

# OP-SF NET – Volume 30, Number 6 – November 15, 2023

The Electronic News Net of the  
SIAM Activity Group on Orthogonal Polynomials and Special Functions

<http://math.nist.gov/opsf>

OP-SF Net is distributed to OPSF Activity Group members and non-members alike through the OP-SF Talk listserv.

If you are interested in subscribing to the Newsletter and/or OP-SF Talk, or if you would like to submit a topic to the Newsletter or a contribution to OP-SF Talk, please send an email to the OP-SF Net Editors.

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## Calendar of Events:

### November 20, 2023

Workshop on Integrable Systems, Random Matrices, and Special Functions at UBI  
University of Beira Interior, Covilhã, Portugal  
<https://asimoe1.wixsite.com/workshop>

### December 14–15, 2023

4<sup>th</sup> Workshop “Two Days of Orthogonal Polynomials”  
University of Almería, Almería, Spain  
Dedicated to Guillermo López Lagomasino for his 75<sup>th</sup> birthday and to  
Andrei Martínez Finkelshtein for his 60<sup>th</sup> birthday.  
<https://w3.ual.es/GruposInv/Tapo/D2PO–2023/D2PO2023.html>

### January 3–6, 2024

2024 [Joint Mathematics Meetings](https://www.jointmathematicsmeetings.org/meetings/national/jmm2024/2300_program.html), American Mathematical Society,  
Moscone Center, San Francisco, California, USA  
[https://www.jointmathematicsmeetings.org/meetings/national/jmm2024/2300\\_program.html](https://www.jointmathematicsmeetings.org/meetings/national/jmm2024/2300_program.html)  
AWM–AMS Noether Lecture: Anne Schilling: [The Ubiquity of Crystal Bases](#)  
AMS Special Session on Numerical Analysis, Spectral Graph Theory, Orthogonal Polynomials,  
and Quantum Algorithms,  
Organized by Anastasiia Minenkova, Gamal Mograby, and Anastasiia Minenkova (SS 92A)  
AMS Special Session on Partition Theory and  $q$ –Series,  
Organized by William Jonathan Keith, Brandt Kronholm, and Dennis Eichhorn (SS 30A)

### May 27–31, 2024

Asymptotics, Randomness, Nonlinearity, and Orthogonality (ARNO 2024)  
ARNO 2024 will also be the 2024 Annual Meeting of the [PIICQ network](#),  
Leuven, Belgium  
<https://gsilva.pages.math.cnrs.fr/arno2024/index.html>

### June 3–6, 2024

International Conference on Analysis and Applications  
in honor of Roderick S. C. Wong’s 80<sup>th</sup> birthday,  
City University of Hong Kong, Hong Kong  
<https://www.cityu.edu.hk/rcms/icaa2024/index.html>

### June 6–9, 2024

The Legacy of Ramanujan 2024  
Celebrating the 85<sup>th</sup> birthdays of George Andrews & Bruce Berndt,  
Penn State University, State College, PA 16801  
<https://sites.psu.edu/ramanujan/>

## June 24–28, 2024

17<sup>th</sup> International Symposium on Orthogonal Polynomials, Special Functions and Applications (OPSFA–17),  
Universidad de Granada, Granada, Spain  
<https://opsfa17.com/>

## July 8–12, 2024

Operator Theory and Approximation 2024  
TU Wien, Vienna, Austria  
<https://haraldworacek.github.io/OTA2024/>

## September 4–7, 2024

Approximation Theory and Special Functions (ATSF 2024)  
TOBB Economics and Technology University, Ankara, Türkiye  
Dedicated to the retirement of George Anastassiou  
<https://sites.google.com/view/atsf2024>

## June 23– 28, 2025

Combinatorics around the  $q$ -Onsager algebra  
A celebration of the 70<sup>th</sup> birthday of Paul Terwilliger  
Kranjska Gora, Slovenia  
<https://conferences.famnit.upr.si/event/15/overview>

Topic #1 ——— OP – SF Net 30.6 ——— November 15, 2023

From: Miguel Piñar ([mpinar@ugr.es](mailto:mpinar@ugr.es))  
Subject: Second Announcement: OPSFA–17

The IMAG Conference on Orthogonal Polynomials, Special Functions and Applications – OPSFA17 will take place from June 24<sup>th</sup> to June 28<sup>th</sup>, 2024, at the Faculty of Sciences of the University of Granada, Spain. The Institute of Mathematics (IMAG) and the Faculty of Sciences will support the organization of the symposium.

Conferences in the OPSFA series provide a forum for specialists in the areas of orthogonal polynomials and special functions to share and discuss recent research results in the field.

Important information about the conference:

- Dates: June 24–28, 2024.
- Website: <https://opsfa17.ugr.es>.
- Registration is open until June 1<sup>st</sup> 2024. End of early registration is April 20<sup>th</sup> 2024
- A limited number of reduced fees and accommodation grants are available for students and participants from developing countries.
- The conference program consists of plenary and contributed talks and posters. Call for contributions will be open from November 1<sup>st</sup> 2023 to March 1<sup>st</sup> 2024.
- The list of plenary speakers is:
  - Rabia Aktas Karaman, Ankara University, Türkiye
  - Marco Bertola, Concordia University, Canada
  - Annie Cuyt, University of Antwerp, Belgium
  - Benjamin Eichinger, TU Wien, Austria

- Ujué Etayo, Universidad de Cantabria, Spain
- Andrei Martínez-Finkelshtein, Baylor University, USA and Universidad de Almería, Spain
- Inés Pacharoni, CIEM-FaMAF, Universidad Nacional de Córdoba, Argentina
- Aron Wennman, KTH Royal Institute of Technology, Sweden

On behalf of the local organizing committee: Miguel A. Piñar

Topic #2 ——— OP – SF Net 30.6 ——— November 15, 2023

From: Mourad E. H. Ismail ([mourad.eh.ismail@gmail.com](mailto:mourad.eh.ismail@gmail.com))

Subject: Announcement: The Legacy of Ramanujan 2024

When & Where: June 6–9, 2024, Penn State University

Plenary Speakers:

- Krishna Alladi, University of Florida George Andrews, Penn State University
- Bruce Berndt, University of Illinois, Urbana–Champaign
- Sylvie Corteel, University of California Berkeley
- Amanda Folsom, Amherst College
- Frank Garvan, University of Florida
- Christian Krattenthaler, University of Vienna, Austria
- Ken Ono, University of Virginia
- Peter Paule, Research Institute for Symbolic Computation and Johannes Kepler University, Austria
- Ole Warnaar, University of Queensland, Australia (To be confirmed)
- Doron Zeilberger, Rutgers University

Further Invited Speakers:

- Scott Ahlgren, University of Illinois Urbana–Champaign
- Cristina Ballantine, College of the Holy Cross
- Alex Berkovich, University of Florida
- Hannah Burson, University of Minnesota
- Song Heng Chan, Nanyang Technological University, Singapore
- Shane Chern, Dalhousie University
- Atul Dixit, IIT Gandhinagar, India
- Howard Cohl, National Institute of Standards and Technology
- Shishuo Fu, Chongqing University, China
- Timothy Huber, University of Texas Rio Grande Valley
- Mourad Ismail, University of Central Florida
- Shashank Kanade, University of Denver
- Soon–Yi Kang, Kangwon National University, Korea
- William Keith, Michigan Technological University
- Byungchan Kim, Seoul National University of Science and Technology, Korea
- Brandt Kronholm, University of Texas Rio Grande Valley
- Kagan Kursungoz, Sabanci University, Turkey
- Matthew Russell, University of Illinois Urbana–Champaign
- Michael Schlosser, University of Vienna, Austria

- Robert Schneider, Michigan Technological University
- James Sellers, University of Minnesota Duluth
- Andrew Sills, Georgia Southern University
- Nicolas Smoot, RISC, Johannes Kepler University, Austria
- Armin Straub, University of South Alabama
- Ali Uncu, University of Bath, UK

#### Special Announcement – Conference Proceedings:

We plan to publish the conference proceedings as a Special Issue of the Ramanujan Journal to honor George Andrews and Bruce Berndt at the occasion of passing the milestone age of 85. More information to come on how to submit an article.

#### Banquet:

A banquet in Bruce's and George's honor is planned for Saturday evening, June 8. More information to follow soon.

## Topic #3 ——— OP – SF Net 30.6 ——— November 15, 2023

From: Mourad E. H. Ismail ([mourad.eh.ismail@gmail.com](mailto:mourad.eh.ismail@gmail.com))

Subject: Announcement: International Conference on Analysis and Applications

#### Objectives:

The purpose of the International Conference on Analysis and Applications 2024 is to provide a platform for researchers, scholars, and practitioners from around the world to gather and discuss the latest advancements in the field of analysis and its applications. This conference aims to foster collaboration, knowledge exchange, and intellectual dialogue among participants.

#### Conference Venue:

The conference will take place at Wong To Yick Tong Lecture Theatre (LT-17), City University of Hong Kong, 4/F Yeung Kin Man Academic Building on CityU campus which is just a few minutes walk from the Mass Transit Railway (MTR) Kowloon Tong Station.

#### Organizing Committee:

- Dan Dai , City University of Hong Kong, Hong Kong (Chair)
- Ya Yan Lu, City University of Hong Kong, Hong Kong
- Chunhua Ou, Memorial University of Newfoundland, Canada
- Lun Zhang, Fudan University, China
- Dingxuan Zhou, The University of Sydney, Australia

#### Plenary Speakers:

- Walter Van Assche, Katholieke Universiteit Leuven, Belgium
- Adri B. Olde Daalhuis, University of Edinburgh, UK
- Huaxiong Huang, Beijing Normal University–Hong Kong Baptist University United International College (UIC), China
- Mourad Ismail, University of Central Florida, USA
- Nalini Joshi, The University of Sydney, Australia
- Dany Leviatan, Tel Aviv University, Israel
- Chun Liu, Illinois Institute of Technology, USA
- Peter Miller, University of Michigan, USA

- Michael Ward, The University of British Columbia, Canada
- Yang Wang, Hong Kong University of Science and Technology, Hong Kong
- Juncheng Wei, The University of British Columbia, Canada
- Jianhong Wu, York University, Canada

Topic #4 ——— OP – SF Net 30.6 ——— November 15, 2023

From: Maria das Neves Rebocho ([mneves@ubi.pt](mailto:mneves@ubi.pt)), Alberto Simoes ([asimoes@ubi.pt](mailto:asimoes@ubi.pt))  
and Nicholas Witte ([n.s.witte@protonmail.com](mailto:n.s.witte@protonmail.com))

Subject: Announcement: Integrable Systems, Random Matrices, and Special Functions at UBI

The Workshop “Integrable Systems, Random Matrices, and Special Functions at UBI” will take place on the 20<sup>th</sup> of November 2023 at University of Beira Interior, Covilhã, Portugal.

This workshop brings together representatives from several communities to discuss interactions and recent developments in Integrable Systems, Random Matrices and Special Functions. The presentations will focus on important developments in these areas, showing recent results and discussing open problems. The intention here is to facilitate communication and collaboration between sub-disciplines that construct sophisticated and powerful tools, with application to novel problems in mathematical physics.

Invited Speakers:

- Amílcar Branquinho, CMUC, University of Coimbra, Portugal
- Rostyslav Kozhan, Uppsala University, Sweden
- Arno Kuijlaars, KU Leuven, Belgium
- Miguel Tierz, Complutense University of Madrid, Spain
- Nicholas Witte, Victoria University of Wellington, New Zealand
- Alexei Zhedanov, Renmin University of China, China

There are no fees to participate, but registration is required. For further information please email the organisers at [issfubi@ubi.pt](mailto:issfubi@ubi.pt). Please fill in the following: [registration form](#) (Until November 15).

For further information, please visit <https://asimoe1.wixsite.com/workshop>.

Organizers:

- Maria das Neves Rebocho
- Alberto Simões
- Nicholas Witte

We are looking forward to seeing you in Covilhã!

Kind regards,

Maria das Neves Rebocho (on behalf of the organisers)

Topic #5 ——— OP – SF Net 30.6 ——— November 15, 2023

From: Mark MacLean ([macleanm@seattleu.edu](mailto:macleanm@seattleu.edu))

Subject: Announcement: Combinatorics around the  $q$ -Onsager algebra in Kranjska Gora, Slovenia

Greetings! We are in the early stages of planning a combinatorics conference for the week of June 23–28, 2025. The title of the conference will be “Combinatorics around the  $q$ -Onsager algebra.” At the

conference we will be celebrating the 70<sup>th</sup> birthday of Paul Terwilliger (as you may recall, this birthday conference was originally scheduled for 2020 but was cancelled due to Covid). The conference will take place in beautiful Kranjska Gora, Slovenia.

The general theme of this conference will be the mathematical topics that Paul has worked on over the years (which all have relationships to the  $q$ -Onsager algebra). These topics include the following:

- Topics in algebraic graph theory, such as distance-regular graphs, association schemes, the sub-constituent algebra, and the  $Q$ -polynomial property;
- Topics in linear algebra, such as Leonard pairs, tridiagonal pairs, billiard arrays, lowering-raising triples, and a linear algebraic approach to the orthogonal polynomials of the Askey scheme;
- Topics in Lie theory, such as the tetrahedron algebra and the Onsager algebra;
- Topics in algebras and their representations, such as the equitable presentation of  $U_q(sl_2)$ , the  $q$ -tetrahedron algebra, the mathematical physics of the  $q$ -Onsager algebra, and the universal Askey-Wilson algebra.

If you are interested in attending, please save the dates of June 23–28, 2025. In 2024 a website will be posted with more details about the conference. If you would be interested in giving a talk at this conference, please contact the organizers at ([q-onsager2025@famnit.upr.si](mailto:q-onsager2025@famnit.upr.si)).

