Errata and comments to the book "Representations of Lie groups and special functions, Vols. 1,2,3" by N. Ja. Vilenkin and A. U. Klimyk (Kluwer, 1991, 1993, 1992)

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Volume 1

title page: interchange addresses of the two authors

pp. xviii, 549, 550, 605: replace "Eberlane" by "Eberlein"

p.xix, l.−18: replace "lead" by "led"

p.xxi, l.7,8: replace "Gordon" by "Gordan"

p.1, l.8: replace "different" by "differential"

p.3, l.8: These coordinate surfaces are not always symmetric spaces

p.12, l.18: add $e \circ x = x$

p.13, l.9: replace 1.0.9 by 1.0.10

p.15, l.5: replace "differentiation" by "derivation"

p.17, l.2: replace $(Ax)(y)$ by $(Ay)(x)$.

p.17, l.−11: replace "homomorphism" by "anti-homomorphism"

p.28, l.14: replace "measureable" by "measurable"

p.29, l.−15: replace $(\phi, \psi)_+k$ by $(\phi, \psi)_k$

p.29, l.−13: replace "countable-Hilbert" by "countably Hilbert"

p.35, formula (1): replace $\mathbf{x}$ by $\mathbf{z}$

p.39, l.5: replace $\mathbf{a}$ by $\mathbf{A}$

p.42, l.4: replace "preserve all of a Lie algebra" by "are Lie algebra homomorphisms"

p.42, l.10: replace "form" by "forms"

p.42, l.11: replace "of real" by "of the real"

p.42, l.−14: replace "the" by "a"

p.42, l.−10: Replace "In" by "For"

p.68, l.18: Continuity of a representation in the operator topology (as in the definition given here) is usually too strong, for instance in Example 6 at p.69. Better write after "implies": $\lim \|T(g_n)v - T(g)v\| = 0$ for all $v \in \mathcal{L}$.

p.70, l.7: replace $\delta;$ by δ

p.75, middle of page; p.607: replace “Gärding” or “Garding” by “Gårding”

p.81, l.10: For closed subspaces one needs a topology on $\mathcal{L}$ (a Banach or Hilbert space?)

p.95, l.2: What kind of group is G? (locally compact or Lie?)

p.95, l.−13: replace g by G

p.95, l.−12: replace “to the” by “to an”

p.103, l.14: skip “called”

p.135, (4): the lower limit of the integral should be $c_1 - i\infty$

p.139, (1): replace e^{-xu} by e^{-xt}

p.146, (3) and (4): replace the argument $\frac{z}{1-z}$ of F by $\frac{z}{z-1}$

p.155, l.−6: add a factor $z^{-\frac{1}{2}}$ to the expression defining $D_p(z)$

p.156, (4′): add “where $C_n^0(z) := \lim_{\alpha\to\infty} \alpha^{-1} C_n^\alpha(z)$”

p.264, §5.5.8, formula(1): replace $L - n$ by L_n

p.288, (1′): replace $(-1)^{m+n}$ by $(-1)^{(m-n)}$

p.330, l.8: replace 4.4.8 by 4.3.8

p.334, (2): insert ‘)’ before last squarfe bracket

p.337, l.1: replace the part in brackets by “with suitable replacements of l, m, n”

p.338, l.2 of §6.7.4: replace 6.7.3 by 6.3.7

p.338, §6.7.4, formula (3): replace right-hand side by

$$\tfrac{1}{2}(n+\alpha+1)(1+z)P_n^{(\alpha,\beta+1)}(z) - \tfrac{1}{2}(n+\beta+1)(1-z)P_n^{(\alpha+1,\beta)}(z)$$

p.339, l.4: replace (1) by (1′)

p.339, (6): replace this formula by

$$(n+\alpha)P_n^{(\alpha-1,\beta)}(z) + (n+\beta)P_n^{(\alpha,\beta-1)}(z) = (n+\alpha+\beta)P_n^{(\alpha,\beta)}(z).$$

p.350, (1): on the right-hand side replace first semicolon by comma

p.351, (6): replace $\frac{\Gamma(\gamma+x+c)}{\Gamma(\gamma+z)}$ by $\frac{\Gamma(\gamma+x+s)}{\Gamma(\gamma+x)}$. The formula is much easier derived from (7) of §6.8.2, (5) of §6.3.8 and (1) of §3.5.8

p.351, (7): easier derived from (1) of §6.8.2 and (1) of §3.5.8

p.352, l.−3: replace (11) by (15)

p.376, §7.1.3, l.1: replace 6.1.4 by 6.4.1

p.465, (1): On the left-hand side replace “$)f$” by “$f)$”. Replace the right-hand side by $e^{-t/4} f(e^{-\frac{1}{2}t}x)$.

p.473, (1): replace the factor $(-1-ix)^n$ by $(-1-it)^n$

p.519, (8): replace ${}^3F_2$ by ${}_3F_2$

p. 540, l.3 of §8.4.9: replace "index c" by "index $\frac{1}{2}$"

p.547, first line after (4): A reference to the introduction of raising factorials $(a)_n$ on p.141 would be suitable here. A consistent use of raising factorials, also here for Hahn polynomials, would be preferable.

p.548, (9): On the right-hand side, in front of Q_n, one should interchange α and β.

p.536, (5): on second line replace the numerator factor $(l_1+l_2+l_3+1)$ by $(l_1+l_2+l_3+l+1)!$.

p.556, (22): On the right-hand side, in the deniminator replace $(\beta+\delta)$ by $(\beta+\delta+1)$. Also add that $\lambda(x)$ on the right-hand side equals $x(x+\gamma+\delta+2)$. This formula is much quicker obtained from the expression of a Racah polynomial as a ${}_4F_3$ given in (5).

p.583, (20): On the right-hand side replace $\Gamma(-a-ix)$ by $\Gamma(a+ix)$.

p.587, l.−1: Replace $l-2$ by l_2.

p.599–608: probably throughout in subject index, for subjects occurring in Chapters 2–8, add 1 to the given page number.

