Scenario based First Passage Model. Working Paper, available at www.damianobrigo.it/cdsscenario1p.pdf Also in: *Proceedings of the Counterparty Credit Risk 2005 C.R.E.D.I.T. conference*, Venice, Sept 22-23, Vol 1.

References XII

[40] Burgard, C., Kjaer, M. (2010). PDE Representations of Options with Bilateral Counterparty Risk and Funding Costs Available at ssrn.com.

[41] Burgard, C., Kjaer, M. (2011). In the Balance Available at ssrn.com.

[42] Canabarro, E., and Duffie, D.(2004). Measuring and Marking Counterparty Risk. In Proceedings of the Counterparty Credit Risk 2005 C.R.E.D.I.T. conference, Venice, Sept 22–23, Vol 1.

[43] Canabarro, E., Picoult, E., and Wilde, T. (2005). Counterparty Risk. *Energy Risk*, May issue.

[44] Castagna, A. (2011). Funding, Liquidity, Credit and Counterparty Risk: Links and Implications, Available at http://ssrn.com/abstract=1855028

References XIII

[45] G. Cesari, J. Aquilina , N. Charpillon, Z. Filipovic, G. Lee and I. Manda (2010). Modelling, Pricing, and Hedging Counterparty Credit Exposure: A Technical Guide, Springer Verlag, Heidelberg.

[46] Cherubini, U. (2005). Counterparty Risk in Derivatives and Collateral Policies: The Replicating Portfolio Approach. In: *ALM of Financial Institutions* (Editor: Tilman, L.), Institutional Investor Books.

[47] Collin-Dufresne, P., Goldstein, R., and Hugonnier, J. (2002). A general formula for pricing defaultable securities. Econometrica 72: 1377-1407.

[48] Crépey, S. (2011). A BSDE approach to counterparty risk under funding constraints. Available at `grozny.maths.univ-evry.fr/pages_perso/crepey`

References XIV

[49] Crépey S., Gerboud, R., Grbac, Z., Ngor, N. (2012a). Counterparty Risk and Funding: The Four Wings of the TVA. Available at arxiv.org.

[50] Crouhy M., Galai, D., and Mark, R. (2000). A comparative analysis of current credit risk models. *Journal of Banking and Finance* 24, 59-117

[51] Danziger, J. (2010). Pricing and Hedging Self Counterparty Risk. Presented at the conference "Global Derivatives", Paris, May 18, 2010.

[52] Duffie, D., and Huang, M. (1996). Swap Rates and Credit Quality. Journal of Finance 51, 921–950.

References XV

[53] Duffie, D., and Zhu H. (2010). Does a Central Clearing Counterparty Reduce Counterparty Risk? Working Paper, Stanford University.

[54] Ehlers, P. and Schoenbucher, P. (2006). The Influence of FX Risk on Credit Spreads, ETH working paper, available at defaultrisk.com

[55] El Alaloui, A., and Reghai, A. (2012). A Consistent Pricing Framework with Counterparty Risk and Collateral Agreements. SSRN

[56] Fries, C. (2010). Discounting Revisited: Valuation Under Funding, Counterparty Risk and Collateralization. Available at SSRN.com

[57] Fujii, M., Shimada, Y., and Takahashi, A. (2010). Collateral Posting and Choice of Collateral Currency. Available at `ssrn.com`.

References XVI

[58] J. Gregory (2009). "Being two faced over counterparty credit risk", *Risk Magazine* 22 (2), pages 86-90.

[59] J. Gregory (2010). Counterparty Credit Risk: The New Challenge for Global Financial Markets, Wiley, Chichester.

[60] Hull, J., White, A. (2012). The FVA debate *Risk Magazine*, 8

[61] ISDA. "Credit Support Annex" (1992), 'Guidelines for Collateral Practitioners"(1998), "Credit Support Protocol" (2002), "Close-Out Amount Protocol" (2009), "Margin Survey" (2010), "Market Review of OTC Derivative Bilateral Collateralization Practices" (2010). Available at `http://www.isda.org`.

[62] Jamshidian, F. (2002). Valuation of credit default swap and swaptions, FINANCE AND STOCHASTICS, 8, pp 343–371

References XVII

[63] Jones, E. P., Mason, S.P., and Rosenfeld, E. (1984). Contingent Claims Analysis of Corporate Capital Structure: An Empirical Investigation. *Journal of Finance* 39, 611-625

[64] Jorion, P. (2007). Value at Risk, 3-d edition, McGraw Hill.

[65] Keenan, J. (2009). Spotlight on exposure. *Risk* October issue.

[66] Kenyon, C. (2010). Completing CVA and Liquidity: Firm-Level Positions and Collateralized Trades. Available at `arXiv.org`.

[67] McNeil, A. J., Frey, R., and P. Embrechts (2005). Quantitative Risk Management: Concepts, Techniques, and Tools, Princeton university press.

References XVIII

[68] Leung, S.Y., and Kwok, Y. K. (2005). Credit Default Swap Valuation with Counterparty Risk. The Kyoto Economic Review 74 (1), 25–45.

[69] Lipton A, Sepp A, Credit value adjustment for credit default swaps via the structural default model, The Journal of Credit Risk, 2009, Vol:5, Pages:123-146

[70] Lo, C. F., Lee H.C. and Hui, C.H. (2003). A Simple Approach for Pricing Barrier Options with Time-Dependent Parameters. *Quantitative Finance* 3, 98-107

[71] Merton R. (1974) On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *The Journal of Finance* 29, 449-470.