Cheers,

Mark MacLean (Seattle University)

Stefko Miklavic (University of Primorska)

Topic #6 ——— OP – SF Net 30.6 ——— November 15, 2023

From: Oktay Duman ([okitayduman@gmail.com](mailto:okitayduman@gmail.com))

Subject: Announcement: Approximation Theory and Special Functions (ATSF 2024) in Ankara, Türkiye

[ATSF 2024](#) Conference – 8<sup>th</sup> Series

TOBB Economics and Technology University

September 4–7, 2024

Ankara – Türkiye

ATSF is an international conference series organized to bring together researchers from ALL areas of Approximation Theory and Special Functions to discuss new ideas and new applications. This conference is dedicated to the valuable contributions to mathematics of our esteemed colleague [Prof. George Anastassiou](#) and to his upcoming retirement years. This organization, which has been held seven times so far as mini-symposia, has grown gradually over the years and will be held for the eighth time on September 4–7, 2024, hosted by [TOBB Economics and Technology University](#) (Ankara, Türkiye).

After taking a break for a few years due to the pandemic, we are coming together again in 2024 with a big organization. This time, we have tried to keep the conference research and presentation topics as broad as possible. In addition to paper presentations, we will also welcome poster presentations and special session proposals. So we expect a big turnout for this series. Click [here](#) to see the conference poster.

The highlighted topics are (but not limited to):

- **Approximation Theory:** Classical approximation, statistical approximation, fuzzy approximation, approximation in the complex plane, best approximation, linear and nonlinear approximation, multivariate approximation, neural networks, numerical approximation, rational approximation, ... and their applications.

- **Summability Theory:** Statistical convergence, regular summability methods, power series methods, sequence spaces, divergent series, Tauberian theorems, summation processes, matrix and integral methods for summability, ... and their applications.
- **Applied Mathematics:** ODEs, PDEs, difference equations, mathematical modeling, dynamical systems, numerical analysis, oscillation, stability theory, control theory, financial mathematics, probability and statistics, stochastic process, graph theory, ... and their applications.
- **Special Functions:** Orthogonal polynomials, hypergeometric series, generating functions, general orthogonal systems, umbral calculus, recurrence relations, matrix-valued polynomials, Fourier series, special orthogonal functions, special polynomials, special functions in mathematical physics,... and their applications.
- **Analysis and Functions Theory:** Harmonic analysis, functional analysis, fuzzy analysis, operator theory, spectral theory, fixed point theory, fractional calculus,  $q$ -analysis, number theory, combinatorics, cryptography and coding theory, inequalities, time scales, ... and their applications.

For your special session proposals in connection with the above topics, please contact the Organizing Committee by email: [atsf2024@gmail.com](mailto:atsf2024@gmail.com)

Topic #7 ——— OP – SF Net 30.6 ——— November 15, 2023

From: Steven Damelin ([steve.damelin@gmail.com](mailto:steve.damelin@gmail.com))

Subject: Announcement: *Near Extensions and Alignment of Data in  $\mathbb{R}^n$*  book by **Steve Damelin**

It may be of interest to some members of the OPSFA mailing list to know about the following new book:

### Near Extensions and Alignment of Data in $\mathbb{R}^n$ :

Whitney extensions of near isometries, shortest paths, equidistribution, clustering and non-rigid alignment of data in Euclidean space

Steven B. Damelin

Wiley, ISBN: 978-1-394-19677-7. 192 Pages. November 2023, [link to Wiley](#).

This monograph, for fixed integers  $n \geq 1$  and  $d \geq 2$ , provides a modern treatment of Whitney extensions of near isometries in  $\mathbb{R}^n$ , non-rigid alignment of data in  $\mathbb{R}^d$ , interpolation by near isometries in  $\mathbb{R}^d$ , continuum limits of shortest paths in  $\mathbb{R}^d$  with several intersecting topics in pure and applied harmonic analysis and data science for example, approximation of near distortions by elements of the orthogonal group  $O(d)$  using the space of functions of bounded mean oscillation (BMO), clustering of data in  $\mathbb{R}^d$ , equidistribution and discrepancy via minimal energy and finite field techniques, tensor methods in Whitney theory, approximation theory in algebraic geometry, quantum lattices and covers of the special unitary group  $SU(2)$ , techniques for counting linear independent vectors over finite fields and the maximum distance separable (MDS) conjecture.

The monograph is structured as follows. Chapter 1 introduces the near Whitney extension problem. For finite sets of distinct data in  $\mathbb{R}^d$  with varying geometries, the chapter interprets this extension problem as an interpolation, non-rigid alignment problem of distinct data in  $\mathbb{R}^d$  and discusses rigid and non-rigid data alignment problems. The monograph, then moves to machinery to analyze the near Whitney extension problem in Chapter 1, from various perspectives for finite sets of distinct points with varying geometries. Chapter 2 introduces the idea of near distorted diffeomorphism extensions which agree with Euclidean motions in  $\mathbb{R}^d$ . Slow Twists and Slides are introduced as examples of near distorted diffeomorphisms. Chapter 5 deals with clustering methods of data in  $\mathbb{R}^d$  and studies continuum limits of



shortest paths. Chapter 7 deals with tensor methods, Chapter 8 studies the use of approximation theory for varieties in algebraic geometry and Chapter 9 introduces near reflection theory. Chapter 10 studies approximation of near distortions by elements of the orthogonal group  $O(d)$ , using the space of functions of bounded mean oscillation (BMO) and the John Nirenberg inequality.

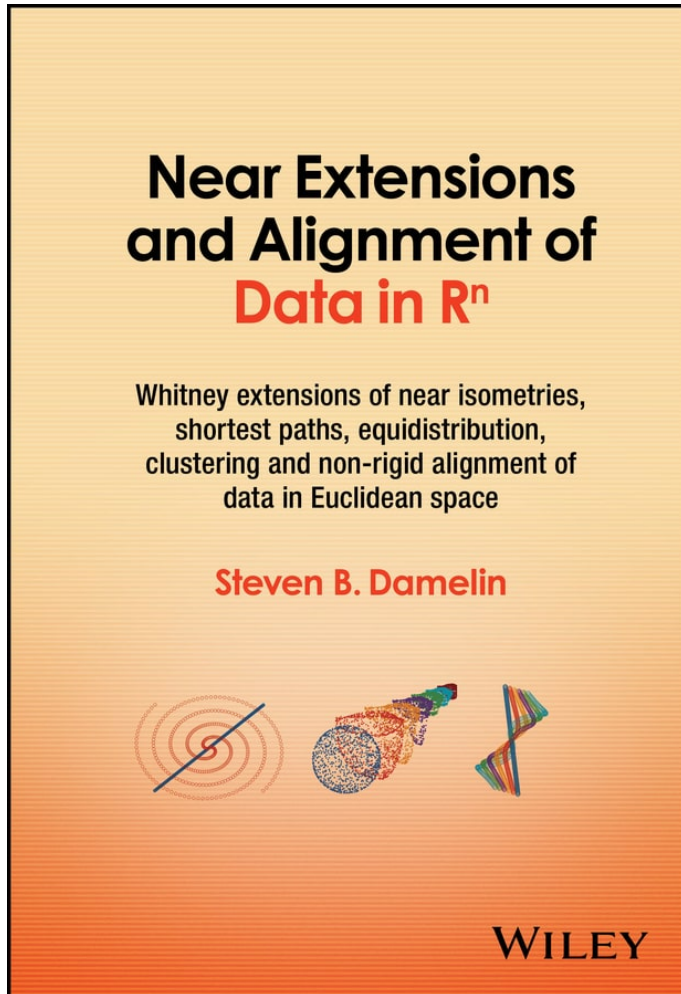


Figure 1: Book cover

Chapters 3, 6, 11–12 introduce Gluing techniques, partitions of unity, further Whitney machinery and finite principles. Chapter 4 deals with manifold learning and the Johnson–Lindenstrauss theorem. Chapters 13–18 deal with the analysis of the near Whitney extension problem for compact sets in open sets in  $\mathbb{R}^n$ . This chapter introduces Whitney techniques such as Whitney cubes and regularization. It provides near distortions agreeing with Euclidean motions in  $\mathbb{R}^n$ . Chapter 19 deals with equidistribution and studies minimal energy on  $n$ -dimensional compact sets embedded in  $\mathbb{R}^{n+1}$  via extremal Newtonian like configurations. It studies group invariant discrepancy, finite field discrepancy, combinatorial designs, techniques for counting linear independent vectors over finite fields, and discusses the maximum distance separable (MDS) conjecture. Chapter 20 deals with quantum lattices and covers of the special unitary group  $SU(2)$ . Finally Chapter 21 deals with the near unlabeled data alignment problem and the related optimal transport problem.

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From: OP–SF Net Editors

Subject: Remembrances of **Diego Dominici** (1972–2023)

**Remembrances of  
Diego Ernesto Dominici  
(February 18, 1972—October 16, 2023)**

Below are remembrances of Diego Dominici from some of his colleagues:

**Paco Marcellán** and **Walter Van Assche**; **Juan J. Moreno–Balcázar**; **Mourad Ismail**; **Chris Howls**;  
**Adri Olde Daalhuis**; **Peter Paule**; **Yuan Xu**; **Andrei Martínez–Finkelshtein**; and **Howard Cohl**.

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**Obituary for Diego Dominici**

Paco Marcellán ([pacomarc@ing.uc3m.es](mailto:pacomarc@ing.uc3m.es)) and Walter Van Assche ([walter.vanassche@kuleuven.be](mailto:walter.vanassche@kuleuven.be))

We were informed by Veronika Pillwein that Diego Dominici passed away on October 16, 2023 in Linz, Austria at the age of 51. He leaves behind his wife Veronika and daughter Malena and many friends from the OPSF community. The picture shows him at the grave of Stirling in Greyfriars Kirkyard, Edinburgh (photo taken from the DLMF website<sup>1</sup>).

Diego Dominici received the degree of Licenciado (Bachelor+Master of Sciences) in Pure Mathematics in 1998 from the Universidad de Buenos Aires in Argentina. In 2003 he defended his doctoral dissertation *Asymptotic analysis of an elliptic PDE, arising from a queuing model, and its discrete analog* at the University of Illinois in Chicago under the supervision of Charles Knesl. Diego was a Full Professor at the State University of New York (SUNY) in New Paltz until May of this year where he was the head of the department of mathematics for some years. There he received the Provost’s Research Award 2005–2006 and the Mentor of the Year Award in 2014. In 2004 he received the Richard C. DiPrima Prize (SIAM) for outstanding research in applied mathematics. The Alexander von Humboldt Foundation awarded Diego a Humboldt Research Fellowship for experienced researchers (2008–2010). These awards are a good indication of the recognition of his research activity. For the past few years he was a senior researcher at the Research Institute for Symbolic Computation (RISC) of the Johannes Kepler University in Linz, Austria.

According to MathSciNet the most cited paper by Diego is *Asymptotic analysis of the Hermite polynomials from their differential-difference equation*, J. Difference Equ. Appl. **13** (2007), No. 12, 1115–1128. In this contribution he obtained nice asymptotic approximations for the Hermite polynomials using the raising operator associated with them (the differential–difference equation satisfied by these polynomials). By applying the leading term of a WKB–type solution to the above differential–difference equation, the associated eikonal and transport equations are solved by an application of the method of characteristics. This yields the desired asymptotic approximation for the range  $|x| > \sqrt{2n}$ . The treatment of the situation when  $x = \pm\sqrt{2n}$ , where the above WKB form breaks down, is also discussed. The approximation near these points is obtained in terms of the Airy function. Finally, an approximation in the oscillatory region  $|x| < \sqrt{2n}$  is derived, together with approximations for the zeros of the Hermite polynomial of  $n$ –th degree. Numerical examples illustrate the estimates obtained in the different regions.

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<sup>1</sup><https://dlmf.nist.gov/about/bio/DDominici>.

In a more general framework, the paper *Zero distribution of polynomials satisfying a differential-difference equation*, Anal. Appl. (Singap.) **12** (2014), No. 6, 635–666, (D. Dominici, W. Van Assche) investigates the asymptotic distribution of the zeros of polynomials satisfying a first-order differential–difference equation and several examples of orthogonal and non-orthogonal families are presented. The study of real zeros of hypergeometric polynomials has also attracted his attention. An algorithm that computes the zeros of these polynomials as well as the conditions under which these zeros are real and simple is described in the paper *Real zeros of  ${}_2F_1$  hypergeometric polynomials*, J. Comput. Appl. Math. **247** (2013), 152–161 (D. Dominici, S. J. Johnston, K. Jordaan).

The different fields of Diego’s interest are a good sample of his very deep background in classical analysis and its applications in approximation theory and analytic properties of classical special functions related to ordinary linear differential and difference equations. He has developed an intensive international cooperation by attending the most relevant meetings on orthogonal polynomials and special functions. We would like to point out his remarkable leadership as organizer of special sessions in the framework of events of the AMS, SIAM, European Consortium on Mathematics in Industry (ECMI) as well as in our biennial international OPSFA symposia. He was involved in the Scientific Committee of the editions of OPSFA–14 and OPSFA–15, hosted at the University of Kent in Canterbury, UK in 2017, and at the Research Institute for Symbolic Computation (RISC) in Hagenberg, Austria in 2019, and he was one of the two principal organizers of OPSFA–13 at NIST in Gaithersburg, Maryland USA in 2015, together with Daniel Lozier. He has also been involved in the Organizing Committee of the SIAM annual meetings 2013 (San Diego) and 2020 (Toronto). In 2015 he joined the Digital Library of Mathematical Functions team as an associate editor for Chapter 9 on Airy and related functions, and Chapter 10 on Bessel functions.

Diego was very active in the SIAM activity group on Orthogonal Polynomials and Special Functions (SIAG–OPSF) since 2010. He acted as Program Director in the period 2011–2016, he was OPSF–Talk moderator from May 15, 2010 through January 15, 2022 for 12 years with Bonita Saunders and he was a co–editor for OPSF–Net for 9 years from January 15, 2006 to January 15, 2015 (Volume 13, Number 1 – Volume 22, Number 1), with Martin Muldoon.

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## In Memory of Diego Dominici

Juan J. Moreno–Balcázar ([balcazar@ual.es](mailto:balcazar@ual.es))

I was informed about the death of our colleague and friend Diego Dominici by Paco Marcellán on October 18<sup>th</sup>. Diego had passed away two days before. When I received this very sad news, I was shocked. Twelve days later I could attend his funeral in Linz via Zoom, and I’m still shocked by this unexpected and tragic event.