Volume 2

p.227, line 3 of §10.5.3: replace 10.4.3 by 10.4.2

p.227, l.−4: replace by "by formula (5) of Section 10.4.2"

p.508, l.−5: insert "the" after "then"

p.509, (5): replace $w_{-i}+$ by $w_{-i}-$

p.512, 2 lines after (6"): is an analog of $\frac{\Gamma(a+n)}{\Gamma(a)\Gamma(n+1)}$

p.512, (7'): in denominator replace $q^{n(N-n)}$ by $q^{nN-\frac{1}{2}n(n-1)}$

p.512, (8): The expression $(a;q)_{-n}$ has not been earlier defined. Write:

$$(a;q)_{-n} \equiv \frac{(a;q)_\infty}{(q^{-n}a;q)_\infty}$$

p.512, (9): replace the right-hand side by $\dfrac{(a;q)_{n(k+1)}}{(a;q)_{nk}}$.

p.513, (14): It is unfortunate that the book does not follow the conventions for q-hypergeometric series notation of the book by Gasper & Rahman [3.71].

p.514, l.1: It is not true that the series converges for all z if $m \le n$.

p.514, (16): On the left-hand side insert semicolon between q^{-1} and z. It is confusing that this formula is given over base q^{-1}, while other formulas on the same page are given over base q.

p.514, (18)–20): On the right-hand side of (20) add factor a^k. With this correction, it is formula (20) rather than (18) which is immediately implied by (17). Then (19) (rather over base q) is obtained by combination of (20) and the formula

$$ {}_2\phi_1\left(\begin{matrix} q^{-n}, a \\ b \end{matrix} \middle| q; z\right) = (-1)^n \, q^{-\frac{1}{2}n(n+1)} \, \frac{(a;q)_n}{(b;q)_n} \, z^n \, {}_2\phi_1\left(\begin{matrix} q^{-n}, q^{-n+1}b^{-1} \\ q^{-n+1}a^{-1} \end{matrix} \middle| q; \frac{q^{n+1}b}{az}\right). $$

Finally (18) follows from (19) and (20). It is next promised that proofs of these formulas will be given in Volume 3. I could not find the proofs there.

p.515, l.7: if $0 < q < 1$

p.516, l.8 and p.607: replace "Eberlane" by "Eberlein"

p.606: Meijer G-functions

p.607: The page numers for the first four index items are not correct.

p.607: "Representations of". Next "the Heisenberg group 410" should be on the next line, entabbed.

Volume 3

p.1, (1): add: if $a \neq q^{-n}, q^{-n-1}, \ldots$

p.1, (3): on right hand side replace $(q;q)_r$ by $(q;q)_N$ and replace /4 by /2

pp.2–3, (16), (17): These formulas are not correct.

p.5, line after (2'): The condition $|x| < 1$ is not needed in (2).

p.7, (8): In the last exponent on the right hand side replace $+j$ by $-j$.

p.7, (9), (10): In fact, (10) is derived with the help of (9).

pp.8–9, (10, (5): Mention that these formulas give definitions for the expressions on their left hand sides.

p.9, (3): This also follows straight from 14.1.2 (6).

p.9, (3), second line: On the right hand side replace $b[[a]]$ by $b[[-a]]$ and replace $1 - bqx$ by $1 - bx$.

p.12, (16): Twice replace $x - a$ by $qx - a$

p.12, l.3–4: Rather say: $[[m]]! = [[1]]\,[[2]]\,\ldots\,[[m]]$, where $[[k]]$ is the same as in (3).

p.12, (1): It is unfortunate that the book does not follow the standard notation for q-exponentials as in Gasper & Rahman [3.71].

p.13, l.5: Twice replace $x + y$ by $x + qy$

p.14, (1): The q-integral does not converge for $0 < q < 1$

p.14, (4): An indication of the (easy) proof would have been appropriate.

p.15, (10): In fact, $\exp(-C_q) = (q;q)_\infty$.

p.16, (13): Observe that this formula is formula (6) of p.6, rewritten in different form.

p.17, (3),(4): These formulas should read as follows:

$$D_q\{x^a\,{}_2\phi_1(q^a,b;c;q,x)\} = \frac{1-q^a}{1-q}\,x^{a-1}\,{}_2\phi_1(q^{a+1}.b;c;q,x),$$
$$D_q\{x^{c-1}\,{}_2\phi_1(a,v;q^c;q,x)\} = \frac{1-q^{c-1}}{1-q}\,x^{c-2}\,{}_2\phi_1(a,b;q^{c-1};q,x).$$

In some formulas on p.18, similar corrections should be made.

p.67, (1) and (2): replace in both formulas the last ${}_3\Phi_2$ by ${}_2\Phi_1$

p.69, l.9: replace "Aberlane" by "Eberlein"

p.81, §14.7: Common usage is "Askey-Wilson polynomials" rather than "q-Askey-Wilson polynomials".

p.273, l.3: replace "subgroup" by "group"

p.620, references 305–308: replace "Vratare" by "Vretare"

p.622, Chapter 13: A reference to [3.71] (as given for Chapter 14) would have been natural here.