References XIX

[72] Moreni, N. and Pallavicini, A. (2010). Parsimonious HJM Modelling for Multiple Yield-Curve Dynamics. Accepted for publication in Quantitative Finance.

[73] Moreni, N. and Pallavicini, A. (2012). Parsimonious Multi-Curve HJM Modelling with Stochastic Volatility, in: Bianchetti and Morini (Editors), Interest Rate Modelling After The Financial Crisis, Risk Books.

[74] Morini, M. (2011). Understanding and Managing Model Risk. Wiley.

[75] Morini, M. and Brigo, D. (2011). No-Armageddon Measure for Arbitrage-Free Pricing of Index Options in a Credit Crisis, Mathematical Finance, 21 (4), pp 573–593

References XX

[76] Morini, M. and Prampolini, A. (2011). Risky Funding: A Unified Framework for Counterparty and Liquidity Charges, Risk Magazine, March 2011 issue.

[77] Pallavicini, A. (2010). Modelling Wrong-Way Risk for Interest-Rate Products. Presented at the conference “6th Fixed Income Conference”, Madrid, 23-24 September 2010.

[78] A. Pallavicini, and D. Brigo (2013). Interest-Rate Modelling in Collateralized Markets: Multiple curves, credit-liquidity effects, CCPs. SSRN.com and arXiv.org

[79] Pallavicini, A., Perini, D., and Brigo, D. (2011). Funding Valuation Adjustment consistent with CVA and DVA, wrong way risk, collateral, netting and re-hypotecation. Available at SSRN.com and arXiv.org

References XXI

[80] Pallavicini, A., Perini, D., and Brigo, D. (2012). Funding, Collateral and Hedging: uncovering the mechanics and the subtleties of funding valuation adjustments. Available at SSRN.com and arXiv.org

[81] Parker E., and McGarry A. (2009) The ISDA Master Agreement and CSA: Close-Out Weaknesses Exposed in the Banking Crisis and Suggestions for Change. Butterworths Journal of International Banking Law, 1.

[82] Picoult, E. (2005). Calculating and Hedging Exposure, Credit Value Adjustment and Economic Capital for Counterparty Credit Risk. In Counterparty Credit Risk Modelling (M. Pykhtin, ed.), Risk Books.

References XXII

[83] Piterbarg, V., (2010). Funding beyond discounting: collateral agreements and derivatives pricing. Risk Magazine, February 2010.

[84] Pollack, Lisa (2012). Barclays visits the securitisation BISTRO. Financial Times Alphaville Blog, Posted by Lisa Pollack on Jan 17, 11:20.

[85] Pollack, Lisa (2012b). The latest in regulation-induced innovation – Part 2. Financial Times Alphaville Blog, Posted by Lisa Pollack on Apr 11 16:50.

[86] Pollack, Lisa (2012c). Big banks seek regulatory capital trades. Financial Times Alphaville Blog, Posted by Lisa Pollack on April 29, 7:27 pm.

References XXIII

[87] Reghai, A., and Messaoud, M. (2013). Credit Risk in Equity Deals: Non Linear PDE Solving. Natixis working paper, SSRN.

[88] Rosen, D., and Pykhtin, M. (2010). Pricing Counterparty Risk at the Trade Level and CVA Allocations. Journal of Credit Risk, vol. 6 (Winter 2010), pp. 3-38.

[89] Sorensen, E.H., and Bollier, T. F. (1994). Pricing Swap Default Risk. *Financial Analysts Journal*, 50. 23–33.

[90] Watt, M. (2011). Corporates fear CVA charge will make hedging too expensive. Risk Magazine, October issue.

[91] Weeber, P., and Robson E. S. (2009) Market Practices for Settling Derivatives in Bankruptcy. ABI Journal, 9, 34–35, 76–78.

References XXIV

[92] F. M. Ametrano and M. Bianchetti.
Bootstrapping the illiquidity: Multiple yield curves construction for market coherent forward rates estimation.
In F. Mercurio, editor, *Modeling Interest Rates: Latest Advances for Derivatives Pricing*. Risk Books, 2009.

[93] M. Brunnermeier and L. Pedersen.
Market liquidity and funding liquidity.
The Review of Financial Studies, 22 (6), 2009.

[94] J. Eisenschmidt and J. Tapking.
Liquidity risk premia in unsecured interbank money markets.
Working Paper Series European Central Bank, 2009.

References XXV

[95] M. Fujii and A. Takahashi.
Clean valuation framework for the usd silo.
Working Paper, 2011a.
URL `ssrn.com`.

[96] M. Fujii and A. Takahashi.
Collateralized cds and default dependence.
Working Paper, 2011b.
URL `ssrn.com`.

[97] D. Heller and N. Vause.
From turmoil to crisis: Dislocations in the fx swap market.
BIS Working Paper, 373, 2012.

References XXVI

[98] M. Henrard.
The irony in the derivatives discounting part ii: The crisis.
ssrn, 2009.

[99] A. Pallavicini and M. Tarenghi.
Interest-rate modelling with multiple yield curves.
2010.

[100] C. Pirrong.
The economics of central clearing: Theory and practice.
ISDA Discussion Papers Series, 2011.