In this brief note I am not going to write about Diego’s successful career as a mathematician. An obituary of Diego has been done by Paco Marcellán and Walter Van Assche. I only pretend to remember some moments of my relationship with him. He was a good mathematician and a good person.

I do not remember exactly when I first met Diego, but I am sure that it was in some conference in Europe. Years later he visited Universidad de Almería (Spain) and he invited me to visit Technische Universität Berlin (Germany) for a research stay. This was in the framework of Olga Holtz’s project in this university since she had received the Sofia Kovalevskaya Award from Alexander von Humboldt Foundation in 2006. We were working hard on some problems related to nonstandard orthogonal polynomials, but as it happens sometimes, we did not obtain the results that we had expected. We left some open problems.

Anyway, we continued keeping in touch. It was easy: he was Argentinian, and I am Spanish. In Hagenberg in 2019 during the conference OPSFA15 we talked about some problems related to the asymptotics of orthogonal polynomials. Some years later we finished a paper, our unique paper together, “Asymptotic



Figure 2: Top: From left to right: Diego, Maxim Derevyagin, Maria das Neves Rebocho, and Juan J. in Berlin (2010); Bottom: From right to left: Andrei Martínez–Finkelshtein, Juan F. Mañas–Mañas, Diego, Veronika Pillwein, and Juan J. in Almería (2022).

analysis of a family of Sobolev orthogonal polynomials related to the generalized Charlier polynomials” in the *Journal of Approximation Theory* (2023) [1]. It was finished in July 2022 when he visited Almería with his wife Veronika and their nice daughter Malena. We were planning to tackle some new problems as well as another visit to Almería in the summer of 2024, which unfortunately will not be possible anymore.

He was a good colleague and friend.

*Requiescat in pace.*

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## In Memory of Diego Dominici

Mourad Ismail ([mourad.eh.ismail@gmail.com](mailto:mourad.eh.ismail@gmail.com))

I do not remember when I first met Diego but it must have been very early in his career. I was using asymptotics to determine orthogonality measures of polynomials defined by three term recurrence relations. He was intrigued by the quasi periodic potential and the application to non uniform lattices. When he was visiting Olga Holtz in Berlin on sabbatical they invited me for a week and I greatly enjoyed the scientific activity and the hospitality of Olga and Diego. He and his wife took me around in Berlin and I will remember their kindness and generosity forever. We discussed many mathematical issues and I truly enjoyed the visit. Since then I met Diego on many occasions and he was always cheerful, friendly, and just a super nice person. He will be greatly missed.



Figure 3: From left to right: Mourad, Diego, and Diego's wife Raquel in Berlin, Germany (2010).

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## In Memory of Diego Dominici

Chris Howls ([C.J.Howls@soton.ac.uk](mailto:C.J.Howls@soton.ac.uk))

The first words that Diego ever spoke to me were “You British are bastards!” It was the start of a warm friendship that lasted the next 18 years until his untimely early passing.

I somehow knew that an academic conference was going to be doubly inspiring if Diego was attending.

First there was the precise mathematical presentation, beautifully constructed and delivered by Diego with natural humbling self-deprecation. Then later would come the search for dinner, sometimes for a meticulously pre-researched restaurant, other times requiring several hours to traverse a circular route, only to end up eating at the first place we had looked at!

On other days there would be a hike to historical sites, for example, on one occasion a mathematical tour of Edinburgh, to have Diego’s DLMF Chapter photo taken: <https://dlmf.nist.gov/about/bio/DDominici>

Often we were so engrossed in conversation that the group seemed to get lost, either collectively, or just Diego. Indeed we needed eyes in the back of our head when out shopping with him. One minute you would be walking together, engrossed in deep mathematical conversation or perhaps political debate, the next Diego would have completely vanished, having spotted a bargain in a shop 200m distant across the other side of the road.

We spent the Fall of 2022 in Cambridge together at the Isaac Newton Institute. When he was not working he was out looking for gifts for his daughter. Loving and devoted, his family was never far from his thoughts.

Every few weeks I would receive emails from Diego relating to mathematics, current affairs, with photos or postcards of his trips. Occasionally mystery packages would show up, with gifts reflecting shared interests and his truly generous nature. His last postcard to me, only a month before his passing simply said “Wish you were here!”. I wish I had been.

There are so many places I had been planning to visit with him together, and with our families. I am just so sorry that we will never have the chance now to do that.

Diego, farewell my friend: every minute with you was time well spent.

\* \* \*

## In Memory of Diego Dominici

Adri Olde Daalhuis ([A.OldeDaalhuis@ed.ac.uk](mailto:A.OldeDaalhuis@ed.ac.uk))

The first time that I met Diego was in Santander in 2005 at a large meeting for Nico Temmes’s 65<sup>th</sup> birthday. Diego immediately became a good friend. Since then we have met at many international conferences/workshops. There was always time for historical walks and very interesting meals, especially when it was just the two of us. Once he boarded a flight from Edinburgh to Madrid with very wet feet and shoes, because there was just enough time for one more walk in the Roslin Glen.

In the last decade Diego hardly had any time for research, and this explained why he often disappeared at conferences to do research instead of listening to talks, but ultimately he would show up and insist on going to a new place for dinner. It still surprises me that he kept on producing many interesting papers even with the lack of research time. I will especially remember the visits in which both of our families were involved.





Figure 4: Left: Diego at Stirling Castle. Right: Dick Askey and Diego Dominici, Washington, DC, 2011. Photos courtesy of Adri Olde Daalhuis.

Diego always had a camera with him, and there are not many outdoor pictures of him in which he is not holding a camera. He must have taken millions of pictures. I include my favourite picture of him at Stirling castle, Stirling, Scotland.

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## Diego Ernesto Dominici: a fine mathematician and a gentle soul

Peter Paule ([Peter.Paule@risc.uni-linz.ac.at](mailto:Peter.Paule@risc.uni-linz.ac.at))  
Director of RISC (2009–2023)  
Johannes Kepler University Linz, Linz, Austria

I am writing this memorial contribution as a former colleague and friend of Diego. In my function as director of RISC (Research Institute for Symbolic Computation), I got Diego's valuable collaboration as a special gift, "brought in" by Veronika Pillwein. Diego has played a substantial role also at another institution of the Johannes Kepler University (JKU): the Doctoral Program "Computational Mathematics", an excellence program funded by the Austrian Science Funds (FWF) for a period of more than 12 years. Under the directorship of Veronika, the key objective was to set up a training program which enables young Ph.D. students to combine algorithmic aspects of two areas of mathematics, usually considered as being different: numerical analysis and symbolic computation. Diego's research interests were much on similar lines; as a consequence, he was able to contribute to our Doctoral Program in essential ways: scientifically, but also with regard to teaching and personal interaction with students.

Diego's interests in combining aspects of pure and applied are already reflected in his early career: Diego earned a Master's degree in Pure Mathematics (Universidad de Buenos Aires); for his PhD studies, he joined the University of Illinois at Chicago, where in 2003 he defended his PhD thesis in Applied Mathematics. This combination of pure and applied mathematics is still rare to find, but constituted another match with the interests of Veronika.

Right after his PhD thesis, Diego accepted the position of an Assistant Professor at the State University of New York (SUNY) at New Paltz. Soon after his promotion to associated professor, Diego was asked to serve as department chair; he served in this duty for several years. After his promotion to full professor, Diego retired only recently from the math department in New Paltz.

Some words on Diego Dominici's awards: In 2004 Diego received the Richard C. DiPrima Prize for outstanding research in applied mathematics, awarded by SIAM (Society for Industrial and Applied Mathematics). For the period 2008–2010 Diego was awarded with an Alexander von Humboldt Research Fellowship which he spent at the Technical University Berlin. To this day a Humboldt Fellowship is a very prestigious distinction. It was during his Humboldt period when I met Diego the first time. Young RISC colleagues had met Diego at conferences. They found his research very interesting, so we invited him to visit RISC where he gave a brilliant talk.

Another essential milestone in Diego's career happened in 2015 when he was invited to serve as Associate Editor for the highly visible and freely online-accessible Digital Library of Mathematical Functions (DLMF) edited by NIST (U.S. National Institute of Standard and Technology). Diego has been in charge for two DLMF Chapters: Airy Functions (9) and Bessel Functions (10). Searching online for Diego Dominici in the DLMF, one finds the following entry: "Dominici has published numerous papers in asymptotics and special functions, organized numerous meetings and conferences in the area, and served several terms as an officer of the SIAM Activity Group on Orthogonal Polynomials and Special Functions (OPSF)."

Indeed, Diego was a highly committed member of this OPSF Activity Group: Steering Committee (2014–2017), Program Director (2011–2016), Talk Moderator (2010–2020). In this context, one should mention that Diego was very active as organizer of minisymposia and special sessions. The first special session he organized was at the occasion of an AMS Sectional Meeting, October 8–9, 2005: [https://www.ams.org/meetings/sectional/2120\\_program\\_ss7.html](https://www.ams.org/meetings/sectional/2120_program_ss7.html).

Diego was also member of a committee which presented the OPSFA15 Lifetime Achievement Award to Dick Askey at the occasion of the 15th International Symposium on Orthogonal Polynomials, Special Functions and Applications (Hagenberg, Austria, July 22–26, 2019): <https://wis.kuleuven.be/events/archive/OPSFA/OPSFA15>.

Diego Dominici's excellent international standing is underlined by numerous invitations to important scientific events. A recent example (September to December 2022) is the Special Interdisciplinary Program "Applicable resurgent asymptotics: towards a universal theory" at the Isaac Newton Institute of Mathematical Sciences, Cambridge.

In addition to research, Diego received various teaching awards; e.g., in 2014 the "Faculty Mentor Award" at SUNY, New Paltz. In personal communication, Veronika confirmed that Diego has put lots of effort and energy in mentoring young students; the corridors of the Math Department at New Paltz were filled with posters of student projects done under his advisorship. She also found the following remarkable entry of the type "RateMyProfessor" on the web: there a student describes Diego as a "gentle soul".

Finally, some personal reaction and words: A reaction from a former Ph.D. student of the DK: "Diego was one of my first professors at JKU. His radiating kindness made my transition to Austria easier and his friendly face is always a pleasure to see at conferences. For these things, I will always be grateful. May he rest in peace".

Last but not least, a remark on my personal side: Diego was so kind to contribute to my "retirement book" which was handed over to me beginning of this October. Sadly, destiny did not allow sufficient time to express my thanks to him. Nevertheless, let me conclude by sharing part of his contribution, a poem by William Blake (1757–1827): "He who binds to himself a joy Does the winged life destroy He who kisses the joy as it flies Lives in eternity's sunrise."

Diego, we will badly miss your "gentle soul"!

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## Memorial contribution for Diego

Yuan Xu ([yuan@uoregon.edu](mailto:yuan@uoregon.edu))

I don't remember when I met Diego for the first time, but that is hardly surprising for people who knew his unassuming demeanor. Diego was kind and considerate, someone you would like to have as your colleague and friend. He was also an excellent citizen and a champion of our community. At the 2013 SIAM Annual Meeting, he was on the Organizing Committee, and he was also the program director of our SIAM activity group, so it was easy to guess that he was the one behind my invitation to give a plenary talk. When I thanked him at the conference, he just smiled, neither took the credit nor acknowledged it. Diego was a gentle soul. He will be deeply missed.

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## In Memory of Diego Dominici

Andrei Martínez-Finkelshtein ([andrei@ual.es](mailto:andrei@ual.es))  
Waco, Texas, USA



Figure 5: Mourad Ismail and Diego Dominici at OPSFA-15 in Linz, Austria in 2019. Photo courtesy of Andrei Martínez-Finkelshtein.

Diego was a colleague who seemed to have been a part of my mathematical life forever. Beyond our shared enthusiasm for the asymptotics of orthogonal polynomials, we connected through our Latin American roots and the common experience of immigration. He was such an easy person to interact with. Although he spoke about his research in a self-deprecating tone, Diego always had interesting ideas and projects to pursue. Regrettably, the plans we envisioned together were left unrealized—a common oversight I now lament. Diego's youth and vibrant energy deceived us into thinking time was abundant. His sudden departure is an immeasurable loss. Diego, you will be profoundly missed.

## Diego's impact on my mathematical life

Howard Cohl ([howard.cohl@nist.gov](mailto:howard.cohl@nist.gov))  
Mission Viejo, California, USA

I first met Diego when I was a transplanted math PhD student at the University of Auckland, New Zealand in December, 2007. I was attending the [first joint meeting of the American Mathematical Society and New Zealand Mathematical Society](#). A special session entitled "Special Functions and Orthogonal Polynomials" was organized by Ole Warnaar, Shaun Cooper, and Diego Dominici. This was one of the first great meetings for me. I was able to overlap, perhaps for the first time, with some of my current personal heroes including Richard Askey, Walter Van Assche, Bruce Berndt, Mourad Ismail, Michael Schlosser, Dennis Stanton and Ole Warnaar. I also learned more about the fascinating work on special functions going on in New Zealand by Ernie Kalnins (master of separable ellipsoidal coordinates for linear PDEs on Riemannian manifolds and longtime principal collaborator of Willard Miller, Jr.), Shaun Cooper (one of Dick Askey's ingenious PhD students), Nicholas Witte, and Garry Tee, and the late Ross Barnett who computed the gamma function accurately to 200D. Not to mention that I also got to meet for the first time, other important figures in OPSF such as Edmund Chiang, Michael Hirschhorn and Wenchang Chu.

