Tutorial #2

Exercise 1

For each of the two games shown below, determine whether it is a potential game.

(a)

	L	R
T	6, 6	5, 6
B	3, 2	4, 4

(b)

	L	R
T	7, 4	2, 0
B	2, 0	5, 5

Exercise 2

We saw a number of different solution concepts for normal-form games in class and discussed how they relate to each other. In particular, we established the following 'inclusions':

$$\text{Equilibria in Strictly Dominant Strategies} \subseteq \text{Pure NE} \subseteq \text{NE} \subseteq \text{Correlated Equilibria}$$

What does the symbol '$\subseteq$' represent here? Does it represent the same kind of relationship in all three cases? Once you have clarified this matter, for each of the three 'inclusions', provide a simple and intuitive argument for why the claim being made here is indeed correct.

Exercise 3

To show that the solution concept for normal-form games provided by the iterated elimination of strictly dominated strategies is well-defined, we had to prove that it does not matter in which order you eliminate strategies (in those cases where there is more than one strategy that could be eliminated): we always arrive at the same irreducible game. To help you understand this result, review the following details of the proof presented in class:

(a) If you are unsure what it means for a binary relation to have the Church-Rosser property, look it up. Then write down a definition for the relation $\twoheadrightarrow$ being Church-Rosser using the usual first-order notation (and variable names G, G', etc.). For example, asymmetry of $\twoheadrightarrow$ can be defined like this: $\forall G.\forall G'.(G \twoheadrightarrow G') \rightarrow \neg(G' \twoheadrightarrow G)$.

(b) One of the steps in the proof is established by reference to a diagram. Which step is this? How does the diagram illustrate the correctness of that step?

(c) What does the ad-hoc notation $G \overset{a_i}{\twoheadrightarrow} G'$, used on the slides, represent?

(d) On the slides, there is the claim that, in order to show that the relation $\twoheadrightarrow$ is Church-Rosser, it is sufficient to show that the following is the case:

$$\text{if } G \overset{a_i}{\twoheadrightarrow} G' \text{ and } G \overset{b_j}{\twoheadrightarrow} G'', \text{ then } G' \overset{b_j}{\twoheadrightarrow} G''' \text{ for some } G'''$$

At first sight, this might not be obvious. Why do we not also have to show that G''' can be reached from G'' as well (and not just from G')?

(e) The proof on the slides only covers the case where the player playing the first action is different from the player playing the second action (i.e., the case where $i \neq j$). Indeed, for $i = j$ it would not make sense to speak of partial profiles $\boldsymbol{s}'_{-j}$ with $a_i \notin \mathit{support}(s'_i)$. So, strictly speaking, we still need to prove that the following is the case:

$$\text{if } G \overset{a_i}{\twoheadrightarrow} G' \text{ and } G \overset{b_i}{\twoheadrightarrow} G'', \text{ then } G' \overset{b_i}{\twoheadrightarrow} G''' \text{ for some } G'''$$

Explain why this is (almost trivially) true.