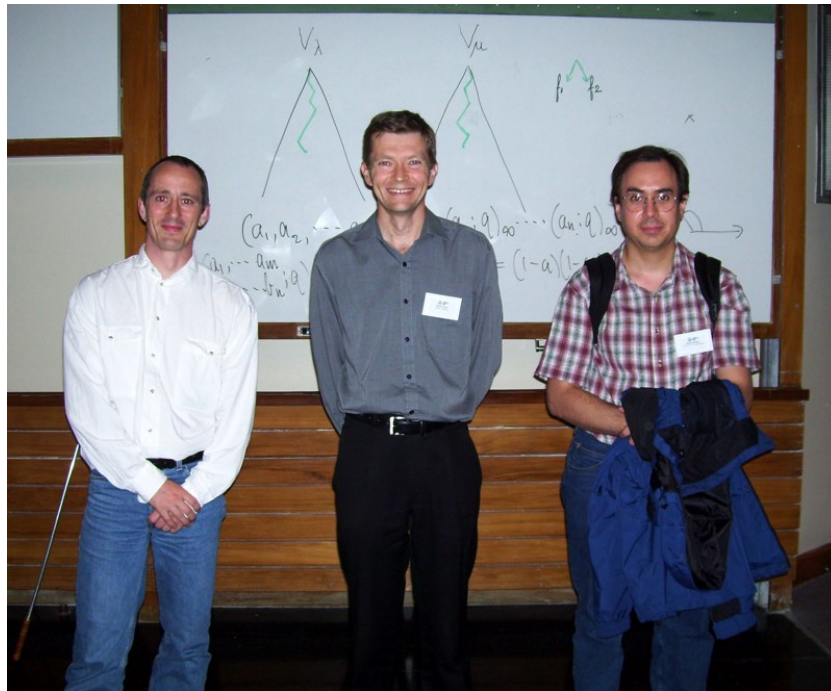


Figure 6: From left to right: Ole Warnaar, Shaun Cooper, Diego Dominici, the organizers of the SFOP Special Session in Wellington, New Zealand, December 12–15, 2007.

Diego was extremely pleasant during the meeting. The week following the meeting, Diego came to the University of Auckland having been invited by Shayne Waldron. Diego and I discussed my current research and there was an open problem that I was working on regarding the computation of a Fourier expansion in  $x$  for the important binomial kernel  $(z - x)^{-\mu}$  where  $\mu$  is complex and  $z \notin (-\infty, 1]$ . After our discussion, Diego said he had some ideas and not long after he came up with what I thought was a brilliant solution to the problem which involved a double-index replacement in a double infinite series over  $n, m \in \mathbb{N}_0$ ,  $0 \leq m \leq n$ , with

$$\begin{pmatrix} n' \\ m' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} n \\ m \end{pmatrix}, \quad (1)$$

where  $n', m', a, b, c, d \in \mathbb{Z}$ , and the determinant of the  $2 \times 2$  matrix is unity. I had never seen this before and have since been able to exploit it creatively when in a bind! This work ended up being published here [1], which was no doubt due to Diego's knowledge and savvy.



Figure 7: From left to right (back row): Dick Askey, Dennis Stanton, Michael Schlosser, Ole Warnaar; Next to last row: Edmund Chiang, Wenchang Chu, Pee Choon Toh, Walter Van Assche, Mourad Ismail, Nicholas Witte, Song Heng Chan; Second row: Howard Cohl, Shayne Waldron, Norman Wildberger, A. Sri Ranga, Ross Barnett; Front Row: Diego Dominici, Bruce Berndt, Michael Hirschhorn. The group photo for the SFOP Special Session in Wellington, New Zealand, December 12–15, 2007.

This started a long relationship with Diego in which we overlapped several times in Providence, Gaithersburg, Linz, and in Madison. In Madison, we had the great opportunity to meet the Askeys: Dick, Liz (wife), Suzanne (daughter) and Jim Zurlo (son), prior to Dick's passing and shared our deep gratitude with him, his family, his colleagues and former students [2]. Diego presented Dick with the OPSFA Lifetime Achievement Award in a very moving ceremony. Now our interactions with Diego have been abruptly cut short by the recent events. I am deeply saddened by the fact that we will no longer have such valuable interactions.

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From: OP–SF Net Editors  
Subject: Remembrances of **Willard Miller, Jr.** (1937–2023)

**Remembrances of  
Willard Miller, Jr.  
(September 17, 1937—October 29, 2023)**

Below are remembrances of Willard (Bill) Miller, Jr. from some of his colleagues:

**Peter Olver; Tom Koornwinder; Niky Kamran; Sergio Benenti, Claudia Chanu, Giovanni Rastelli;  
George Pogosyan; Steven Damelin; Howard Cohl; Alexander Turbiner; and Sarah Post.**

\* \* \*

**Willard**

Peter Olver ([olver@umn.edu](mailto:olver@umn.edu))

It is with a great sense of loss and sadness that I learned of Willard Miller’s passing on October 29 of this year. Willard was not just a friend, colleague, and researcher with common interests — he had a continual and profound effect on my academic career.

I first met Willard in late February, 1975, when I was still a graduate student, at a memorable meeting devoted to new developments in special functions, that took place in the Mathematics Research Center at the University of Wisconsin. Participants combined the up and coming new generation along with many older experts, including my father, Frank. As the meeting ended, a classic midwest spring snow-storm arrived and stranded many of us for an extra day — my first, but unheeded warning about the perils of living in the upper midwest! The meeting took place soon after Willard’s pivotal year visiting the [Centre de Recherches Mathématiques](#) at the University of Montréal, where he became close friends with Pavel Winternitz and Jiri Patera, both also sadly no longer with us, and where he and Ernie Kalnins began their almost 50 year collaboration. My father and Willard were already mutual admirers — even though my father, being a very classical British applied mathematician, never fully understood what a Lie group was, although he certainly appreciated their power in systematizing the mathematics of special functions. After the meeting, Willard took over from my father as the second managing editor of the SIAM Journal on Mathematical Analysis. (Both were quite peeved by the journal’s later policies, when it ceased to be the go-to venue for OPSF papers.) Willard later served as Program Director of the SIAM Activity Group OPSF from 1996 to 1998.

In 1979, while finishing my postdoc at Oxford, I knew that Willard had become Department Head at the University of Minnesota and hence it would be an excellent place to be. However, based on similar weather experiences, my wife Cheri forbade me from applying to Wisconsin or Minnesota, saying it was too cold and she would never move there. So I had to send him a “secret” application. Those were the days when you did not even have to interview to land a tenure track position, so Cheri never caught on until Willard phoned her at home to say an offer was coming. After fraught discussions, we both decided Minnesota was the place to be, and, despite the ensuing at times severe winters, it has proven to be an outstanding venue to raise a family and develop our respective careers, mine in large part because of Willard’s unfailing mentorship and guidance. Perhaps surprisingly, while Willard and I were close colleagues for over 40 years, and had many research interests in common, we never actually collaborated

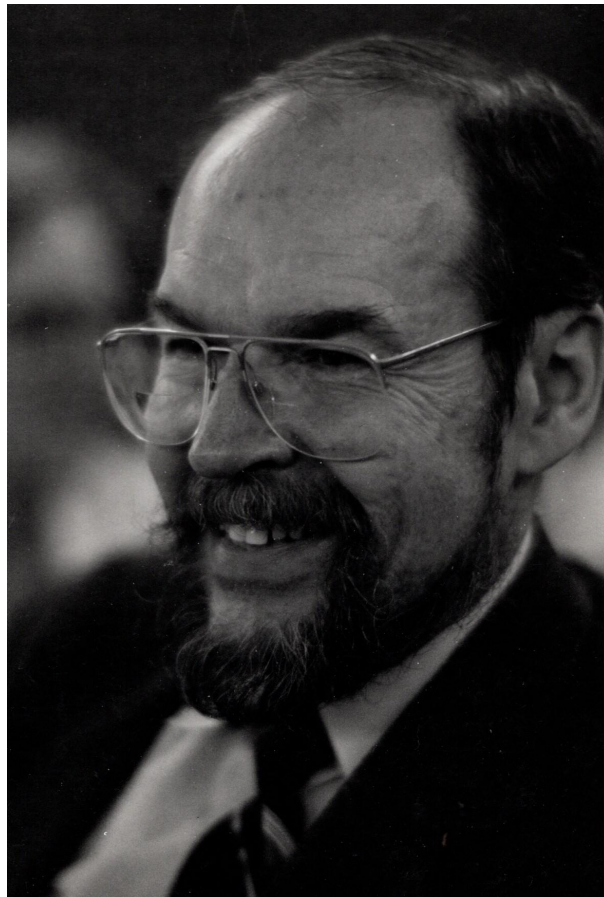


Figure 8: Willard Miller, Jr.

on a paper; it seemed our overlapping interests were temporally incompatible. In fact, he had an impact on all three generations of Olver mathematicians, supervising my son Sheehan Olver's undergraduate research project, which played an important role in his decision to become a mathematician and not a computer scientist, as I had been expecting. Sheehan is now on the faculty of Imperial College, London, and an extremely successful researcher in his own right, specializing on numerical analysis, orthogonal polynomials, and related fields.

After Wigner, and then Vilenkin pioneered the remarkable connections between Lie group representation theory, special functions, and separation of variables, Willard soon became the most prominent researcher in the subject, and a mentor and role model for many younger researchers. He was a prolific author, publishing well over 200 research papers and three influential texts on symmetry and separation of variables, as well as a later book, coauthored with Steve Damelin [1], on signal processing. The vast majority of his papers were coauthored with Ernie Kalnins and, at times, included a wide range of senior and younger researchers. Even in retirement, his research output showed no sign of slowing down, and he was often seen hard at work in his office, at least until covid hit. Indeed, his recent discoveries in the field of superintegrable systems, both classical and quantum, provided compelling new insight into the Askey–Wilson scheme of orthogonal polynomials, and his 2010 paper on the subject, coauthored with Ernie Kalnins and his former student Sarah Post, was awarded the [2012 Journal of Physics A, Best Paper Prize](#) [4]. Over the years, he faithfully attended the Math Physics Seminar that I co-organized with my students, and was always willing to contribute a talk on his latest contributions. In 2010, the School of Mathematics and the Institute for Mathematics and its Applications (IMA) organized the Conference on Symmetry, Separation, Super-integrability and Special Functions ( $S^4$ ) in his honor and to celebrate his retirement (see [link](#) for the SIGMA conference proceedings). The list of organizers and speakers consti-

tuted a “Who’s Who” of the field of symmetry analysis and special function theory, and included many close friends, collaborators, and admirers.

In addition to his research prowess, Willard has an exemplary and almost unparalleled record of service to the University of Minnesota. He was a man who was apparently unable to say no when asked to serve! At various stages in his career, he served as Head of the School of Mathematics, Associate Director and Director of the IMA, and Associate Dean and Acting Dean of the Institute of Technology (now the College of Science and Engineering). During his time as Department Head, he cowrote, with George Sell and Hans Weinberger, an innovative proposal for a new NSF Institute devoted to the applications of mathematics. Although NSF was only planning to fund one institute at that time, Minnesota’s proposal was so novel and compelling that two were founded — [MSRI](#) in Berkeley and the IMA at the University of Minnesota. Thanks to Willard, George, and Hans’ vision, its range of programs, and the number of visitors, both young and old, the IMA had an indelible impact on applied mathematics, in the broadest possible sense. Willard also played a key role fostering Harvey Keynes’ founding of the highly successful University of Minnesota Talented Youth Mathematics Project ([UMTYMP](#)) that continues to offer accelerated classes in mathematics to talented junior high and high school students. In 2005, in recognition of his many years of service and his research prominence, Willard was appointed College of Science and Engineering Distinguished Professor here at the University of Minnesota. He also served as my role model when I reluctantly agreed to be department head in 2008 — we were both talked into the position whilst chairing the search committee — in particular demonstrating that it was indeed possible to serve in administrative capacities while still maintaining a productive research program. He performed all his years of administrative service with absolute professionalism, dedication, and humility (I would remark to close friends that Willard was the only ego-less mathematician I ever knew!), all the while continuing to produce high quality research papers at a rate that would be impressive even for an ordinary faculty member.



Figure 9: Willard from his website

Over the years, Willard and Jane were regular guests at our house, attending many of our gatherings, dinners, and concerts. He and I also attended many international meetings on symmetry methods together, notably our unforgettable 1988 high altitude adventure in Paipa, Colombia. Sadly, the last time I was able to see Willard in person was pre-covid; his illness became increasingly severe in the last couple of years, and he was unable to welcome visitors. I miss him deeply.

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## Remembering Willard Miller

Tom Koornwinder ([thkmath@xs4all.nl](mailto:thkmath@xs4all.nl))

When around 1970 I started working in special functions and Lie theory I had three heroes, all being authors of books on the subject which appeared in 1968: Vilenkin, Talman and Willard Miller [1, 5, 1]. Vilenkin [9] associated special functions with groups and homogeneous spaces, for instance as matrix elements of representations. Talman [8] followed essentially the same approach, but less encyclopedic. Miller [1] worked the other way around. He started with a family of special functions, for instance Jacobi polynomials, and he built a Lie algebra from differential operators acting on the special functions by degree and/or parameter shifts. Then he exponentiated the Lie algebra to a local Lie group, still acting on the special functions, and for special values of the parameters to a global Lie group. I never met Vilenkin and Talman, but I met Willard Miller already in 1972, first at the SIAM meeting in Philadelphia and next at the AMS Summer Research Institute in Williamstown, MA. In Williamstown he was with his wife and kids, camping there if I remember well.

Miller's paper [6] associated with the 1972 Philadelphia meeting introduces the dynamical symmetry algebra associated with the Gauss hypergeometric differential equation. This algebra turns out to be a Lie algebra isomorphic with  $sl(4)$ . Later he extended this approach to, among others, Appell hypergeometric functions. In a transformed shape as so-called canonical systems [2] this anticipated the theory of  $A$ -hypergeometric (or GKZ hypergeometric) functions by Gel'fand and coworkers, as we also pointed out in [4, §1.2.2]. The method of [2] was extended to the  $q$ -case in [1], but no quantum groups or algebras entered. It would be an interesting problem whether a link with quantum groups is still possible here.

While still being a PhD student I already got papers by Willard Miller for refereeing, which was a pleasure to do. He was very productive. Several times a year I received his preprints (of course hardcopy in those days), which were usually written together with Ernie Kalnins. For some period his focus shifted to separation of variables in connection with Lie theory, culminating in a book [3]. While I read this book it struck me that no precise definition of separation of variables was given. It brought me to writing [3].

For many years Willard was involved in administration at the University of Minnesota, but admirably he maintained a large research output. I attended his retirement conference in 2010. But also afterwards he remained active, mainly in superintegrability. The OPSF community loses a dear friend and eminent researcher.

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## Willard Miller Jr.

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Willard Miller was an extraordinary mathematician who made major research contributions to a richly diverse and yet deeply inter-related set of areas comprising Lie theory and special functions, separation of variables,  $q$ -series and superintegrability. His books and papers were written with exemplary care, impeccable scholarship and meticulous attention to detail. They have been enormously influential and have inspired several generations of mathematicians on several continents.

Willard was extremely generous in sharing his ideas with other researchers and encouraging them in their efforts. I was one of the many beneficiaries of his help and support and will forever be grateful to him for his encouragement and guidance.

I will also remember Willard for his exceptional spirit of service to the scientific community. His fairness, integrity and amazing efficiency were second to none. We have lost a giant. He will be deeply missed.

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## A memorial contribution for Willard Miller

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It is since the 1980's that Willard's works have connections with our research, and certainly they will also connect in the future. In several of their works Bill, together with Ernie Kalnins and other collaborators, gave a great contribution to the foundations of a geometric and algebraic theory of integration by separation of variables for Hamilton–Jacobi, Schrödinger, Dirac and other types of differential equations. This provided a reference and stimulus for our research in the field. Later on, our investigations converged on superintegrable Hamiltonian systems, particularly those with polynomial constants of motion of high degree, where Bill's contributions have been of equally outstanding quality. During all these years, we had the opportunity of interacting via scientific publications. Some of us had the pleasure of co-authoring articles with him and all of us enjoyed meeting Bill in person, in workshops and conferences around the world. We like to remember in particular Bill's last visit in Torino in 2015. He was always friendly and happy to share not only his mathematical knowledge, but also his personal views and memories. We will particularly hold dear in our memory those conversations.

## Remembrance of Willard Miller

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 November 13, 2023

On October 29, the famous mathematician, our big friend and coauthor Willard (Bill) Miller Jr., passed away. He was well known as a person selflessly devoted to science and who made enormous contributions to mathematical physics, and especially in the theory of special functions, the theory of separation of variables, symmetry of differential equations, and the theory of superintegrable systems. He is the (co-)author of around 230 articles including numerous review articles in these areas. He also authored several famous monographs such as “Lie Theory and Special Functions” [1] and “Symmetry Groups and Their Applications” [2].

The first time I met Willard Miller was exactly 20 years ago in 2003, at a conference on Symmetry Methods in Physics, which had been organized in Yerevan, Armenia. He came with his wife Jane. In fact, I had known the name Willard Miller for a long time. I had carefully studied his excellent book “Symmetry and Separation of Variables” [3], which had been translated into Russian [4] in the beginning of the 80s. In fact, our scientific cooperation had begun a long time before our first personal meeting in Yerevan.

In the 90s, I invited him and Ernie Kalnins to participate in conferences which we had organized every two years in Dubna, Russia and in Yerevan. Ernie had worked closely with Bill for many years and co-authored many of his articles. Bill was very busy at the time and initially I met in Dubna with Ernie. We often talked in Dubna with Ernie about superintegrable systems, of which the interest in the last few years has slightly decreased. Almost everything in this area was considered to be known. The only interesting lead in this direction was a paper by Evans published several years ago, where he classified all three-dimensional superintegrable systems in the Euclidean plane.

We decided to begin our studies of superintegrable systems with the construction of quadratic symmetry algebras and the calculation of eigenfunctions in all coordinate systems allowing a separation of variables for the Schrödinger equation. This is how our first series of works appeared, devoted to the research of superintegrable systems on spaces of constant curvature: two-dimensional and three-dimensional Euclidean space and on the two-dimensional sphere and hyperboloid.

Later on, we managed to solve the problem of a complete classification of superintegrable systems in two-dimensional complex Euclidean space and on the complex sphere, constructed analogues of the Hurwitz transformation on the sphere, shed light on the nature of quasi-exactly solvable systems and proved their connection with superintegrable systems. We made a good team with Bill and Ernie. Until 2010, we published more than 20 joint works. Subsequently, many studies found their final reflection in the wonderful book “Separation of Variables and Superintegrability. The symmetry of solvable systems” [5], by Ernie Kalnins, Jonathan Kress and Willard Miller, Jr., published in 2018.

In general, for a theoretical physicist to work with good mathematicians is a great success. Bill was a great mathematician, scrupulous about every formula and demanding correct proof. I remember how once, when generalizing quasi-exactly solvable systems, the question of the convergence of eigenfunctions was raised. I offered a simple proof, as physicists say on their fingers, but Bill completely rejected it, saying that the answer was correct but the proof does not inspire confidence. As a result, he sent detailed evidence on three pages, which had to be placed in the appendix due to its large volume.

We will miss Bill Miller greatly and only memories of him will be warming our hearts. **Bibliography.**

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## A memorial contribution for Willard Miller

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I met Willard during the 2005–2006 academic year when I was very fortunate to be a New Directions Professor at the Institute for Mathematics and its Applications (IMA), University of Minnesota for that academic year. Willard was a remarkable person. The IMA proposal to the National Science Foundation was written when he was Head of the School of Mathematics, University of Minnesota (with 2 coauthors Hans Weinberger and George Sell). He served as associate director of the IMA and as its Director for a shorter period. He was also Associate Dean for some time. He was a world authority on superintegrable systems and a CSE Distinguished Professor.

Willard and I became friends and I recall many great mathematical discussions we had in his office. We had an idea to write a text on Signal Processing, an area I was learning about. Our text was published by Cambridge University Press [1] and in 2017, together with H. Guo, we wrote a comprehensive solution manual for our book [2].

Willard’s generosity and friendship to me without doubt changed my career and life in many very positive ways and I am very indebted to him for this. I will always have the fondest memories of him.

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## My lucky memories of Bill Miller

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My mathematical research has been deeply directed by my interactions with the influential author and caring mentor Willard Miller, Jr. Going back to my Physics PhD where I was to study the Newtonian gravitational potential in circular cylindrical coordinates and was driven straight into the theory of the three-dimensional Poisson and Laplace equations, one could barely avoid encountering the work of Willard

Miller, Jr. and his collaborators. At the time, it seemed that the most modern treatment of (simple and  $\mathcal{R}$ ) separable coordinate systems for Laplace's equation (in three dimensions) could be found in the book "Symmetry and Separation of Variables" by Willard Miller, Jr. [1]. I was immediately fascinated by what was presented there, which was a recap and extension of thesis work that had been done previously by the eminent American mathematician Maxime Bôcher [2] under the direction of Felix Klein in which he realized that the separable coordinate systems of Laplace's equation could be broken into 17 conformally inequivalent classes, each with their own particular set of harmonic solutions which were given by higher transcendental functions. In fact, Willard inspired Dick Askey to write a beautiful Foreword in this book and reading this Foreword was my first introduction to Askey, his deep opinion on addition theorems and his major influence on the theory of special functions which has been inspiring me ever since.

Many of these special functions and orthogonal polynomials which appeared in Miller's book were well-known, other special functions, not so much. There was a deep symmetry surrounding these objects and it was all could be viewed through a Lie group theoretic interpretation which was beautifully presented in Bill's book. I learned that most of the material presented on the 3-variable Laplace equation in Bill's book came from a beautiful paper by Boyer, Kalnins and Miller (1976) [3] where apparent notions of Physics (linear momentum, angular momentum, energy, dilatations and generators of special conformal transformations) seemed to bond beautifully with equally abstract notions of symmetry (the local representation theory of the conformal group, symmetry groups, symmetry algebras, Lie algebras). It was like being exposed to an arena of mysticism which connected to the vector and differential calculus of my undergraduate years. I learned in Miller's book that the symmetry group of the three-variable Laplace equation was a Lie group called the *conformal group* which was locally isomorphic to  $SO(4, 1)$  and its Lie algebra is  $so(4, 1)$ . The basis of this Lie algebra was given by differential operators which seemed to be connected to conserved quantities in physics. It was all a mystery to be unlocked. I had no past training or experience in abstract algebra and my introduction was the work of a master of the subject. After all, Bill and collaborators hadn't simply studied Laplace's equation. This was one of a huge number of partial differential equations which they had studied using this incredibly powerful formalism. They had numerous papers which used their algebraic formalism to study the behavior of separable solutions to the Helmholtz equation, the diffusion equation, Schrödinger's equation, the Hamilton-Jacobi equation, and the wave equation, which they studied in arbitrary real and complex dimensions and even on curved Riemannian manifolds where the Laplace operator morphed into its generalization referred to as the Laplace-Beltrami operator. I remember riding my bicycle through Baton Rouge, Louisiana next to the Mississippi river yelling at the top of my lungs, "The Laplace-Beltrami Operator!". I got all of this from Bill's book and barely understood most of it. But I could see the table of separable coordinate systems and entry number two with commuting operators given by  $J_3^2$  and  $P_3^2$  was circular cylindrical coordinates and the eigenfunctions were given by Bessel functions, my favorite special function at that time.

Bill's papers and books were like a Rosetta Stone for me. I was familiar with parts of it (the special function part) and other parts of it (the algebraic part) were a mystery to me. I immediately wanted to meet and discuss my current research on fundamental solutions of Laplace's equation with Willard Miller, Jr., to see if there was also an algebraic interpretation for it. After searching, I realized that he was still alive, unlike so many of the other great figures I had encountered in books I had found in the library up to that point. I started sending him email and as gracious as he was, he responded to my messages with very polite detailed answers (using ASCII mathematics at the time) to my questions. This started a email correspondence that I had with Bill and Ernie henceforth. He was such a great guy in all respects. I visited the University of Minnesota and met Bill and Ernie Kalnins. Ernie came to visit me at the Lawrence Livermore National Laboratory and gave a talk where I was doing a postdoc with Peter Eggleton.

When Bill learned that I was going to New Zealand to study for my math PhD, he emphasized to me what a great opportunity it would be for me to overlap with Ernie Kalnins whose computational skills were incredible. While I was in New Zealand, I managed to interact with Ernie. He visited with me in America several times and even though I was studying at the University of Auckland, I continually visited Ernie at the University of Waikato in Hamilton, New Zealand which was about an hour and a half drive from

Auckland. I took the bus and through the course of my PhD work at Auckland, I made regular stops to Hamilton. During my visits with Ernie, I often had lunch and dinner with Ernie and his wonderful wife Anne. Ernie collaborated on a paper on a fundamental solution of Laplace's equation in hyperboloid model of hyperbolic geometry [4].

I was so lucky to have met Willard Miller, Jr., who opened up my mind to many possibilities. I learned that unlike what I was led to believe, the classical areas of mathematical physics were not stagnant, but were alive and kicking and there were areas that almost nobody had even touched. It was a wide open field of discovery and I was informed of this to a large extent by Willard Miller. Bill then lectured at the New Zealand Mathematics Research Institute (NZMRI) [Thematic Programme on Conformal Geometry and its Applications Summer Workshop on Conformal geometry and Geometric Approaches to PDE](#) in Nelson, New Zealand on January 6–12, 2008 on the beautiful northern shore of the South Island of New Zealand. The topics of his three lectures was: (1) Planetary motion, the hydrogen atom and super-integrability; (2) Structure and classification results for second order superintegrable systems; and (3) Models for irreducible representations of quadratic algebras. Bill visited the mathematics department at the University of Auckland for a week in late January and early February, 2008. I was able to come to an office that he was staying in on a daily basis and pick his mind about anything I could mathematically entertain. In 2010, I completed my mathematics thesis at the University of Auckland working with analyst A. F. M. (Tom) ter Elst and differential geometer A. Rod Gover. During my thesis defense, it became clear that Willard Miller, Jr. and Shaun Cooper were external examiners for my thesis, with Willard Miller being the international referee. I learned later that Bill was an external examiner for 10 PhD students over his career. Willard Miller, Jr. was a mathematical giant who according to his website had 230 publications (each with PDF files) and according to MathSciNet he published 6 epic monographs.

My thesis advisor Tom ter Elst told me upon hearing that Miller had died, that when he was a Masters student, he did a course using Willard's book, "Symmetry Groups and Their Applications" [5] and that it was a massive course! Even my pure analysis advisor knew very-well about Willard Miller from early on. This is just one example of a person who was deeply affected by Prof. Miller. There must also be many hundreds of other people who were and will be equally impacted by Willard's focused lifetime contributions. Over the course of my career, I learned at meetings about the individuals who praised him such as Luc Vinet, Pavel Winternitz (d. 2021), Frank W. J. Olver (d. 2013), Niky Kamran, George Pogosyan, Alexander Turbiner, Sergio Benenti, Giovanni Rastelli, Claudia Channu, Anotoni Sym, Charles Boyer, Peter Olver, Kurt Bernardo Wolf (d. 2022), Tom Koornwinder, Roman Smirnov, Raymond McLenaghan, Alexei Zhedanov, Jonathan Kress, Sarah Post, and all those who were so lucky to have interacted with him.

Thank you Bill for always being such a continual gracious host. You lived your life as a humble servant of an ever growing crescendo of mathematical production. Last Monday night, I attended a recital of the towering organ music of Max Reger in a cathedral which contained the sixth largest pipe organ in the world. It was all I could do to think of Willard Miller's lifetime achievements to remind myself what it is to be alive and to be productive even on a tiny scale when compared to greats like Bill.

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## Willard

Alexander Turbiner ([turbiner@nucleares.unam.mx](mailto:turbiner@nucleares.unam.mx))

In 1991, thanks to Peter Olver, I visited the University of Minnesota to give a course of lectures on the newly discovered quasi-exactly-solvable problems. Peter introduced me to Willard Miller, Jr, to whom with directness typical for Russian-born scientists (in that time) I told that I am surprised to see him alive: I was sure that people of such a caliber lived (and died) in XIXth century! Willard laughed and many years after in some high-profile celebration in his honor told this story publicly. In the lectures he was sitting in the first row, took notes, asked questions, and translated my physics (broken math) language into his rigorous math language – I was impressed! I saw it only once in my lifetime before when I M Gel'fand did the same listening to physicists (or obscure mathematicians).

Willard was fully dedicated to Science, it looked like the world around him was like the decorations on the stage. Only once I saw him upset and truly angry when protesters blocked a highway near the University of Minnesota, and he was unable to get to his office in time when we agreed to meet. Another episode when he was angry beyond limits (went ballistic) when we got a referee report by a Board member on our joint paper and he wrote to the Editor-in-Chief alone stating that he is astonished by the poor mathematical qualification of a Board member of such a respected journal. The editor apologized and the paper was immediately accepted.

After discovery of the Tremblay-Turbiner-Winternitz (TTW) system, he approached me and said: “I am so grateful to you Sasha that higher order super-integrability has been discovered! Frankly speaking, I was sure that I would die only knowing quadratic super-integrability! Thank you very much!” That was not at all typical for Willard.

I could not imagine that I could collaborate with someone who speaks so rigorous a mathematical language. But in 2011, Willard invited me (indirectly) to write a paper to his honor, which I did with pleasure but trembling if he would be a referee, it will be rejected. From that moment we began our collaboration – we published 10 papers together in total!

It was very interesting to collaborate with Willard, he invited me to visit Minneapolis many times and each time entering his office he behaved like I left it yesterday. Pointing to the blackboard he can say: “You wrote this formula in the last time, now I can prove it and moreover I understand what is behind it!” (The formula could have been written 3–6 months ago.) His computational skill was truly outstanding, I do not know anybody who could carry out such complicated MAPLE calculations as Willard managed. Recent years he started to ask to check/repeat his calculations: he was sincerely happy if there were no errors and very sad otherwise. But his errors were NEVER conceptual! We still have several unfinished projects: I am not sure that I want to work on them in Willard's absence.

He can be very tough on mathematics but not on me. Willard generously taught me. He was very glad to see that I understood. I am very grateful to him for his lessons.

The last 10 years we were in regular communications on Skype; he was always, day and night, on video calls. When Pavel Winternitz died, whom he liked very much; Willard was ruined (his words). He stopped making video calls, turning the video off. The last time I saw him was when he made a Zoom presentation (it was pre-recorded) in memorial meeting for Pavel in July 2021. Since July 27, 2021, he stopped answering Skype calls.

Willard made many non-trivial important observations, some, but not all, were properly recognized. Recently, three months ago together with AM Escobar Ruiz we discovered one of those observations. It is

a pity that we were/are unable to share it with Willard.

R.I.P.

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## Memories of Willard

Sarah Post ([spost@hawaii.edu](mailto:spost@hawaii.edu))

I first met Willard during a meeting of the undergraduate math club at the University of Minnesota. I was probably a third-year graduate student at the time and was giving a basic introduction on using algebraic methods to calculate energy levels in physics. Of course, I had no idea there would be such an expert in the audience, but luckily for me he came up after and said something to the effect of “I think you’ll like what I do.”



Figure 10: Sasha Turbiner, Willard Miller Jr. with Milada and Pavel Winterntitz on the right side of the table celebrating the 80<sup>th</sup> birthdays of Pavel and Jiri Patera (opposite), May 30 – June 3, 2016 in Prague.

That was the beginning of a wonderful mentorship in the world of superintegrable systems, representation theory and special functions that has kept me busy throughout the years. Willard was full of new ideas and open problems, usually the perfect combination of computations and theory. His ideas always came with a MAPLE file and showed me the power of the computational software but also the importance of an accompanying proof or hand done computation. I am forever grateful that I was able to work on so many problems with not just Willard but also the large number of collaborators and researchers in his orbit.

There are several other beautiful remembrances here that speak to Willard’s vast contributions to mathematics. I will just mention that he always seemed to me to represent the best of the people in the Twin Cities: humble, direct, civically engaged and kind. His office on the 5<sup>th</sup> floor of Vincent Hall was stuffed with books and papers but of course neatly organized. He was always dressed with a button down white shirt and with a pen and notebook in his pocket (even at dinner as you can see in Fig. 10). While he didn’t give off the air of being especially athletic, he’d always have his sneakers on and I was surprised to learn that he would often run to campus in the morning even on some of the coldest Minnesota days.

It has been a wonderful experience working with Willard and I will miss him dearly.





Figure 11: From left to right: Jonathan Kress, Willard Miller Jr. and Sarah Post, at Willard's retirement conference "Symmetry, Separation, Super-integrability and Special Functions ( $S^4$  for short)", Sept. 17–19, 2010.

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## In Memoriam: Professor Willard Miller, Jr. (1937–2023)

Roman Smirnov ([Roman.Smirnov@Dal.Ca](mailto:Roman.Smirnov@Dal.Ca))

It is with profound sorrow that I received the news of Professor Willard Miller, Jr.'s passing. A distinguished scholar and cherished member of both the University of Minnesota community and the broader scientific realm, Professor Miller has left an enduring legacy. His contributions to the fields of applied mathematics, mathematical physics, and the theory of Lie groups have left an indelible mark, and his influence will undoubtedly continue to inspire generations.

I first met Willard Miller in the late 1990s when I was an NSERC Postdoctoral Fellow, collaborating with Ray McLenaghan at the University of Waterloo. Professor Miller visited Waterloo to deliver a colloquium talk that was inspirational and thought-provoking. At that time, my focus was on the theory of orthogonal separation of variables, and Willard, being one of the foremost global authorities on the subject, played a pivotal role in shaping my understanding of this area.

In the years that followed, our paths frequently intersected at conferences, and workshops, including our mutual visits to Minnesota and Dalhousie, respectively, providing abundant opportunities for engaging discussions with Willard. I am especially grateful that Willard was the external examiner of two of my former Ph.D. students, Jin Yue and Caroline Cochran (nee Adlam). I took immense pleasure in sharing our collective contributions (later coined as the invariant theory of Killing tensors) to the field. Equally enriching were our conversations delving into Willard's own significant results in the area.

One of my most cherished memories with Willard took place in the summer of 2005 during our participation in the conference "Symmetry in Nonlinear Mathematical Physics" held at the Institute of Mathematics in Kyiv, Ukraine. Being a native of Ukraine, I was delighted to guide Willard and his wife, Jane, through my hometown of Kyiv, an experience they thoroughly enjoyed. Subsequently, we decided to embark on a joint journey to Russia for our next conference, "The 2<sup>nd</sup> International Conference on Superintegrable Systems in Classical and Quantum Mechanics," held at the [Bogolyubov Laboratory of Theoretical Physics](#) of the [Joint Institute of Nuclear Research](#) in Dubna.

It is disheartening to reflect on this journey now, considering the current events in Ukraine. Russia's invasion has led to a devastating war, causing loss of life and displacing many innocent people. The

circumstances are unimaginably different from the time of our joint travel.

Accompanied by Professor Manuel Ranada from Zaragoza, Spain, we journeyed to Moscow by plane and embarked on a memorable car ride from Sheremetyevo-2 airport to Dubna. The trip proved to be quite eventful, particularly as three of us (Willard, Jane, and Manuel) were first-time visitors to Russia. Unexpectedly, our car broke down in the middle of nowhere between Moscow and Dubna, prompting a patient wait while our driver adeptly fixed his old GAZ-24 "Volga," probably manufactured around the time when Mikhail Gorbachev came to power in the USSR. This unplanned delay, however, turned out to be a blessing in disguise. On that very day, Dubna experienced a severe storm which we managed to miss due to the delay caused by the car breakdown.

Upon our arrival in Dubna, we encountered streets filled with debris, and none of the city's hotels had power. Compounding the situation, we discovered that there were two hotels under the same name "Dubna," and due to the power outage, there was no way to verify our reservations through the computer system. Fortunately, a receptionist at one of the hotels temporarily accommodated us with two rooms, and I ventured out to purchase some food and water for our party of four since I could speak Russian.

Luckily, the power was restored the same day, and we were able to meet with the organizers and check into our hotel where we had reservations. The conference was a great success, enjoyed by all participants. Remembering that travel with Willard, Jane, and Manuel, I still recall that throughout that adventure, Willard managed to hold his composure and remain a mathematician first and foremost. No matter what was going on around us, Willard remained focused on his beloved mathematics, continuing to discuss recent results in the area and problems that were still to be solved. This was truly an inspiration, especially to a young academic like myself just starting his career at the time.

I feel incredibly blessed that Willard was in my life when I needed his guidance the most, helping to shape my understanding of the topic of orthogonal separation of variables and having such a strong influence on my career as a mathematician. Apart from his scholarly achievements, I will always remember Professor Miller for his kindness, humility, and warm sense of humor.

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## Remembrance of Willard Miller, Jr.

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It was with much sadness that I learned of the passing of Willard Miller. He was a fine mathematician who will be greatly missed by the community. Our research interests overlapped in the area of the theory of separation of variables where he made seminal contributions in particular concerning the Dirac equation. Recently, some collaborators and I employed his general theory of separation of variables for a single  $n$ th order PDE to analyse the non-regular separation of the bi-Helmholtz equation.

I believe that I first met Willard in person at one of the "Symmetry and Perturbation Theory" conferences always held in very nice resorts in Italy. At this and subsequent meetings and conferences we had many productive discussions on various problems of common interest. Willard was unfailingly supportive of my research. He also was kind enough to act as the external examiner for some of my PhD students. I shall keep very good memories of my numerous get-togethers with Willard over many decades.

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## Willard Miller Memorial

Ian Marquette ([i.marquette@uq.edu.au](mailto:i.marquette@uq.edu.au))

I met Willard for the first time at a Workshop at The University of Minnesota as a graduate student in 2006. This is around the time Willard published a series of papers on the structure theory of superintegrability. It appears with time that his rigorous description of superintegrability and different methods to study higher order symmetries have been truly influential.

I meet Willard at different events over the years, and we started to interact more closely. He always was available to discuss mathematics. I visited him in 2019, and had the chances to collaborate closely on different topics of superintegrability and Lie algebras. I will always be grateful that I had the opportunity to work with him and benefit from his insights into those problems. I am sure that his large body of papers and his books on Lie algebras, symmetries and separation of variables will keep influencing mathematicians and physicists.

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## Some Personal Memories of Willard Miller

Jonathan Kress ([j.kress@unsw.edu.au](mailto:j.kress@unsw.edu.au))

When I first encountered Willard Miller's work as a PhD student, I had little idea of the effect he would have on my life. His work on separation of variables with Ernie Kalnins provided a solid foundation on which I could tinker, but I didn't really appreciate what a remarkable mathematician and human being Willard was until a few years later when I had the great fortune to begin working with him in person. My visits to the University of Minnesota to work with Bill are some of my fondest memories. The contrast of the long hot summer days on one visit with the bitter winter cold of another seemed a fitting parallel to the breadth of his knowledge and extent of his generosity to so many. His stamina seemed endless to me. He was able to work from early morning and throughout the day, without a break for lunch, while remaining as sharp as ever. It seemed that every day I spent working with Bill, there was a new advance and a something new to learn. But the fond memories of Bill extend far further than just the mathematics. Bill's compassion for his fellow humans was always clear when we spoke about social issues. I loved his enthusiasm for visits to cultural institutions, whether it was an art gallery or to the Minnesota State Fair. I couldn't have had a better guide. He will be greatly missed.

Topic #10 ——— OP – SF Net 30.6 ——— November 15, 2023

From: OP-SF Net Editors  
Subject: Preprints in arXiv.org

The following preprints related to the fields of orthogonal polynomials and special functions were posted or cross-listed to one of the subcategories of arXiv.org during September and October 2023. This list has been separated into two categories.

### OP-SF Net Subscriber E-Prints

<http://arxiv.org/abs/2309.00487>

Hardinian Arrays  
Robert Dougherty-Bliss, Manuel Kauers

<http://arxiv.org/abs/2309.02630>

Three new  $q$ -Abel transformations and their applications  
Jianan Xu, Xinrong Ma

<http://arxiv.org/abs/2309.02667>

Special function models of indecomposable  $sl(2)$  representations: The Laguerre Case  
Sébastien Bertrand, Ian Marquette, Willard Miller Jr, Sarah Post

<http://arxiv.org/abs/2309.02925>

Collisionless shock region of the KdV equation and an entry in Gradshteyn and Ryzhik  
Tewodros Amdeberhan, Victor Moll, John Lopez Santander, Ken McLaughlin, Christoph Koutschan

<http://arxiv.org/abs/2309.02934>

Universal convexity and range problems of shifted hypergeometric functions  
Toshiyuki Sugawa, Li-Mei Wang, Chengfa Wu

<http://arxiv.org/abs/2309.03191>

Extensions of MacMahon's sums of divisors  
Tewodros Amdeberhan, George E. Andrews, Roberto Tauraso

<http://arxiv.org/abs/2309.03803>

On the integrable structure of deformed sine kernel determinants  
Tom Claeys, Sofia Tarricone

<http://arxiv.org/abs/2309.05970>

Coloured corner processes from asymptotics of LLT polynomials  
Amol Aggarwal, Alexei Borodin, Michael Wheeler

<http://arxiv.org/abs/2309.06083>

On the weighted trigonometric Bojanov–Chebyshev extremal problem  
Béla Nagy, Szilárd Gy. Révész

<http://arxiv.org/abs/2309.06401>

Diagonal operators,  $q$ -Whittaker functions and rook theory  
Samrith Ram, Michael J. Schlosser

<http://arxiv.org/abs/2309.06959>

Turán Colourings in Off-Diagonal Ramsey Multiplicity  
Joseph Hyde, Jae-baek Lee, Jonathan A. Noel

<http://arxiv.org/abs/2309.07234>

On the asymptotic behaviour of the quantiles in the gamma distribution  
Henrik Laurberg Pedersen

<http://arxiv.org/abs/2309.07734>

Asymptotic approximations for the distribution of the product of correlated normal random variables  
Robert E. Gaunt, Zixin Ye

<http://arxiv.org/abs/2309.08446>

Diagonally symmetric alternating sign matrices  
Roger E. Behrend, Ilse Fischer, Christoph Koutschan

<http://arxiv.org/abs/2309.08762>

Explicit Expressions for Moments of the Duration of a 3-Player Gambler's Ruin  
Shalosh B. Ekhad, Doron Zeilberger

<http://arxiv.org/abs/2309.09516>

On the gamma difference distribution  
Peter J. Forrester

<http://arxiv.org/abs/2309.10004>

A probabilistic proof of some integral formulas involving incomplete gamma functions  
Robert E. Gaunt

<http://arxiv.org/abs/2309.10970>

Real roots of hypergeometric polynomials via finite free convolution  
Andrei Martínez-Finkelshtein, Rafael Morales, Daniel Perales

<http://arxiv.org/abs/2309.11364>

Rational extensions of an oscillator-shaped quantum well potential in a position-dependent mass background  
Christiane Quesne

<http://arxiv.org/abs/2309.11459>

New closed forms for a dilogarithmic integral, related integrals, and series  
Abdulhafeez A. Abdulsalam

<http://arxiv.org/abs/2309.11864>

A Golub-Welsch version for simultaneous Gaussian quadrature  
Walter Van Assche

<http://arxiv.org/abs/2309.13652>

On positivity of the two-parameter bivariate kernel built of  $q$ -ultraspherical polynomials and other Lancaster-type expansions of bivariate distribution  
Paweł J. Szabłowski

<http://arxiv.org/abs/2309.15190>

On the Use of the Mellin Transform to Generate Families of Power, Hyperpower, Lambert and Dirichlet Type Series and Some Consequences  
Larry Glasser, Michael Milgram

<http://arxiv.org/abs/2309.15557>

Some results and conjectures about Hankel determinants of sequences which are related to Catalan-like numbers  
Johann Cigler

<http://arxiv.org/abs/2309.15774>

The Icosidodecahedron  
John C. Baez

<http://arxiv.org/abs/2309.16016>

$m$ -distance-regular graphs and their relation to multivariate  $P$ -polynomial association schemes  
Pierre-Antoine Bernard, Nicolas Crampe, Luc Vinet, Meri Zaimi, Xiaohong Zhang

<http://arxiv.org/abs/2309.16091>

A new two-component sasa–satsuma equation: large–time asymptotics on the line  
Xiaodan Zhao, Lei Wang

<http://arxiv.org/abs/2309.16450>

New Perspectives on Torsional Rigidity and Polynomial Approximations of  $z$ -bar  
Adam Kraus, Brian Simanek

<http://arxiv.org/abs/2309.16550>

Connection formulae for the radial Toda equations I  
Martin A. Guest, Alexander R. Its, Maksim Kosmakov, Kenta Miyahara, Ryosuke Odoi

<http://arxiv.org/abs/2310.00326>

The Beauty of Roots  
John C. Baez

<http://arxiv.org/abs/2310.06688>

Interlacing and monotonicity of zeros of Angelesco–Jacobi polynomials  
Andrei Martínez–Finkelshtein, Rafael Morales

<http://arxiv.org/abs/2310.06864>

Hermite, Higher order Hermite, Laguerre type polynomials and Burgers like equations  
Giuseppe Dattoli, Roberto Garra, Silvia Licciardi

<http://arxiv.org/abs/2310.07370>

Orthogonal Random Features: Explicit Forms and Sharp Inequalities  
Nizar Demni, Hachem Kadri

<http://arxiv.org/abs/2310.07541>

Building hierarchies of semiclassical Jacobi polynomials for spectral methods in annuli  
Ioannis P. A. Papadopoulos, Timon S. Gutleb, Richard M. Slevinsky, Sheehan Olver

<http://arxiv.org/abs/2310.08239>

Centrosymmetric and reverse matrices in bivariate orthogonal polynomials  
Cleone F. Bracciali, Glalco S. Costa, Teresa E. Pérez

<http://arxiv.org/abs/2310.09906>

Generalized Schröder paths arising from a combinatorial interpretation of generalized Laurent bi-orthogonal polynomials  
Mawo Ito

<http://arxiv.org/abs/2310.10459>

On Turán inequality for ultraspherical polynomials  
Ilia Krasikov

<http://arxiv.org/abs/2310.11734>

Classification of 2-Orthogonal Polynomials with Brenke Type Generating Functions  
Hamza Chaggara, Abdelhamid Gahami

<http://arxiv.org/abs/2310.12312>

Sobolev orthogonal polynomials: Connection formulae  
Roberto S. Costas–Santos

<http://arxiv.org/abs/2310.12735>

Boundary statistics for the six-vertex model with DWBC

Vadim Gorin, Karl Liechty

<http://arxiv.org/abs/2310.14229>

Bounds for the kernel of the  $(\kappa, a)$ -generalized Fourier transform

Hendrik De Bie, Pan Lian, Frederick Maes

<http://arxiv.org/abs/2310.14362>

Closed form for  $\sum_{k=1}^n k^p$  through the Hermite integral representation of the Hurwitz zeta function

Abdulhafeez A. Abdulsalam

<http://arxiv.org/abs/2310.14898>

On Airy Solutions of  $P_{II}$  and Complex Cubic Ensemble of Random Matrices, I

Ahmad Barhoumi, Pavel Bleher, Alfredo Deaño, Maxim L. Yattselev

<http://arxiv.org/abs/2310.15207>

Dwork-type  $q$ -congruences through the  $q$ -Lucas theorem

Victor J. W. Guo

<http://arxiv.org/abs/2310.15312>

A short proof of the Almkvist-Meurman theorem

Ira M. Gessel

<http://arxiv.org/abs/2310.16016>

Uniform asymptotic expansions for the zeros of Bessel functions

T. M. Dunster

<http://arxiv.org/abs/2310.16674>

Connecting Exceptional Orthogonal Polynomials of Different Kind

Christiane Quesne

<http://arxiv.org/abs/2310.17286>

Mathematical properties and numerical approximation of pseudo-parabolic systems

E. Abreu, E. Cuesta, A. Duran, W. Lambert

<http://arxiv.org/abs/2310.17727>

Cluster algebras and tilings for the  $m=4$  amplituhedron

Chaim Even-Zohar, Tsviqa Lakrec, Matteo Parisi, Ran Tessler, Melissa Sherman-Bennett, Lauren Williams

<http://arxiv.org/abs/2310.18153>

Implementing and Experimenting with the Calabi-Wilf algorithm for random selection of a subspace over a finite field

Shalosh B. Ekhad, Doron Zeilberger

<http://arxiv.org/abs/2310.18294>

Hypergeometric Expressions for Type I Jacobi-Piñeiro Orthogonal Polynomials with Arbitrary Number of Weights

Amílcar Branquinho, Juan EF Díaz, Ana Foulquié Moreno, Manuel Mañas

<http://arxiv.org/abs/2310.19689>

An operational point of view to the theory of multi-variable/multi-index Hermite polynomials

Giuseppe Dattoli, Silvia Licciardi, Elio Sabia

<http://arxiv.org/abs/2310.20219>

An esoteric identity with many parameters and other elliptic extensions of elementary identities  
Gaurav Bhatnagar, Archana Kumari, Michael J. Schlosser

## Other Relevant OP–SF E–Prints

<http://arxiv.org/abs/2309.00405>

A Hamiltonian for the Hilbert–Pólya Conjecture  
Enderalp Yakaboylu

<http://arxiv.org/abs/2309.00409>

Multiple Mellin–Barnes integrals and triangulations of point configurations  
Sumit Banik, Samuel Friot

<http://arxiv.org/abs/2309.00484>

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Elena Boguslavskaya, Elina Shishkina

<http://arxiv.org/abs/2309.00510>

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Xiangqin Yu, Jianfeng Huang, Changjian Liu

<http://arxiv.org/abs/2309.00539>

An integral representation for  $\zeta(4)$   
Jean–Christophe Pain

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Andrés Chirre, Steven M. Gonek

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Yusuke Nemoto

<http://arxiv.org/abs/2309.01117>

GICAR algebras and dynamics on determinantal point processes: discrete orthogonal polynomial ensemble case  
Ryosuke Sato

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Ryosuke Yamaguchi

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Pushpa Kalauni, Prasanta K. Panigrahi



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Ryan C. H. Ang

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Philipp J. di Dio

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Differential Operator Representation of  $sl(2, \mathbb{R})$  as Modules over Univariate and Bivariate Hermite Polynomials  
Manouchehr Amiri

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Xiongming Dai, Gerald Baumgartner

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Carlos A. Sánchez-Villalobos, Bertrand Delamotte, Nicolás Wschebor

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John S. Caughman, Taiyo S. Terada

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Su Hu, Min-Soo Kim

<http://arxiv.org/abs/2309.09201>

Multiple zeta-star values for indices of infinite length

Minoru Hirose, Hideki Murahara, Tomokazu Onozuka

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On polynomial solutions of certain finite order ordinary differential equations

L. M. Anguas, D. Barrios Rolanía

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Rational deformations of the set of multiple zeta-star values

Jiangtao Li

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Perturbations of Fefferman spaces over almost CR manifolds  
Arman Taghavi-Chabert

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Pietro Gallato

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Bernd C. Kellner

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Abdelkader Intissar

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Johan Andersson

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Carolina Melo, Alfredo Nájera Chávez

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Junqing Ji, Lili Ju, Xiaoping Zhang

<http://arxiv.org/abs/2310.03949>

Negative discrete moments of the derivative of the Riemann zeta-function

Hung M. Bui, Alexandra Florea, Micah B. Milinovich

<http://arxiv.org/abs/2310.04115>

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Michael C. H. Choi, Geoffrey Wolfer

<http://arxiv.org/abs/2310.04240>

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Pier Luigi Novi Inverardi, Aldo Tagliani, Jordan M. Stoyanov

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Andrei Teleman

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Yubo Jin

<http://arxiv.org/abs/2310.04543>

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Robert Reynolds

<http://arxiv.org/abs/2310.04544>

A remark on density theorems for Riemann's zeta-function

Janos Pintz

<http://arxiv.org/abs/2310.04642>

On two conjectural series involving Riemann zeta function

Chuanan Wei, Ce Xu

<http://arxiv.org/abs/2310.04968>

Multivariate Meixner polynomials as Birth and Death polynomials

Ryu Sasaki

<http://arxiv.org/abs/2310.05016>

The Dunkl-Fokker-Planck Equation in  $1 + 1$  Dimensions and the Shifted Harmonic Oscillator

R. D. Mota, D. Ojeda-Guillén, M. A. Xicoténcatl

<http://arxiv.org/abs/2310.05017>

The Generalized Fokker–Planck Equation in terms of Dunkl–type Derivatives

R. D. Mota, D. Ojeda–Guillén, M. A. Xicoténcatl

<http://arxiv.org/abs/2310.05112>

Proofs of conjectures on Ramanujan–type series of level 3

John M. Campbell

<http://arxiv.org/abs/2310.05310>

Exact Jacobi elliptic solutions of some models for the interaction of long and short waves

Bruce Brewer, Jake Daniels, Nghiem V. Nguyen

<http://arxiv.org/abs/2310.05533>

The Lagrange top and the fifth Painlevé equation

Holger R. Dullin

<http://arxiv.org/abs/2310.05702>

Condenser capacities and capacitary potentials for unbounded sets, and global  $p$ –harmonic Green functions on metric spaces

Anders Björn, Jana Björn

<http://arxiv.org/abs/2310.05713>

Approximations of the Green’s Function in Multiple Scattering Theory for Crystalline Systems

Xiaoxu Li, Huajie Chen

<http://arxiv.org/abs/2310.06637>

A new approach to weighted Hardy–Rellich inequalities: improvements, symmetrization principle and symmetry breaking

Anh Xuan Do, Nguyen Lam, Guozhen Lu

<http://arxiv.org/abs/2310.06723>

Bounding zeta on the 1–line under the partial Riemann hypothesis

Andrés Chirre

<http://arxiv.org/abs/2310.06809>

Regular primes, non–Wieferich primes, and finite multiple zeta values of level  $N$

Shin–ichiro Seki

<http://arxiv.org/abs/2310.06920>

Stability Analysis of A Single–Species Model with Distributed Delay

Isam Al–Darabsah

<http://arxiv.org/abs/2310.06963>

Plea for diagonals and telescopers of rational functions

S. Hassani, J–M. Maillard, N. Zenine

<http://arxiv.org/abs/2310.06971>

Hypergeometric  $L$ –functions in average polynomial time, II

Edgar Costa, Kiran S. Kedlaya, David Roe

<http://arxiv.org/abs/2310.06986>

High order biorthogonal functions in  $H(\text{curl})$

Tim Haubold, Sven Beuchler, Joachim Schöberl



<http://arxiv.org/abs/2310.07318>

On generalization of duality formulas for the Arakawa–Kaneko type zeta functions  
Kyosuke Nishibiro

<http://arxiv.org/abs/2310.07360>

A note on simple zeros related to Dedekind zeta functions  
Wei Zhang

<http://arxiv.org/abs/2310.07606>

Low-lying zeros of a large orthogonal family of automorphic  $L$ -functions  
Siegfried Baluyot, Vorrapan Chandee, Xiannan Li

<http://arxiv.org/abs/2310.07976>

Higher Jacobian ideals, contact equivalence and motivic zeta functions  
Quy Thuong Lê, Takehiko Yasuda

<http://arxiv.org/abs/2310.08018>

Gromov–Witten Generating Series of Elliptic Curves and Iterated Integrals of Eisenstein–Kronecker Forms  
Jie Zhou

<http://arxiv.org/abs/2310.08126>

A novel Newton method for inverse elastic scattering problems  
Yan Chang, Yukun Guo, Hongyu Liu, Deyue Zhang

<http://arxiv.org/abs/2310.08509>

On the regularity conditions in the CLT for the LUE  
Henry Hu

<http://arxiv.org/abs/2310.08518>

On the five-dimensional non-extremal Reissner–Nordström black hole: Retractions and scalar quasi-bound states  
M. Abu–Saleem, H. S. Vieira, L. H. C. Borges

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Bicomplex generalized hypergeometric functions and their applications  
Snehasis Bera, Sourav Das, Abhijit Banerjee

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Painlevé kernels and surface defects at strong coupling  
Matijn François, Alba Grassi

<http://arxiv.org/abs/2310.09286>

Percolation with invariant Poisson processes of lines in the 3-regular tree  
Guillaume Blanc

<http://arxiv.org/abs/2310.09295>

On the impact of insurance on households susceptible to random proportional losses: An analysis of poverty trapping  
Kira Henshaw, Jorge M. Ramirez, José M. Flores–Contró, Enrique A. Thomann, Sooi–Hoe Loke, Corina Constantinescu

<http://arxiv.org/abs/2310.09334>

On the Virasoro fusion kernel at  $c = 25$

Sylvain Ribault, Ioannis Tsiaras

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The cokernel of a polynomial push-forward of a random integral matrix with concentrated residue

Gilyoung Cheong, Yifeng Huang

<http://arxiv.org/abs/2310.09548>

General birationality and Hyperelliptic Theta divisors

Fabrizio Catanese

<http://arxiv.org/abs/2310.09579>

Existence and asymptotic behavior of entire large solutions for Hessian equations

Xiang Li, Jiguang Bao

<http://arxiv.org/abs/2310.09595>

Tree size distribution as the stationary limit of an evolutionary master equation

Szabolcs Kelemen, Máté Józsa, Tibor Hartel, György Csóka, Zoltán Nédá

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Convergence rates for Chernoff-type approximations of convex monotone semigroups

Jonas Blessing, Lianzi Jiang, Michael Kupper, Gechun Liang

<http://arxiv.org/abs/2310.10019>

KPZ exponents for the half-space log-gamma polymer

Guillaume Barraquand, Ivan Corwin, Sayan Das

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The Uniform Distribution Modulo One of Certain Subsequences of Ordinates of Zeros of the Zeta Function

Fatma Çiçek, Steven M. Gonek

<http://arxiv.org/abs/2310.10234>

Decomposition of global 2-SLE for  $\kappa \in (4, 8)$  and an application for critical FK-Ising model

Yu Feng, Mingchang Liu, Hao Wu

<http://arxiv.org/abs/2310.10257>

Asymptotics of the Green function of killed random walks in a cone of  $\mathbb{Z}^d$

Irina Ignatiouk-Robert

<http://arxiv.org/abs/2310.10415>

Non-ergodicity of the geodesic flow on Cantor tree surfaces

Michael Pandazis

<http://arxiv.org/abs/2310.10456>

Radial bounded solutions for modified Schrödinger equations

Federica Mennuni, Addolorata Salvatore

<http://arxiv.org/abs/2310.10526>

A spectrally accurate step-by-step method for the numerical solution of fractional differential equations

L. Brugnano, K. Burrage, P. Burrage, F. Iavernaro

<http://arxiv.org/abs/2310.10617>

Biasymptotics of the Möbius- and Liouville-signed partition numbers  
Taylor Daniels

<http://arxiv.org/abs/2310.10846>

Clebsch-Gordan coefficients for Macdonald polynomials  
Aritra Bhattacharya, Arun Ram

<http://arxiv.org/abs/2310.10926>

Destabilization of synchronous periodic solutions for patch models: a criterion by period functions  
Shuang Chen, Jicai Huang

<http://arxiv.org/abs/2310.11408>

Sum of the  $GL(3)$  Fourier coefficients over quadratics and mixed powers  
Himanshi Chanana, Saurabh Kumar Singh

<http://arxiv.org/abs/2310.11443>

Operations on the set of scalar and matrix-valued quiddity sequences  
Raúl Felipe

<http://arxiv.org/abs/2310.11468>

Explicit estimates for the sum  $\sum_{k=0}^n k! \binom{n}{k}^2 (-1)^k$   
Anne-Maria Ernvall-Hytönen, Tapani Matala-aho

<http://arxiv.org/abs/2310.11625>

The Einstein-Hilbert functional and the Donaldson-Futaki invariant  
Abdellah Lahdili, Eveline Legendre, Carlo Scarpa

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Hardy spaces and Szegő projection on quotient domains  
Liwei Chen, Yuan Yuan

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Local limits in  $p$ -adic random matrix theory  
Roger Van Peski

<http://arxiv.org/abs/2310.12354>

Rational Catalan Numbers for Complex Reflection Groups  
Weston Miller

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Globality of the DPW construction for Smyth potentials in the case of  $SU(1,1)$   
Tadashi Udagawa

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Large zeta sums  
Zikang Dong, Weijia Wang, Hao Zhang

<http://arxiv.org/abs/2310.13534>

On Weighted Generalized Gauss Quadratures for Müntz Systems  
Huaijin Wang, Chuanju Xu

<http://arxiv.org/abs/2310.13542>

On the generalized interlacing property for the zeros of Bessel functions  
Seok-Young Chung, Sujin Lee, Young Woong Park

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Matrix solutions of the cubic Szegő equation on the real line  
Ruoci Sun

<http://arxiv.org/abs/2310.14372>

Geometric quantization results for semi-positive line bundles on a Riemann surface  
George Marinescu, Nikhil Savale

<http://arxiv.org/abs/2310.14415>

A New Discriminant for the Hardy  $Z$ -Function and the Corrected Gram's law  
Yochay Jerby

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On the density hypothesis for  $L$ -functions associated to holomorphic cusp forms  
Bin Chen, Gregory Debruyne, Jasson Vindas

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Optimal bounds for the Dunkl kernel in the dihedral case  
Jean-Philippe Anker, Bartosz Trojan

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Hermite-Padé approximation, multiple orthogonal polynomials, and multidimensional Toda equations  
Adam Doliwa

<http://arxiv.org/abs/2310.15536>

Non-smoothness of the fundamental solutions for Schrödinger equations with super-quadratic and spherically symmetric potential  
Keiichi Kato, Wataru Nakahashi, Yukihide Tadano

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Infinite product formulae for semilocal Solomon zeta functions  
Sean B. Lynch

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A multiparametric Murnaghan-Nakayama rule for Macdonald polynomials  
Naihuan Jing, Ning Liu

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Three dimensional quotient singularity and 4d  $\mathcal{N} = 1$  AdS/CFT correspondence  
Yuanyuan Fang, Jing Feng, Dan Xie

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On the joint second moment of zeta and its logarithmic derivative

Alessandro Fazzari

<http://arxiv.org/abs/2310.15973>

Explicit Formulas of Fractional GJMS operators on hyperbolic spaces and sharp fractional Poincaré–Sobolev and Hardy–Sobolev–Maz'ya inequalities

Guozhen Lu, Qiaohua Yang

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Distributed Uncertainty Quantification of Kernel Interpolation on Spheres

Shao-Bo Lin, Xingping Sun, Di Wang

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A note on the zeros of  $L$ -functions associated to fixed-order Dirichlet characters

C. C. Corrigan

<http://arxiv.org/abs/2310.16940>

Optimal approximation of infinite-dimensional holomorphic functions II: recovery from i.i.d. pointwise samples

Ben Adcock, Nick Dexter, Sebastian Moraga

<http://arxiv.org/abs/2310.17039>

The limiting distribution of a random variable transformed by Chebyshev Polynomials

Javier Chico Vazquez, Andrew J. Horning

<http://arxiv.org/abs/2310.17118>

Representation theory of  $\mathfrak{sl}(2, \mathbb{R}) \simeq \mathfrak{su}(1, 1)$  and a generalization of non-commutative harmonic oscillators

Ryosuke Nakahama

<http://arxiv.org/abs/2310.17362>

Intermediate Macdonald Polynomials and Their Vector Versions

Philip Schلösser

<http://arxiv.org/abs/2310.17482>

Orthonormal representations, vector chromatic number, and extension complexity

Igor Balla

<http://arxiv.org/abs/2310.17756>

A note on the map expansion of Jack polynomials

Houcine Ben Dali

<http://arxiv.org/abs/2310.17850>

The Barnes–Hurwitz zeta cocycle at  $s = 0$  and Ehrhart quasi-polynomials of triangles

Milton Espinoza

<http://arxiv.org/abs/2310.17853>

Rahman polynomials

Ryu Sasaki

<http://arxiv.org/abs/2310.18423>

Zeta zeros and prolate wave operators

Alain Connes, Caterina Consani, Henri Moscovici

<http://arxiv.org/abs/2310.18649>

Weighted  $L^p$ -norm inequality of multi-parameter fractional integration  
Chuhan Sun, Zipeng Wang

<http://arxiv.org/abs/2310.18666>

An efficient stochastic particle method for high-dimensional nonlinear PDEs  
Zhengyang Lei, Sihong Shao, Yunfeng Xiong

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On a class of algebro-geometric solutions to the Ernst equation  
Eddy B. de Leon

<http://arxiv.org/abs/2310.19141>

Optical STAR-RIS-Aided VLC Systems: RSMA Versus NOMA  
Omar Maraqa, Sylvester Aboagye, Telex M. N. Ngatched

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The Cauchy problem for the Novikov equation under a nonzero background: Painlevé asymptotics in two transition zones  
Zhaoyu Wang, Xuan Zhou, Engui Fan

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Koopman representations for positive definite functions  
Sohail Farhangi

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On the Hankel Transform of Bessel Functions on Complex Numbers and Explicit Spectral Formulae over the Gaussian Field  
Zhi Qi

<http://arxiv.org/abs/2310.19501>

Irregular Riemann-Hilbert Correspondence and Enhanced Subanalytic Sheaves  
Yohei Ito

<http://arxiv.org/abs/2310.19979>

Using Fibonacci Numbers and Chebyshev Polynomials to Express Fox Coloring Groups and Alexander-Burau-Fox Modules of Diagrams of Wheel Graphs  
Anthony Christiana, Huizheng Guo, Jozef H. Przytycki

<http://arxiv.org/abs/2310.20628>

Arithmetic properties and asymptotic formulae for  $\sigma_o \text{mex}(n)$  and  $\sigma_e \text{mex}(n)$   
Gurinder Singh, Rupam Barman

<http://arxiv.org/abs/2310.20702>

A simple range characterization for spherical mean transform in odd dimensions and its applications  
Divyansh Agrawal, Gaik Ambartsoumian, Venkateswaran P. Krishnan, Nisha Singhal

Topic #11 ——— OP – SF Net 30.6 ——— November 15, 2023

From: OP-SF Net Editors

Subject: Submitting contributions to OP-SF NET and SIAM-OPSF (OP-SF Talk)

To contribute a news item to OP–SF NET, send e–mail to one of the OP–SF Editors [howard.cohl@nist.gov](mailto:howard.cohl@nist.gov), or [spost@hawaii.edu](mailto:spost@hawaii.edu).

Contributions to OP–SF NET 31.1 should be sent by January 1, 2024.

OP–SF NET is the electronic newsletter of the SIAM Activity Group on Special Functions and Orthogonal Polynomials (SIAG/OPSF). We disseminate your contributions on anything of interest to the special functions and orthogonal polynomials community. This includes announcements of conferences, forthcoming books, new software, electronic archives, research questions, and job openings as well as news about new appointments, promotions, research visitors, awards and prizes. OP–SF Net is transmitted periodically through a post to OP–SF Talk which is currently managed and moderated by Howard Cohl ([howard.cohl@nist.gov](mailto:howard.cohl@nist.gov)). Anyone wishing to be included in the mailing list (SIAG/OPSF members and non–members alike) should send an email expressing interest to him. Bonita Saunders also posts the Newsletter through SIAM Engage (SIAG/OPSF) which is received by all SIAG/OPSF members.

OP–SF Talk is a listserv associated with SIAG/OPSF which facilitates communication among members, non–members and friends of the Activity Group. To post an item to the listserv, send e–mail to [howard.cohl@nist.gov](mailto:howard.cohl@nist.gov).

WWW home page of this Activity Group:

<http://math.nist.gov/opsf>

Information on joining SIAM and this activity group: [service@siam.org](mailto:service@siam.org)

The elected Officers of the Activity Group (2020–2022<sup>\*</sup>) are:

Peter Alan Clarkson, Chair

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Andrei Martínez–Finkelshtein, Program Director

Teresa E. Pérez, Secretary and SIAM Engage (SIAG/OPSF) moderator

The appointed officers are:

Howard Cohl, OP–SF NET co–editor

Sarah Post, OP–SF NET co–editor

Bonita Saunders, Webmaster and SIAM Engage (SIAG/OPSF) moderator

<sup>\*</sup>As of the date of the publication of OP–SF NET 30.6, the SIAG/OPSF elections have not occurred.

From: OP–SF Net Editors

Subject: Thought(s) of the Month by **Diego Dominici** and **Willard Miller, Jr.–Ernie Kalnins**

“We sincerely hope that more and more professors and their students will discover the beauty contained in the (very!) Special Functions, among which  $\Gamma(z)$  is, without doubt, a prima donna.”

**Diego Dominici**, *Variations on a theme by James Stirling*. Note di Matematica. **28** (2008), no. 1, 1–13, Conclusion.

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“Among the most important contributions of Carl Gustav Jacobi to mechanics was his introduction of the “remarkable change of variables”, the generalized elliptical coordinates  $x_j$  in  $n$  dimensions, [1]. These can be defined by the relations

$$1 + \sum_{k=1}^n \frac{q_k^2}{z - e_k} = \frac{\prod_{j=1}^n (z - x_j)}{\prod_{k=1}^n (z - e_k)}, \quad (2)$$

where the  $q_k$  are Cartesian coordinates and the  $e_k$  are distinct constants. An equivalent definition is

$$q_k^2 = \frac{\prod_{j=1}^n (e_k - x_j)}{\prod_{j \neq k} (e_j - e_k)}, \quad (3)$$

where  $e_1 < x_1 < e_2 < \dots < e_n < x_n$  and  $k = 1, \dots, n$ . In the case that  $n = 3, 4$  the elliptic coordinates admit expression in terms of Jacobi elliptic functions [2], [3].”

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[2] E. T. Whittaker, G. N. Watson. A Course of Modern Analysis. Fourth Edition reprinted. Cambridge University Press. 1958.

[3] F. M. Arscott. Periodic Differential Equations. Macmillan, New York. 1964.

**Willard Miller, Jr., Ernie G. Kalnins**, *Jacobi elliptic coordinates, functions of Heun and Lamé type and the Niven transform*. Regular and Chaotic Dynamics, **10** (2005), no. 4, 487–508, Introduction.